The Federal Reserve and the Cross-Section of Stock Returns *

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Preliminary. Comments Welcome!

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Abstract

We analyze the effects of monetary policy on the equity premium and the cross-section of stock returns in a general equilibrium framework. Monetary policy is conducted using an interest-rate policy rule reacting to inflation and output. Product-price rigidities in the production sector generate an equity premium that depends on the policy. The model predicts that (i) industries with lower price rigidities earn higher expected returns than industries with higher price rigidities and (ii) the difference in expected returns declines with more aggressive monetary policies. We provide an explanation for these results based on countercyclical markups. Markups of industries with low price rigidities are less variable than markups of industries with high price rigidities. When the marginal utility of consumption is high, markups in industries with high rigidities increase by more than markups in industries with low rigidities. As a result, profits of industries with low rigidities are more sensitive to policy shocks, and investors require a higher compensation for holding stocks on these industries. When the response of monetary policy to inflation is more aggressive, the markup variability reduces, and the difference in expected returns between high and low rigidity industries decreases. We find empirical evidence supporting the model's predictions.

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1 Introduction

The Federal Reserve conducts monetary policy to promote effectively the goals of price stability, i.e., control inflation, and maximum employment. This mandate implies the idea that monetary policy can influence real economic activity and suggests that real returns on financial assets can be affected by the policy. Therefore, monetary policy is potentially helpful to understand asset-pricing facts. This paper provides a theoretical analysis of the effects of monetary policy on the cross-section of stock returns and presents empirical evidence supporting the predictions of the theory.

We model an economy where the effects of monetary policy on stock returns are the result of price rigidities in production, as in Woodford (2003). Differences in returns across stocks are explained by different degrees of price rigidity across industries, and the responsiveness of the policy to inflation.[1] The policy is conducted setting a short-term interest rate using a policy rule. This rule responds to the level of inflation and a measure of output, and is affected by policy shocks. We show that stocks carry a risk premium associated to policy shocks in an economy with homogeneous price rigidity across industries. The magnitude of the risk premium increases with the degree of price rigidity and decreases with the elasticity of intertemporal substitution of consumption and labor, and the response to inflation in the policy rule. For an economy with heterogenous price rigidities across industries, we show that industries with high price rigidities should earn lower expected returns than industries with low price rigidities, and the difference in returns decreases with a more aggressive response to inflation and output in an interest-rate policy rule.

We provide a consumption-based explanation for the policy-related differences in stock returns. Industries with low price rigidities earn higher expected returns because their profits are more correlated to aggregate consumption than industries with high rigidities. Policy shocks induce a positive correlation between consumption and inflation in the model. As a result, a policy shock that reduces inflation, decreases profits in the industry with more flexible prices by more than the reduction in profits in the industry with more price rigidities. Simultaneously, the shock increases

[1] There is ample evidence of infrequent adjustments in the prices of goods and services and significant differences in the degree of price rigidity across industries. Bils and Klenow (2004) analyze 350 product categories. They report a median duration of prices between 4 and 6 months and the standard deviation is around 3 months. Nakamura and Steinsson (2007) exclude price changes related to sales and adjust this duration upwards to a range between 8 and 11 months. Blinder et al. (1998) conduct surveys on firms' pricing policies and summarize different theories for the existence of price rigidities based on the nature of costs, demand, contracts, market interactions and imperfect information.

marginal utility because aggregate consumption is low. Therefore, investors require an additional compensation for holding stocks on industries with more flexible product prices.

The dependence of profits on the degree of price rigidity can be understood as the result of countercyclical markups induced by the rigidity. When prices are flexible, monopolistic competitors choose a level of production and a price that ensure an optimal constant markup over the marginal cost. When a producer does not adjust the product price, production depends on aggregate demand. During bad times, aggregate demand is low, labor demand declines and nominal wages decrease. Since prices are sticky, real wages also decline and the difference between a unit of production and the real labor cost increases. That is, the markup increases during bad times. The opposite happens during good times, and the markup is compressed with respect to the optimal constant markup.

Monetary policy affects asset returns because it determines the distortions in markups generated by price rigidities. When inflation is low, differences between the optimal product price and the "sticky" price are small, the variability in markups is low and investors do not require high compensations for inflation risk. On the other hand, if monetary policy is conducted in such a way that inflation is volatile, markups are volatile and thus high compensations for claims on profits are required.

When there are differences in prices rigidities across industries, markups for different industries have different sensitivities to shocks in the economy. Industries with more flexible prices have implied markups that are closer to the optimal constant markup than the markups for industries with less flexible prices. As a result, the markups of rigid-price industries expand more than those of flexible-price industries during bad times and decrease less during good times. Investors effectively perceive stocks on rigid-price firms as less risky than stocks on flexible-price firms and require lower returns. When monetary policy implies low inflation, the distortions caused by price rigidities in the two industries are small and, therefore, differences in expected returns in the two industries are small too.

Our theoretical results are complemented with empirical evidence supporting the predictions of the model. We sort industries into 10 deciles on price rigidity and form 10 portfolios using firms within the same deciles. We then form a hedge portfolio, defined as the price rigidity portfolio, that longs the portfolio with lowest price rigidity and shorts the portfolio with highest price rigidity. For the sample period from 1970 to 2006, we find that the price rigidity portfolio earns positive abnormal returns on average and this return is not explained by the market, size, book-to-market,

and momentum factors. In addition we find that the average return of the price rigidity portfolio is much higher from 1970 to 1979, than from 1980 to 2006. This finding is also consistent with the model's predictions since there is evidence of a significantly more aggressive response to inflation in monetary policy after 1980 than during the 70's.

The paper is organized as follows. Section 2 describes the economic model. Section 3 presents the stock-pricing implications of the model. For comparison purposes we present results for three different economies: an economy with flexible prices and economies with homogeneous and heterogeneous price rigidities across industries, respectively. The model is solved numerically. Section 4 presents the empirical evidence and Section 5 conclude. The appendix contains all the proofs.

2 The Model

We model a production economy where households derive utility from the consumption of a basket of two goods and disutility from supplying labor for the production of these goods. The two goods are produced in two different industries characterized by monopolistic competition and nominal price rigidities. We allow for heterogenous degrees of price rigidity in the two industries to learn about the effects of different rigidities on the cross section of stock returns.

Nominal rigidities generate real effects of monetary policy. When some producers are not able to adjust prices optimally, inflation generates distortions in relative prices that affect production decisions. Since inflation is determined by monetary policy, different policies have different implications for real activity. We model monetary policy as an interest-rate policy rule that reacts to inflation and deviations of output from a target. Important derivations are provided in the appendix.

2.1 Households

Assume a representative infinitely-lived household, maximizing its expected total utility

$$\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^{t}\left(\frac{C_{t}^{1-\gamma}}{1-\gamma}-\frac{N_{t}^{1+\omega}}{1+\omega}\right)\right], \tag{1}$$

where C_t is the consumption of a final good and N_t is the supply of labor at time t. The final good is a basket of two intermediate goods produced in two industries. We refer to these industries as $I=\{H,L\}$, where H and L are the industries with high and low price rigidities, respectively. The

consumption of each industry's good is $C_{I,t}$ and the production of the final good is given by

$$C_t = \left[\varphi^{1/\theta} C_{H,t}^{\frac{\theta-1}{\theta}} + (1-\varphi)^{1/\theta} C_{L,t}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \tag{2}$$

where φ is the weight of industry H in the basket and $\theta > 1$ is the elasticity of substitution between industry goods. Each industry good is a Dixit-Stiglitz aggregate of a continuum of differentiated goods, defined as

$$C_{I,t} = \left[\int_0^1 C_{I,t}(j)^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}}, \tag{3}$$

where, for simplicity, the elasticity of substitution across differentiated goods is the same as across industries.

Households supply labor $N_{I,t}(j)$ for the production of the differentiated good j in industry I. The total labor supplied to industry I is aggregated as,

$$N_{I,t} = \left[\int_0^1 N_{I,t}(j)^{1+\omega} dj\right]^{\frac{1}{1+\omega}},$$

for $I \in \{H, L\}$, where ω is the inverse of the elasticity of intertemporal substitution of labor. Aggregate labor can be written as

$$N_t = \left[\varphi^{-\omega} N_{H,t}^{1+\omega} + (1-\varphi)^{-\omega} N_{L,t}^{1+\omega}\right]^{1/(1+\omega)}.$$

The intertemporal budget constraint faced by households is

$$\mathbb{E}\left[\sum_{t=0}^{\infty} M_{0,t}^{\$} P_t C_t\right] \leq \mathbb{E}\left[\sum_{t=0}^{\infty} M_{0,t}^{\$} \sum_{I\in\{H,L\}} \left(\int_0^1 w_{I,t}(j) N_{I,t}(j) dj + P_t \int_0^1 \Psi_{I,t}(j) dj\right)\right], \tag{4}$$

where $M_{0,t}^{\$} > 0$ is the nominal pricing kernel that discounts nominal cash flows at time t to time 0, P_t is the price of the final good, and $w_{I,t}(j)$, and $\Psi_{I,t}(j)$ are the nominal wage and the firm's real profit related to the production of the differentiated good j in industry I, respectively.

The maximization of (1) subject to (4) provides us with the intertemporal marginal rates of substitution of consumption for the economy. The intertemporal marginal rates of substitution of

consumption between period t and period $t+n$ in real and nominal terms are

$$M_{t,t+n} = \beta^n \left(\frac{C_{t+n}}{C_t} \right)^{-\gamma}, \tag{5}$$

and

$$M^{\$}_{t,t+n} = \beta^n \left(\frac{C_{t+n}}{C_t} \right)^{-\gamma} \left(\frac{P_{t+n}}{P_t} \right)^{-1}, \tag{6}$$

respectively. From these two equations we can compute the real and nominal (continuously compounded) one-period risk-free rates as

$$r_t = -\log \left(\mathbb{E}_t \left[M_{t,t+1} \right] \right), \tag{7}$$

and

$$i_t = \log \left(\mathbb{E}_t \left[M^{\$}_{t,t+1} \right] \right), \tag{8}$$

respectively. The real risk-free rate will be important to compute excess real returns on stocks. The one-period nominal risk-free rate is the instrument of monetary policy.

The intratemporal marginal rate of substitution between labor and consumption is

$$\frac{w_{I,t}(j)}{P_t} = \varphi_I^{-\omega} N_{I,t}(j)^{\omega} C_t^{\gamma}, \tag{9}$$

where $\varphi_H = \varphi$ and $\varphi_L = 1 - \varphi$. This equation provides us with real wages once we determine the levels of labor and production from the production problem.

2.2 Firms

The production of differentiated goods is characterized by monopolistic competition and price rigidities in two different industries. Producers have market power to set the price of their differentiated goods within a Calvo (1983) staggered price setting. At each point of time, the producer is unable to change the price with some positive probability. We allow for different probabilities across industries to capture heterogeneous degrees of price rigidities.

The probability of not changing the price of a differentiated good at a particular time in industry I is α_I. When the producer is able to set a new price for the differentiated good, the

price is set such that it maximizes the expected present value of all future profits that depend on today's price. The maximization problem is

$$\max_{\{P_{I,t}(j)\}} \mathbb{E}_t \left[\sum_{T=t}^{\infty} \alpha_I^{T-t} M_{t,T}^{\$} \left(P_{I,t}(j) Y_{I,T|t}(j) - w_{I,T|t}(j) N_{I,T|t}(j) \right) \right] \tag{10}$$

subject to the demand function (see appendix A for its derivation)

$$P_{I,t}(j) = P_{I,T} \left(\frac{Y_{I,T|t}(j)}{Y_{I,T}} \right)^{-1/\theta},$$

and the production function

$$Y_{I,T|t}(j) = A N_{I,T|t}(j), \tag{11}$$

where $Y_{I,T|t}(j)$ is the level of output of firm j in industry I at time T, when the last time that the price was reset was at t. A similar definition applies to $N_{I,T|t}(j)$ and $w_{I,T|t}(j)$. We assume constant labor productivity, A, to isolate the effects of price rigidities from changes in productivity.

The output of industry I is $Y_{I,t}$ and the aggregate output of the final good is Y_t. We denote deviations in aggregate output from the flexible-price output, or "output gap", by

$$x_t \equiv \log Y_t - \log Y^f,$$

where Y^f is the constant aggregate output when prices are perfectly flexible. Its equilibrium value is presented in section 3.

Inflation in industry I is

$$\pi_{I,t} \equiv \log P_{I,t+1} - \log P_{I,t}$$

and the relative price between the two industry goods is

$$p_{R,t} \equiv \log P_{H,t} - \log P_{L,t}.$$

Appendix A shows that the solution to the firm's maximization problem implies

$$\pi_{I,t} = \kappa_I x_t + \kappa_I \zeta^{-1} \varphi_{-I} p_{R,t} + \beta \mathbb{E}_t[\pi_{I,t+1}], \tag{12}$$

where φ_{-I}, κ_I, and ζ are constants defined in the appendix. From the industry inflations we can

obtain the inflation in the aggregate price index, $\pi_t \equiv \log P_{t+1} - \log P_t$, given by

$$\pi_t = \bar{\kappa} x_t + b_\varphi p_{R,t} + \beta \mathbb{E}_t[\pi_{t+1}], \tag{13}$$

and the relative price equation

$$b_R p_{R,t} = \underline{\kappa} x_t + p_{R,t-1} + \beta \mathbb{E}_t[p_{R,t+1}], \tag{14}$$

where $\bar{\kappa}$, b_φ, and b_R are constants defined in the appendix. Equations (13) and (14) summarize the optimality conditions for the production sector of the economy.

2.3 Monetary Authority

We model a monetary authority that sets the level of a short-term nominal interest rate. Monetary policy is described by the policy rule

$$i_t = \bar{\imath} + \imath_\pi \pi_t + \imath_x x_t + u_t,$$

where the one-period nominal interest rate , i_t, is set responding to aggregate inflation, the output gap, and a policy shock u_t. The shock follows the process

$$u_{t+1} = \phi_u u_t + \sigma_u \varepsilon_{u,t+1}, \tag{15}$$

with $\varepsilon_u \sim \mathcal{N}(0,1)$. Policy shocks are the only source of uncertainty in the economy and, therefore, expected returns on financial assets only reflect compensations for this risk.

3 Equilibrium

We describe in this section the equilibrium of the model and analyze its macroeconomic and asset pricing implications. We analyze two simplified economies before the analysis of an economy with differentiated price rigidities across industries. The economies are an economy with flexible prices and an economy with the same level of price rigidity across the two industries. The economy with flexible prices provides a benchmark to understand the effects of price rigidities. The economy with the same level of price rigidity across industries allows us to understand the implications of price rigidities on the aggregate stock market.

In order to find allocations and prices for the economy, we need to solve the system of equations

with the relevant optimality conditions for households, firms and the exogenous monetary policy rule obtained in section 2. We summarize the system as

$$
\begin{aligned}
e^{-i_t} &= \mathbb{E}_t\left[\exp(\log\beta - \gamma\Delta x_{t+1} - \pi_{t+1})\right], && (16)\\
\pi_t &= \bar{\kappa}x_t + b_\varphi p_{R,t} + \beta\mathbb{E}_t[\pi_{t+1}], && (17)\\
b_R p_{R,t} &= \underline{\kappa}x_t + p_{R,t-1} + \beta\mathbb{E}_t[p_{R,t+1}], && (18)\\
i_t &= \bar{\imath} + \imath_x x_t + \imath_\pi \pi_t + u_t, && (19)\\
\text{and} \qquad u_t &= \phi_u u_{t-1} + \sigma_u \varepsilon_{u,t},
\end{aligned}
$$

where (16) is the households' optimality condition (8), equations (17) and (18) are results (13) and (14) of profit maximization in the production sector, equation (19) describes the monetary policy rule, and the last equation is the assumed process for the policy shocks. The market clearing conditions, $C_{I,t} = Y_{I,t}$, and $C_t = Y_t$, apply in equilibrium.

Appendix B shows that equilibrium implies processes for inflation, the relative price and the output gap depending on the lagged relative price and the policy shock, given by

$$
\begin{aligned}
\pi_t &= \bar{\pi} + \pi_p p_{R,t-1} + \pi_u u_t, && (20)\\
p_{R,t} &= \bar{\rho} + \rho_p p_{R,t-1} + \rho_u u_t, && (21)\\
\text{and} \qquad x_t &= \bar{x} + x_p p_{R,t-1} + x_u u_t\,. && (22)
\end{aligned}
$$

The coefficients $\{\bar{\pi}, \pi_p, \pi_u, \bar{\rho}, \rho_p, \rho_u, \bar{x}, x_p, x_u\}$ depend on preference, production and policy parameters as described in the appendix.

Equation (5) and the equilibrium process for the output gap in equation (22) imply that the log of the real pricing kernel, $m_{t,t+1}$, can be written in terms of the relative price and the policy shock as

$$m_{t,t+1} = \log\beta - \gamma x_p \Delta p_{R,t} + \gamma x_u(1-\phi_u)u_t - \gamma x_u \sigma_u \varepsilon_{u,t+1}.$$

It can be observed that policy shocks are a source of risk in the real pricing kernel. From the equation above, the compensation per unit of this risk, or market price of risk, is given by

$$\lambda = \gamma x_u \sigma_u\,.$$

Since the sensitivity of the output gap to the policy shocks (x_u) depends on the policy rule, the compensation for risk in the economy is determined by the responses of the monetary authority

to inflation ($\imath_\pi$) and the output gap ($\imath_x$). Figure 1 plots the market price of risk as a function of the response of monetary policy to inflation, using the parameter values in Table 1. It can been seen that a weak response to inflation in monetary policy leads to a higher risk premium in the economy.

As a result of price rigidities, the real short-term rate is affected by monetary policy. It is given by

$$r_t = -\log\beta - \frac{1}{2}\gamma^2 x_u^2 \sigma_u^2 + \gamma x_p \Delta p_{R,t} - \gamma x_u (1-\phi_u) u_t.$$

We define industry stocks as financial claims on all future profits in the industry. The real stock price for industry I at time t, $S_{I,t}$ is given by

$$S_{\Psi,I,t} = \mathbb{E}_t \left[\sum_{n=1}^{\infty} M_{t,t+n} \Psi_{I,t+n} \right], \tag{23}$$

with associated one-period real return

$$r_{\Psi,I,t+1} = \log\left(\frac{\Psi_{I,t+1} + S_{\Psi,I,t+1}}{S_{\Psi,I,t}} \right). \tag{24}$$

We analyze the characteristics of the expected stock returns for industries H and L below.

3.1 Flexible-Price Economy

Production decisions are completely unlinked from policy shocks when prices are flexible ($\alpha_H = \alpha_L = 0$). Aggregate output is constant, given by

$$Y^f = \left[\frac{A^{1+\omega}}{\mu} \right]^{1/(\omega+\gamma)},$$

where

$$\mu = \frac{\theta}{\theta - 1}$$

is the constant markup resulting from monopolistic competition.

Profit maximization implies that labor income and profits are constant shares of production. In particular, real profits in industry I are given by

$$\Psi_{I,t}^f = \frac{\varphi_I}{\theta} Y^f,$$

and the real stock price is

$$S^f_{\Psi,I,t} = \frac{\varphi_I}{\theta}\mathbb{E}_t\left[\sum_{n=1}^{\infty} M_{t,t+n}\Psi^f_{I,t+n}\right] = \frac{\varphi_I}{\theta}\frac{\beta}{1-\beta}Y^f\,.$$

Real stock prices do not depend on policy shocks in the absence of price rigidities and, therefore, no compensations for risk are required to hold stocks. It follows that real stock returns in all industries are equal to the real risk-free rate. That is,

$$r^f_{\Psi,I,t} = r^f_t = -\log\beta\,,$$

for all I, and t.

3.2 Homogeneous Price Rigidity Across Industries

The analysis of an economy where the two industries have the same degree of price rigidity ($\alpha_H = \alpha_L$) allows us to gain some insights into the effect of price rigidities on the equity premium. Since the only difference between the two industries is the degree of price rigidity, in this economy the two industries share the same dynamics. In particular, the relative price between the two industries $p_{R,t}$ does not play a role in equilibrium, and both inflation and output gap are linear functions of policy shocks only. Inflation in the aggregate price index (and in the two industries) is

$$\pi_t = \bar{\pi} + \pi_u u_t,$$

where

$$\bar{\pi} = \frac{\kappa}{\kappa(1-\imath_\pi) - \imath_x(1-\beta)}\left[\log\beta + \bar{\imath} + \frac{1}{2}\left(\frac{\gamma}{\kappa}(1-\beta\phi_u)+1\right)^2\pi_u^2\sigma_u^2\right],$$

and

$$\pi_u = -\frac{\kappa}{\kappa(\imath_\pi - \phi_u) + \imath_x(1-\beta) + \gamma(1-\beta\phi_u)(1-\phi_u)}, \tag{25}$$

where $\kappa \equiv \bar{\kappa} = \kappa_H = \kappa_L$. The output gap is

$$x_t = \frac{1}{\kappa}(1-\beta)\bar{\pi} + \frac{1}{\kappa}(1-\beta\phi_u)\pi_u u_t.$$

The effect of policy shocks on inflation and output decreases when monetary policy responds more aggressively to inflation and the output gap. From the equation for the real pricing kernel,

$m_{t,t+1} \equiv M_{t,t+1}$, we find that the market price of risk is

$$\lambda = \frac{\gamma}{\kappa}(1 - \beta\phi_u)\pi_u\sigma_u \,.$$

It becomes clear from the value of π_u in equation (25) that the magnitude of the market price of risk decreases as the responses of monetary policy to inflation and the output gap increase.

The real one-period short-term rate, r_t, only depends on the policy shock and is given by

$$r_t = -\log\beta - \frac{1}{2}\left(\frac{\gamma}{\kappa}(1 - \beta\phi_u)\pi_u\sigma_u\right)^2 - \frac{\gamma}{\kappa}(1 - \beta\phi_u)(1 - \phi_u)\pi_u u_t.$$

Strong responses to inflation and the output gap in the policy rule decrease the reaction of the real risk-free rate to policy shocks.

In order to understand the implications of price rigidities and monetary policy on the equity premium, it is convenient to analyze the aggregate markup in this economy. This markup is not longer constant when prices are not perfectly flexible. Appendix D shows that real aggregate profits, Ψ_t, can be written in terms of aggregate production as

$$\Psi_t = \left(1 - \frac{1}{\mu_t}\right) Y_t,$$

where

$$\mu_t = \mu X_t^{-(\omega+\gamma)} \tag{26}$$

is the time-varying markup in production. Time variation in the markup is the result of distortions in production caused by the policy shocks. The distortions in the markup depend on the elasticities of intertemporal substitution of consumption and labor, γ^{-1} and ω^{-1}, respectively. When this elasticities are low, the markup volatility is high. In addition, the markup is countercyclical with respect to the output gap as a result of price rigidities. High markups are observed when the output gap is low. This characteristic plays an important role in determining the properties of stock returns in this economy.

We can use the affine framework in appendix E to analyze the returns of claims on real profits (stocks). In particular, we can analyze "one-period" claims which only pay off at some future time $t + n$. Claims on all future real profits can be considered as portfolios of the one-period claims for all n.

Let $r^{(n)}_{\Psi,t+1}$ be the one-period real return of a claim on aggregate profits at time $t+n$. The expected excess return of this claim over the risk-free rate r_t is

$$\mathbb{E}_t\left[r^{(n)}_{\Psi,t+1} - r_t\right] = -\frac{1}{2}\text{var}_t\left(\Delta\psi_{t+1} + d^{(n-1)}_{\Psi,t+1}\right) - \text{cov}_t\left(m_{t,t+1}, \Delta\psi_{t+1} + d^{(n-1)}_{\Psi,t+1}\right),$$

where $\Delta\psi_{t+1} \equiv \log\Psi_{t+1} - \log\Psi_t$, and $d^{(n)}_{\Psi,t+1}$ is the price-profit ratio associated to the claim with payoff at time n. It can be shown that the covariance term above can be approximated by

$$-\text{cov}_t\left(m_{t,t+1}, \Delta\psi_{t+1} + d^{(n-1)}_{\Psi,t+1}\right) = \gamma\left[\gamma + (1+\omega-\theta(\omega+\gamma))\phi_u^{n-1}\right]\text{var}_t\left(x_{t+1}\right).$$

This term is proportional to the variability of the output gap and depends on the elasticities of substitution of consumption, labor, and across goods. Policy rules that stabilize output reduce the equity premium since policy shocks generate less distortions in the production sector and, as a result, investors require lower compensations for risk to hold stocks. Low elasticities of intertemporal substitution of consumption and labor may generate negative expected excess returns for holding some of these "one-period" claims. This is the result of countercyclical aggregate markups. A negative distortion in output caused by price rigidities generates an increase in the markup. When the elasticities of substitution are low, the increase in the markup can be substantial, such that aggregate profits can increase while aggregate production decreases. Since the output is low, the marginal rate of substitution of consumption is high. Therefore, when consumption is valuable, these claims may have a high return. Effectively, these claims can act as a hedge for consumption risk and investors are willing to hold them for expected returns that are lower than the real risk-free rate. A similar reasoning will be applied to understand the differences in the cross-section of returns implied by price rigidities.

3.3 Heterogenous Price Rigidity Across Industries

We analyze in this section the difference in expected stock returns of industries with high and low price rigidities. Since there are no analytical solutions for stock returns in this economy are not available, we rely on numerical solutions to conduct the analysis. However, we provide first some intuition for the results analyzing the differences in profits in the two industries, and the expected returns on simplified claims on these profits.

Real profits in industry I, $\Psi_{I,t}$, can be written as

$$\Psi_{I,t} = \left(1 - \frac{1}{\mu_{I,t}}\right) Y_{I,t}^{real} .$$

where

$$\mu_{I,t} = \mu_t e^{-(1+\theta\omega)\varphi_{-I} p_{R,t}}$$

is the time-varying markup for the industry,

$$Y_{I,t}^{real} = \frac{P_{I,t} Y_{I,t}}{P_t} = \varphi_I Y^f e^{x_t + (\theta-1)\varphi_{-I} p_{R,t}}$$

is the real output in industry I, and μ_t is the markup for the aggregate production as in (26). It follows that the difference in markups in the two industries is given by

$$\frac{\mu_{H,t}}{\mu_{L,t}} = e^{(1+\theta\omega)p_{R,t}} , \tag{27}$$

and the difference in real output is

$$\frac{Y_{H,t}^{real}}{Y_{L,t}^{real}} = \frac{\varphi}{1-\varphi} e^{-(\theta-1)p_{R,t}} . \tag{28}$$

It can be seen from these two equations that differences in real profits in the two industries can be explained in terms of the relative price. When the product price in industry H is higher than the product price in industry L, the markup in industry H is higher than the markup in industry L and its real output is lower.[2] Real profits in industry H are thus higher than real profits in industry L as long as the positive markup effect dominates the negative output effect. Simultaneously, the product price in industry H is higher than the product price in industry L, $p_R > 0$, when there are negative distortions in output. That is, when the marginal utility of consumption is high. It follows that markups in industry H that are more countercyclical than markups in industry L can induce stock returns for industry L that are lower than stock returns for industry H when the marginal utility of consumption is high. In that case, investors will require a higher expected return to hold stocks on industry L than those required to hold stocks on industry H. This is

[2]This is the result of differences in price rigidities. For instance, a positive policy shock increases the nominal short-term interest rate. If prices are perfectly flexible, firms adjust their prices down and keep their markup and real output unchanged. If there are price rigidities, prices are kept high, production and consumption decrease, real wages decrease and markups expand. Since prices in industry H are more rigid than those in industry L, p_H is higher than p_L, and $p_{R,t}$ increases. Therefore, the aggregate output gap and the relative price are negatively correlated.

exactly the result that we obtain using numerical solutions.

The result can be further illustrated for claims on real profits that pay off only one period in the future, as shown in the appendix. Let $r^{(1)}_{\Psi,I,t+1}$ be the one-period return of a claim on real profits at time $t+1$ of the good produced in industry I. The expected excess return of this claim on industry H over a claim on industry L (up to the Jensen's inequality terms) can be approximated by

$$\begin{aligned}\mathbb{E}_t[r^{(1)}_{\Psi,L,t+1} - r^{(1)}_{\Psi,H,t+1}] &\approx -\text{cov}_t(m_{t,t+1}, \Delta\psi_{L,t+1} - \Delta\psi_{H,t+1}) \\ &= \gamma(1-\theta)\theta\omega\text{cov}_t(x_{t+1}, p_{R,t+1}),\end{aligned}$$

which is always positive, given the negative correlation between the output gap and the relative price. That is, at least for these "one-period" claims, the markup effect always dominates the product effect above, meaning that claims on profits of the industry with more rigid product prices are less risky than those on profits of the industry with more flexible product prices. The numerical solution shows that the result extends to claims on all future profits.

3.3.1 Numerical Solution of the Model

We analyze in this section the implications on expected excess returns for stocks in the two industries using numerical solutions, and conduct a comparative statics analysis. The details of the numerical procedure are presented in appendix F. The comparative statics allow us to see the implications on the difference in expected returns of policies with different responses to inflation and the output gap.

Given the equilibrium processes for inflation, the relative price, and the output gap inequations (20)-(22), we obtain stock prices and expected returns for both industries using a recursive approach. The real value of industry I can be written recursively as[3]

$$V_I(p_{R,t}, u_t) = \Psi_{I,t}(p_{R,t}, u_t) + \mathbb{E}_t\left[M_{t,t+1}V_I(p_{R,t+1}, u_{t+1})\right], \tag{29}$$

where the state variables are the current period's relative price and the policy shock $(p_{R,t}, u_t)$. The first two terms summarize the real profit of industry I and the last term is the continuation value.

[3]This value reflects the stock price plus the current period profits.

Expected real stock returns are

$$\mathbb{E}[r_{\Psi,I,t+1}] \;=\; \mathbb{E}\left[\log\left(\frac{V_I(p_{R,t+1},u_{t+1})}{V_I(p_{R,t},u_t)-\Psi_{I,t}(p_{R,t},u_t)}\right)\right],$$

for $I=\{H,L\}$. Table 1 shows the parameter values used in the exercise.

Figure 2 plots the differences in expected returns between the low and high rigidity industries for claims on consumption, labor income and profits, for different parameter values. The difference in expected returns for claims on profits increase as the elasticities of consumption and labor decrease, the price rigidity in industry H increases and the persistence of the policy shock increases. More aggressive responses to inflation and the output gap in the policy rule reduce the difference in expected returns.

Figure 3 shows impulse responses to a positive policy shock. This shock represents bad news for the economy since it induces a negative output gap. Simultaneously, it increases the relative price, production in the industry with the more sticky price is negatively affected while production in the one with more flexible price is positively affected. The value of claims on consumption and labor decline, and the claims in industry H are more negatively affected. However, the values of the claims in labor income in the two industries are less affected than the values of the respective claims in consumption, reflecting expanded markups in the two industries. Since the expansion in markups in industry H is larger than in L, profits in L are more negatively affected than profits in H, resulting in higher expected returns on a stock in industry L over the expected return for industry H.

4 Empirical Results

We test the predictions of the model using the data of publicly traded firms. The stock market data is from the Center for Research in Security Prices (CRSP). The price rigidities for individual industries are from Bils and Klenow (2004), which provides the monthly frequency of price changes for 350 categories of consumer goods and services comprising around 70% of consumer expenditures from 1995 to 1997. Using the 49-industry classification from Kenneth French's web site, we obtain the frequencies of price changes for 31 industries, used as our proxy for price rigidity.[4] Table 2 lists the summary statistics of the price rigidities for 31 industries.

[4]The frequency of price changes for a particular industry is the average of the frequencies of price changes of consumer goods and services within this industry.

We sort industries into 10 deciles according to their price rigidities in descending order. Firms within the industries of the same decile are used to form both value-weighted and equal-weighted portfolios. We then run Carhart four-factor model for each of the 10 portfolios and the hedge portfolio, defined as the price-rigidity portfolio, that longs the portfolio with the lowest price rigidity (decile 10) and shorts the portfolio with the highest price rigidity (decile 1). Tables 3 and 4 present the regression results for two sample periods: $1970-1980$ and $1980-2006$. The selection of the two periods was based on Clarida, Galí and Gertler (2000). They find that the response of the short-term interest rate to inflation is significantly stronger after 1980 than for the $1970-1980$ period. The model predicts that profits of industries with low price rigidity earn higher expected returns than industries with high price rigidities. This difference decreases with the response of the interest rate to inflation.

Table 3 shows the regression results using the Carhart four-factor model and the data for the first sample period. For value-weighted returns, portfolio 10 (firms with lowest price rigidity) earns 77 basis points more than portfolio 1 (firms with highest price rigidity) monthly, controlling for market, size, book-to-market, and momentum factors. The difference increases to 117 basis points for equal-weighted portfolios. The t-stats are 2.32 and 2.85, respectively. Therefore, industries with low price rigidities earn significantly higher returns than industries with high price rigidities from 1970 to 1980.

Table 4 shows the results for the second period. For value-weighted returns, portfolio 10 earns 3.9 basis points more than portfolio 1, controlling for market, size, book-to-market, and momentum factors. And 2.4 basis points for equal-weighted portfolios. The t-stats are 0.12 and 0.07, respectively. Although industries with low price rigidities still earn higher average returns, the difference is much smaller after 1980 compared to that during the 1970's and is not statistically significant.

In summary, the empirical results provide strong support for the predictions of the model. A weak response of the central bank to inflation increases expected excess returns and industries with high price rigidities earn higher expected returns than industries with low price rigidities.

5 Conclusions

This paper provides a theoretical framework for the analysis of the effects of monetary policy on stock returns. We use this framework to analyze the implications of monetary policy on the equity premium and the cross-section of returns. Monetary policy has effects on stock returns because

firms are not able to adjust their product prices every period. This nominal rigidity generates an equity premium for inflation risk, which depends on the elasticities of substitution of consumption and labor, the degree of price rigidity, and the reaction of the policy to inflation and output. In the cross-section, expected returns are higher for industries with more flexible product prices. Countercyclical markups for these industries are less sensitive to inflation risk and, as result, their returns are more sensitive to this risk. Therefore, investors require an additional compensation for holding stocks on these industries.

We find empirical evidence supporting the model predictions. The return difference between low and high price rigidity industries is positive and significant for a period in the US monetary policy characterized by a weak response to inflation. This difference in returns can not be explained by market, value, size and momentum factors. The theoretical approach suggests a potential role for relative prices across industries and/or industry-specific inflation to explain this difference. It also presents a potential explanation for the empirical results on industry concentration and stock returns in Hou and Robinson (2006).

Table 1: Baseline parameter values.

Parameter	Description	Value
β	Subjective discount factor	0.974
γ	Inverse of EIS of consumption	0.8
ω	Inverse of EIS of labor	0.4
α_H	Price rigidity in industry H	0.5
α_L	Price rigidity in industry L	0.05
θ	Elasticity of substitution of goods	1.2
ϕ_u	Autocorrelation of policy shock	0.5
σ_u	Conditional volatility of policy shock	0.05
$\bar{\imath}$	Constant in the policy rule	0.029
$\imath_\pi$	Response to inflation in the policy rule	1.1
$\imath_x$	Response to output gap in the policy rule	0

Table 2: **Summary Statistics**

This table reports the average frequencies of price changes and the standard deviation for products in each industry. We divide firms into 49 industries according to the classification from Ken French's web site.

Industry Number	Industry	Number of Products	Avg. Freq.	STD of Freq.
2	Food Product	81	34.27	11.97
3	Candy and Soda	9	27.39	8.92
4	Beer and Liquor	4	17.43	3.19
5	Tobacco Product	3	20.07	2.92
6	Recreation	12	23.15	8.34
7	Entertainment	6	11.12	6.03
8	Printing and Publishing	7	9.53	4.65
9	Consumer Goods	54	19.54	6.48
10	Apparel	43	32.72	11.17
11	Healthcare	5	6.76	2.71
12	Medical Equipment	3	8.10	2.94
13	Pharmaceutical Products	3	14.77	1.76
14	Chemicals	3	19.43	10.62
16	Textiles	1	17.00	N/A
17	Construction Materials	8	12.40	4.47
21	Machinery	4	26.25	10.61
22	Electrical Equipment	1	19.40	N/A
23	Automobiles and Trucks	6	26.18	11.13
30	Petroleum and Natural Gas	8	56.45	20.81
31	Utilities	4	30.30	28.45
32	Communication	4	12.60	5.53
33	Personal Services	47	8.64	5.46
34	Business Services	1	10.00	N/A
35	Computer Hardware	4	18.90	13.02
36	Computer Software	1	16.50	N/A
37	Electric Equipment	2	13.45	6.15
39	Business Supplies	2	9.25	3.04
41	Transportation	11	21.05	22.84
44	Restaurants, Hotels, Motels	8	11.63	10.77
45	Banking	3	12.77	13.80
46	Insurance	2	12.50	4.24

Table 3: **Performance-Attribution Regressions for Portfolios with Different Price Rigidities**

This table reports the Fama-French-Carhart four-factor regression: $R_t = \alpha + \beta_1 * RMRF_t + \beta_2 * SMB_t + \beta_3 * HML_t + \beta_4 * Momentum_t + \epsilon_t$, where R_t is the excess return relative to risk-free rate of the 10 decile portfolios and the hedge portfolio at month t, α is the monthly abnormal return, $RMRF_t$ is the excess return of value-weighted market portfolio, and SMB_t, HML_t, and $Momentum_t$ are the month t returns on the zero-investment factor-mimicking portfolios that capture size, book-to-market, and momentum effects, respectively.

Sample period: $1970 - 1980$

	Value-weighted						Equal-weighted					
Portfolio	α	$RMRF$	SMB	HML	$Momentum$	$Adj\ R^2$	α	$RMRF$	SMB	HML	$Momentum$	$Adj\ R^2$
1	0.33	1.07	0.24	0.06	-0.03	0.96	0.44	0.94	1.17	0.37	-0.08	0.96
	2.98	43.1	6.72	1.54	-1.1		4.24	40.81	35.17	9.98	-2.91	
2	0.38	0.99	0.09	-0.17	0.08	0.73	0.64	0.93	1.34	0.22	-0.05	0.82
	1.26	14.72	0.9	-1.58	0.97		4.6	30.02	29.91	4.51	-1.24	
3	0.35	1.17	0.15	0.26	-0.06	0.88	0.5	1.02	1.22	0.48	-0.14	0.69
	1.64	24.12	2.14	3.41	-1.11		2.6	23.6	19.57	6.91	-2.66	
4	0.68	0.79	0.08	-0.03	-0.02	0.83	0.98	1.06	1.35	0.32	-0.01	0.81
	3.8	19.85	1.46	-0.44	-0.47		5.61	27.36	24.11	5.21	-0.14	
5	0.62	1.04	-0.19	-0.48	0.03	0.86	0.52	1.02	1.13	0.25	0	0.75
	2.95	22.06	-2.82	-6.39	0.6		3.35	29.2	22.46	4.46	0.1	
6	0.5	1.01	-0.04	-0.12	0	0.88	0.73	1.02	0.85	0.3	-0.06	0.84
	2.78	25.2	-0.62	-1.8	-0.03		5.92	37.24	21.51	6.93	-1.91	
7	0.38	1.07	0	0.19	0.03	0.87	0.44	0.95	1.06	0.36	-0.09	0.85
	1.95	24.81	0.04	2.8	0.58		2.65	25.52	19.64	6	-1.95	
8	0.59	0.96	0.22	0.2	-0.15	0.87	0.56	0.99	1.07	0.35	-0.02	0.87
	3.01	21.74	3.4	2.83	-2.79		4.49	35.69	26.73	7.89	-0.52	
9	0.65	0.85	0.11	-0.08	-0.18	0.86	0.51	0.86	1.04	0.36	-0.14	0.82
	3.53	20.55	1.79	-1.21	-3.53		3.2	23.98	20.09	6.25	-3.29	
10	1.1	0.97	-0.34	-0.02	0.16	0.69	1.61	1.09	0.62	0.03	0.32	0.52
	3.94	15.5	-3.82	-0.22	2.07		4.04	12.27	4.82	0.23	2.96	
H-L	0.77	-0.1	-0.58	-0.08	0.19	0.36	1.17	0.15	-0.55	-0.34	0.4	0.3
	2.32	-1.4	-5.47	-0.7	2.12		2.85	1.68	-4.13	-2.28	3.59	

Table 4: **Performance-Attribution Regressions for Portfolios with Different Price Rigidities**

This table reports the Fama-French-Carhart four-factor regression: $R_t = \alpha + \beta_1 * RMRF_t + \beta_2 * SMB_t + \beta_3 * HML_t + \beta_4 * Momentum_t + \epsilon_t$, where R_t is the excess return relative to risk-free rate of the 10 decile portfolios and the hedge portfolio at month t, α is the monthly abnormal return, $RMRF_t$ is the excess return of value-weighted market portfolio, and SMB_t, HML_t, and $Momentum_t$ are the month t returns on the zero-investment factor-mimicking portfolios that capture size, book-to-market, and momentum effects, respectively.

Sample period: $1980 - 2006$

	Value-weighted						Equal-weighted					
Portfolio	α	$RMRF$	SMB	HML	$Momentum$	$Adj\ R^2$	α	$RMRF$	SMB	HML	$Momentum$	$Adj\ R^2$
1	0.36	1.06	0.21	-0.1	-0.03	0.98	0.42	0.92	0.93	0.34	-0.07	0.91
	4.01	45.26	5.62	-2.37	-0.82		3.42	28.37	18.14	5.82	-1.51	
2	0.54	0.95	0.1	-0.45	-0.25	0.97	0.76	0.99	1.08	0.03	-0.07	0.9
	2.85	19.17	1.22	-5.03	-3.6		4.85	24.11	16.63	0.37	-1.29	
3	0.37	1.19	0.55	0.05	-0.11	0.94	0.24	1.01	1.19	0.52	-0.03	0.73
	1.15	14.31	4.16	0.35	-0.93		0.85	13.71	10.2	3.88	-0.3	
4	0.37	0.92	-0.23	0.19	0.15	0.96	0.59	1.02	0.93	0.34	-0.01	0.9
	2.32	22.02	-3.51	2.56	2.52		4.07	26.83	15.36	4.98	-0.19	
5	1.11	0.99	0.29	-0.35	0.24	0.96	1.23	1.11	1.38	0.08	-0.03	0.81
	4.39	15.01	2.81	-2.89	2.63		4.62	15.96	12.44	0.63	-0.29	
6	0.73	1.07	-0.17	-0.78	-0.14	0.97	0.72	1.09	1.23	-0.19	-0.19	0.84
	3.53	19.79	-2.03	-8	-1.89		3.03	17.6	12.49	-1.67	-2.21	
7	0.49	1.06	-0.01	-0.19	-0.03	0.95	0.51	0.96	1.03	0.15	-0.02	0.87
	2.86	23.93	-0.13	-2.37	-0.48		2.97	21.34	14.33	1.83	-0.35	
8	0.55	1.13	0.06	0.14	-0.05	0.97	0.39	1.06	0.95	0.48	-0.04	0.88
	3.48	27.55	0.93	1.82	-0.95		2.43	25.26	14.19	6.25	-0.74	
9	0.52	0.99	-0.05	-0.24	0.1	0.94	0.59	0.98	0.93	0.21	-0.03	0.89
	2.81	20.38	-0.63	-2.77	1.51		4	25.14	15.05	2.95	-0.56	
10	0.4	0.87	-0.27	0.47	0.05	0.7	0.45	0.85	0.51	0.6	0.14	0.41
	1.4	11.8	-2.29	3.5	0.44		1.19	8.63	3.24	3.36	0.99	
H-L	0.04	-0.18	-0.48	0.57	0.07	0.29	0.02	-0.07	-0.43	0.26	0.2	0.1
	0.12	-2.24	-3.65	3.81	0.62		0.07	-0.7	-2.77	1.47	1.52	

Figure 1: **Market Price of Risk**

The figure plots the market price of risk as a function of the response of monetary policy to inflation, using the calibrated parameters in Table 1.

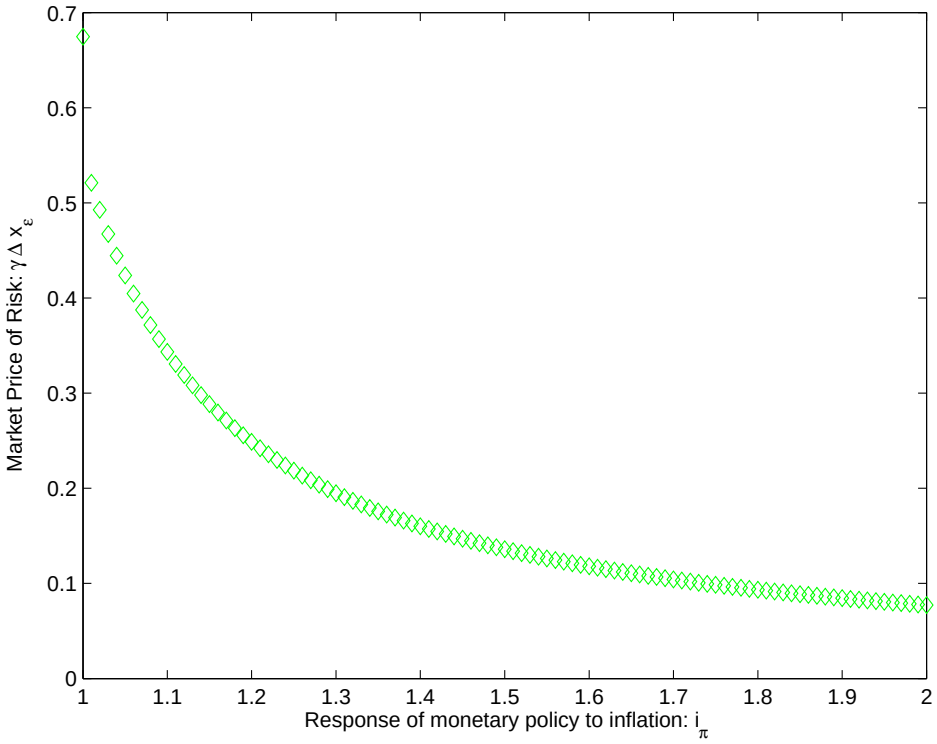


Figure 2: **Expected Return Differences between Low and High Rigidity Industries**

The figure plots the differences in expected returns for claims on real consumption, real labor income and real profits between low and high rigidity industries as a function of different values for different model parameters, for an economy affected by monetary policy shocks.

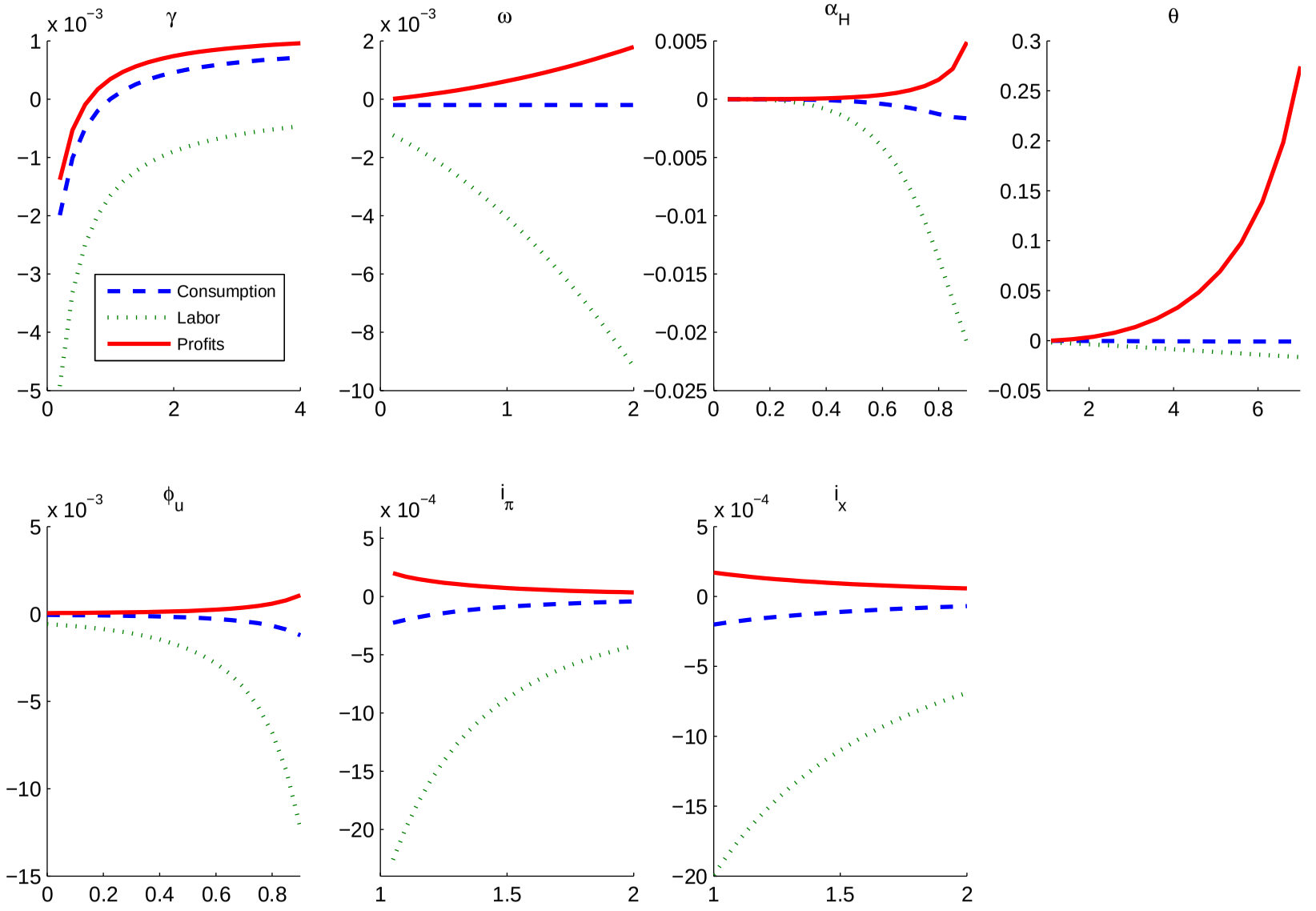
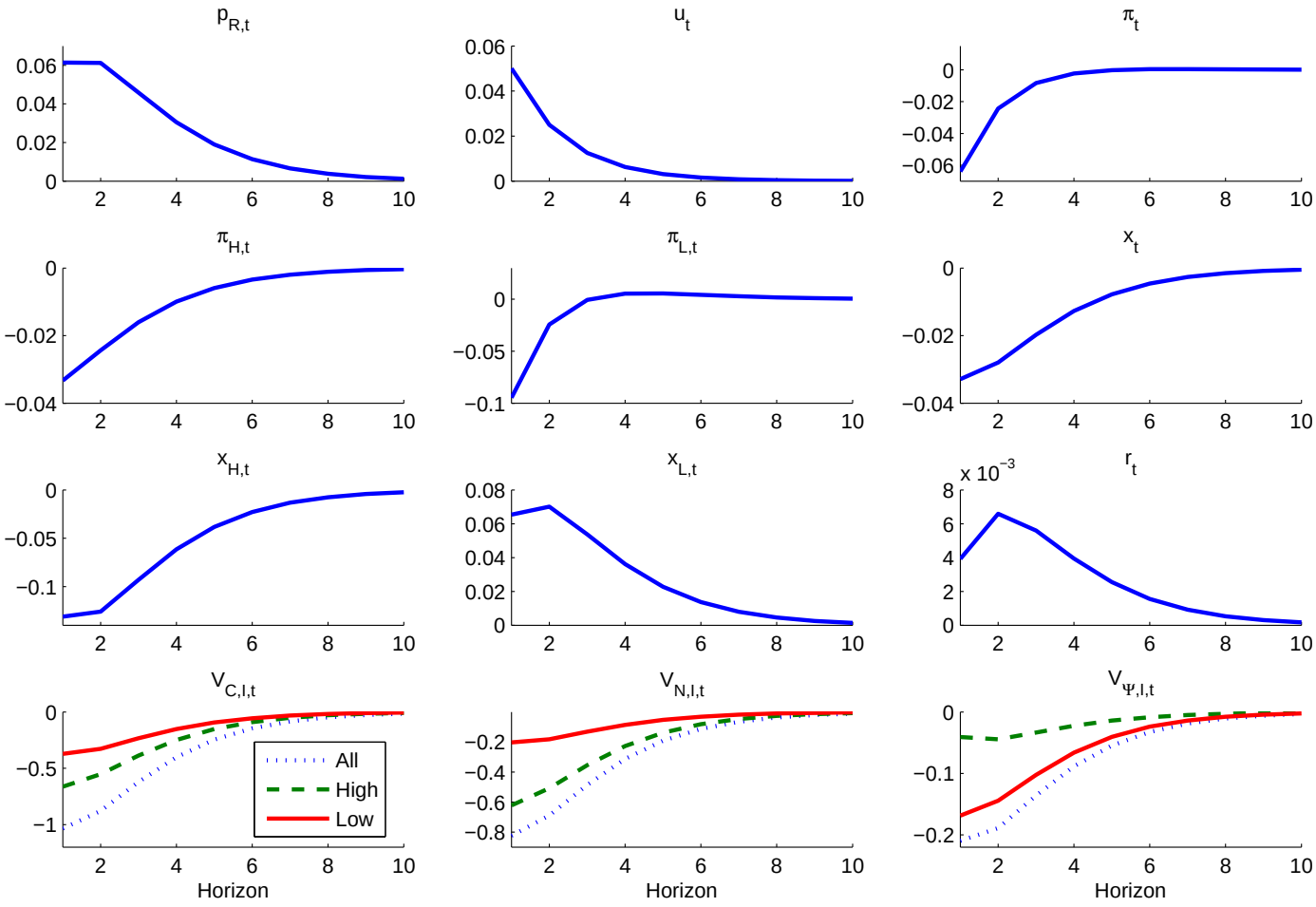


Figure 3: **Impulse Responses to a Positive Policy Shock**

The figure plots impulse responses for different macroeconomic variables, the one period real interest rate and the value of claims to real consumption, real labor income and real profits. "All", "High" and "Low" refer to the aggregate economy, the industry with high price rigidity and the industry with low price rigidity, respectively.



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Appendix

A Profit Maximization under Price Rigidities

Consider the Dixit-Stiglitz aggregate (3) as a production function, and a competitive "producer" of the industry good facing the problem

$$\max_{\{C_{I,t}(i)\}} P_{I,t}C_{I,t} - \int_0^1 P_{I,t}(i)C_{I,t}(i)di$$

subject to (3). Solving the problem, we find the demand function

$$P_{I,t}(i) = P_{I,t}\left(\frac{C_{I,t}(i)}{C_{I,t}}\right)^{-1/\theta} \tag{30}$$

The zero-profit condition implies

$$P_{I,t}C_{I,t} = \int_0^1 P_{I,t}(i)C_{I,t}(i)di = \int_0^1 P_{I,t}C_{I,t}\left(\frac{P_{I,t}(i)}{P_{I,t}}\right)^{-\theta} di.$$

Solving for $P_{I,t}$, it follows that

$$P_{I,t} = \left[\int_0^1 P_{I,t}(i)^{1-\theta}di\right]^{\frac{1}{1-\theta}} \tag{31}$$

which can be written as the demand function for each differentiated good in sector I

$$C_{I,t}(i) = \left(\frac{P_{I,t}(i)}{P_{I,t}}\right)^{-\theta} C_{I,t}\,. \tag{32}$$

Similarly, we can solve the profit maximization problem of the final good industry, which use goods from industry H and L as inputs. The demand function for industry I good is

$$C_{I,t} = \varphi_I\left(\frac{P_{I,t}}{P_t}\right)^{\theta} C_t \tag{33}$$

where P_t is the final good price, defined as the aggregate price index. The zero profit condition of the final goods production implies

$$P_t = \left[\varphi P_{H,t}^{1-\theta} + (1-\varphi) P_{L,t}^{1-\theta}\right]^{1/(1-\theta)} .$$

Notice that these relations imply that consumption in both sectors is related by

$$C_{H,t} = \frac{\varphi}{1-\varphi}\left(\frac{P_{H,t}}{P_{L,t}}\right)^{-\theta} C_{L,t}.$$

Therefore, when prices are flexible, prices of the sector goods are the same and consumptions in the two sectors are proportional.

The profit maximization problem (10) is solved relying on a linear approximation around a "steady state". The steady state is defined as the solution of the profit maximization problem in an economy with perfectly flexible prices. It is convenient to analyze this problem for the hypothetical flexible economy first and then show the solution for the actual economy.

$$\max_{\{P_{I,t}(i)\}} P_{I,t}(i) Y_{I,t}^f(i) - w_{I,t}(i) N_{I,t}(i)$$

subject to (30) and (11). The solution to this problem implies

$$\frac{P_{I,t}(i)}{P_t} = \mu s_{I,t}(i)$$

where the markup $\mu = \frac{\theta}{\theta-1}$ over the real marginal cost $s_{I,t}(i) \equiv \frac{1}{P_t}\frac{\partial(w_{I,t}(i)N_{I,t}(i))}{\partial Y_{I,t}(i)}$ is the result of monopolistic power. By using the production function (11) and the marginal rate of substitution (9) we can write the real marginal production cost as

$$s_{I,t}(i) = \frac{\varphi_I^{-\omega}}{Y_{I,t}(i)}\left(\frac{Y_{I,t}(i)}{A}\right)^{1+\omega} Y_t^{\gamma}. \tag{34}$$

Since prices are flexible and firms are identical, $P_t(i) = P_t$, $Y_t(i) = Y_t$. As a result, production in the flexible-price economy can be written as

$$y_t^f = \log Y_t^f = \frac{1}{\omega+\gamma}\left[(1+\omega)\log A - \log \mu\right]. \tag{35}$$

Since the production is a constant, we will drop the subscription t in Y_t^f. Real wage in industry

I is the same for every firm

$$\frac{w_{I,t}}{P_t} \;=\; \varphi_I^{-\omega} A^{-\omega} Y_{I,t}^{\omega} C_t^{\gamma} \;=\; A^{-\omega} \left(Y^f\right)^{\omega+\gamma} .$$

The real profit of industry I is then given by

$$\begin{aligned} \Psi_I^f &= \varphi_I Y^f \left(1 - \frac{1}{\mu(1+\omega)}\right) \\ &= \frac{\varphi_I(1+\theta\omega)}{\theta(1+\omega)} Y^f \end{aligned}$$

The flexible-price output provides us with a "point" to approximate the solution to the profit maximization problem in the sticky price economy.

Denote $M_{t,T}^{\$} = \beta^{T-t}\Lambda_T$, $S_{I,t} = P_t s_{I,t}$. Consider the derivative

$$\frac{\partial \Psi_{I,T|t}(i)}{\partial P_{I,t}(i)} = Y_{I,T|t}(i) \frac{1-\theta}{P_{I,t}(i)} \left[P_{I,t}(i) - \mu S_{I,T|t}(i)\right].$$

Therefore, the first order condition to the profit maximization problem (10) is

$$\mathbb{E}_t \left[\sum_{T=t}^{\infty} (\alpha_I \beta)^{T-t} \Lambda_T Y_{I,T|t}(i) P_{I,t}^*(i)\right] = \mathbb{E}_t \left[\sum_{T=t}^{\infty} (\alpha_I \beta)^{T-t} \Lambda_T Y_{I,T|t}(i) \mu S_{I,T|t}(i)\right]. \tag{36}$$

Since all producers in industry I who can change prices at t face the same optimization problem, $Y_{I,T|t}(i) = Y_{I,T|t}$, $P_{I,t}^*(i) = P_{I,t}^*$ and $S_{I,T|t}(i) = S_{I,T|t}$. Applying the Taylor expansion $a_t b_t = \bar{a}\bar{b} + \bar{b}(a_t - \bar{a}) + \bar{a}(b_t - \bar{b})$ to both sides of the equation around a steady-state with $\bar{P} = \mu\bar{S}$, we have for the left hand side of the equation

$$\begin{aligned} \mathbb{E}_t \left[\sum_{T=t}^{\infty} (\alpha_I \beta)^{T-t} \Lambda_T Y_{I,T|t} P_{I,t}^*\right] &= \overline{\Lambda Y}\; \overline{P} \sum_{T=t}^{\infty} (\alpha_I \beta)^{T-t} + \overline{P}\; \mathbb{E}_t \left[\sum_{T=t}^{\infty} (\alpha_I \beta)^{T-t} \left(\Lambda_T Y_{I,T|t} - \overline{\Lambda Y}\right)\right] \\ &+ \overline{\Lambda Y} \left(P_{I,t}^* - \overline{P}\right) \sum_{T=t}^{\infty} (\alpha_I \beta)^{T-t} \end{aligned}$$

and for the right hand side

$$\mathbb{E}_t\left[\sum_{T=t}^{\infty}(\alpha_I\beta)^{T-t}\Lambda_T Y_{I,T|t}\mu S_{I,T|t}\right] = \mu\overline{\Lambda Y}\,\overline{S}\sum_{T=t}^{\infty}(\alpha_I\beta)^{T-t} + \mu\overline{S}\,\mathbb{E}_t\left[\sum_{T=t}^{\infty}(\alpha_I\beta)^{T-t}\left(\Lambda_T Y_{I,T|t} - \overline{\Lambda Y}\right)\right] + \mu\overline{\Lambda Y}\,\mathbb{E}_t\left[\sum_{T=t}^{\infty}(\alpha_I\beta)^{T-t}\left(S_{I,T|t} - \overline{S}\right)\right].$$

Noting that the first and second terms in both sides of the equation are the same, equation (36) becomes

$$\frac{1}{(1-\alpha_I\beta)}P^*_{I,t} = \mathbb{E}_t\left[\sum_{T=t}^{\infty}(\alpha_I\beta)^{T-t}\mu S_{I,T|t}\right].$$

Since $S_{T|t} = s_{T|t}P_T$, replacing equation (34) in the equation above and re-arranging terms, we obtain

$$\frac{1}{(1-\alpha_I\beta)}\left(P^*_{I,t}\right)^{1+\theta\omega} = \mathbb{E}_t\left[\sum_{T=t}^{\infty}(\alpha_I\beta)^{T-t}\mu P_T^{1+\theta\omega}Y_T^{\omega+\gamma}A^{-(1+\omega)}\right].$$

Dividing by $\bar{P}^{1+\theta\omega}$, the equation can be written in terms of the output gap $x_t = y_t - y_t^f$ as

$$\frac{1}{(1-\alpha_I\beta)}\left(\frac{P^*_{I,t}}{\bar{P}}\right)^{1+\theta\omega} = e^{(\omega+\gamma)x_t}\left(\frac{P_t}{\bar{P}}\right)^{1+\theta\omega} + \frac{\alpha_I\beta}{1-\alpha_I\beta}\mathbb{E}_t\left[\left(\frac{P^*_{I,t+1}}{\bar{P}}\right)^{1+\theta\omega}\right].$$

Letting $p^*_{I,t} = \log\frac{P^*_{I,t}}{\bar{P}}$ and using the approximation $e^x \approx 1+x$, we obtain

$$\frac{1}{(1-\alpha_I\beta)}\left(1+(1+\theta\omega)p^*_{I,t}\right) = 1+(\omega+\gamma)x_t + (1+\theta\omega)p_t + \frac{\alpha_I\beta}{1-\alpha_I\beta}\mathbb{E}_t\left[1+(1+\theta\omega)p^*_{I,t+1}\right] \quad (37)$$

that simplifies to

$$p^*_{I,t} = \frac{\omega+\gamma}{1+\theta\omega}x_t + p_t + \frac{\alpha_I\beta}{1-\alpha_I\beta}\mathbb{E}_t\left[p^*_{I,t+1} - p^*_{I,t}\right]. \quad (38)$$

Since there are infinitely many firms in each industry, at each period, a measure α_I of firms will keep the last period's price and a measure $1-\alpha_I$ of firms will set a new price by solving the

above maximization problem. The aggregate price index for industry I is

$$P_{I,t} = \left[(1-\alpha_I) \left(P^*_{I,t} \right)^{1-\theta} + \alpha_I P^{1-\theta}_{I,t-1} \right]^{\frac{1}{1-\theta}} .$$

A first order Taylor approximation of the price index results in

$$p_{I,t} = (1-\alpha_I) p^*_{I,t} + \alpha_I p_{I,t-1}.$$

It implies

$$p^*_{I,t} = \frac{\alpha_I}{1-\alpha_I} \pi_{I,t} + p_{I,t} \quad \text{and} \quad p^*_{I,t+1} - p^*_{I,t} = \frac{1}{1-\alpha_I} \pi_{I,t+1} - \frac{\alpha_I}{1-\alpha_I} \pi_{I,t}.$$

Replacing these equations in equation (38), we obtain

$$\pi_{I,t} = \kappa_I x_t + \frac{\kappa_I}{\zeta} (p_t - p_{I,t}) + \beta \mathbb{E}_t[\pi_{I,t+1}],$$

where $\kappa_I \equiv \frac{(1-\alpha_I\beta)(1-\alpha_I)}{\alpha_I}\zeta$ and $\zeta \equiv \frac{\omega+\gamma}{1+\theta\omega}$. We can write

$$p_t - p_{I,t} = \varphi_{-I} p_{R,t}$$

where $\varphi_{-H} \equiv -(1-\varphi)$ and $\varphi_{-L} = \varphi$.

Equation (12) can be written in terms of aggregate inflation, the output gap and the relative price. Inflation in the aggregate price index, $\pi_t \equiv \log P_{t+1} - \log P_t$, can be written in terms of industry inflations as

$$\pi_t = \varphi \pi_{H,t} + (1-\varphi) \pi_{L,t} \, .$$

As a result, by adding up the two equations (weighted by the industry weights) we obtain

$$\pi_t = \bar{\kappa} x_t + b_\varphi p_{R,t} + \beta \mathbb{E}_t[\pi_{t+1}] \, .$$

where

$$\bar{\kappa} = \varphi \kappa_H + (1-\varphi)\kappa_L \quad , \quad \underline{\kappa} = \kappa_H - \kappa_L \quad \text{and} \quad b_\varphi = -\frac{\varphi(1-\varphi)}{\zeta} \underline{\kappa}.$$

Therefore, if the degree of price rigidities in the two industries is the same ($\underline{\kappa} = 0$), aggregate inflation does not depend on the relative price between the two industries. In order to obtain an expression for the evolution of the relative price, we can subtract one of the equations (12) from

the other one and obtain

$$b_R p_{R,t} = \underline{\kappa} x_t + p_{R,t-1} + \beta \mathbb{E}_t[p_{R,t+1}],$$

where

$$b_R = 1 + \beta + \frac{1}{\zeta}\left[(1-\varphi)\kappa_H + \varphi\kappa_L\right].$$

This equation describes the evolution of the relative price in terms of the output gap, the one-period lag and the expected future relative prices.

B Equilibrium

$$\begin{aligned} e^{-i_t} &= \mathbb{E}_t\left[\exp(\log\beta - \gamma(\Delta y^f + \Delta x_{t+1}) - \pi_{t+1})\right], \\ \pi_t &= \bar{\kappa} x_t + b_\varphi p_{R,t} + \beta \mathbb{E}[\pi_{t+1}], \\ b_R p_{R,t} &= \underline{\kappa} x_t + p_{R,t-1} + \beta \mathbb{E}_t[p_{R,t+1}], \\ i_t &= \bar{\imath} + \imath_\pi \pi_t + \imath_x x_t + u_t \\ \text{and} \qquad u_t &= \phi_u u_{t-1} + \sigma_u \varepsilon_{u,t}. \end{aligned}$$

Where $b_\varphi = -\frac{\varphi(1-\varphi)}{\zeta}\underline{\kappa}$, $\bar{\kappa} = \varphi\kappa_H + (1-\varphi)\kappa_L$, $\underline{\kappa} = \kappa_H - \kappa_L$ and

$$b_R = 1 + \beta + \frac{1}{\zeta}[(1-\varphi)\kappa_H + \varphi\kappa_L].$$

Equation (17) can be written as

$$x_t = \frac{1}{\bar{\kappa}}\left[\pi_t - b_\varphi p_{R,t} - \beta \mathbb{E}_t[\pi_{t+1}]\right] \tag{39}$$

and its first difference as

$$\Delta x_{t+1} = \frac{1}{\bar{\kappa}}\left[\Delta\pi_{t+1} - b_\varphi \Delta p_{R,t+1} - \beta\left(\mathbb{E}_{t+1}[\pi_{t+2}] - \mathbb{E}_t[\pi_{t+1}]\right)\right]. \tag{40}$$

Replacing (39) in (18) we obtain

$$b_R p_{R,t} - p_{R,t-1} = \mathbb{K}\left[\pi_t - b_\varphi p_{R,t} - \beta \mathbb{E}_t[\pi_{t+1}]\right] + \beta \mathbb{E}_t[p_{R,t+1}]. \tag{41}$$

where $\mathbb{K} = \frac{\underline{\kappa}}{\bar{\kappa}}$.

Guess solutions for inflation and the relative price of the form

$$\pi_t = \bar{\pi} + \pi_p p_{R,t-1} + \pi_u u_t \quad \text{and} \quad p_{R,t} = \bar{\rho} + \rho_p p_{R,t-1} + \rho_u u_t,$$

respectively. Replacing this solution in equation (41) and matching coefficients we obtain the sub-system of equations

$$\begin{aligned} b_\pi \bar{\rho} &= \mathbb{K}(1-\beta)\bar{\pi} + \beta\bar{\rho}, && (42)\\ b_\pi \rho_p &= 1 + \mathbb{K}\pi_p, && (43)\\ b_\pi \rho_u &= \mathbb{K}(1-\beta\phi_u)\pi_u + \beta\rho_u\phi_u, && (44) \end{aligned}$$

where

$$b_\pi = b_R + b_\varphi \mathbb{K} + \beta\mathbb{K}\pi_p - \beta\rho_p.$$

To complete the system of equations, replace (40) and (19) in (16). The guessed solutions imply log-normal distributions for all variables and therefore we obtain

$$\begin{aligned} -\bar{\imath} - \imath_\pi \pi_t - \imath_x x_t - u_t &= \log\beta - \frac{\gamma}{\bar{\kappa}}[(\pi_p - b_\varphi\rho_p - \beta\pi_p\rho_p)(\bar{\rho} + (\rho_p - 1)p_{R,t-1} + \rho_u u_t)\\ &- (1-\phi_u)(\pi_u - b_\varphi\rho_u - \beta\pi_p\rho_u - \beta\pi_u\phi_u)u_t]\\ &- \bar{\pi} - \pi_p(\bar{\rho} + \rho_p p_{R,t-1} + \rho_u u_t) - \pi_u\phi_u u_t\\ &+ \frac{1}{2}\text{var}_t\left(\frac{\gamma}{\bar{\kappa}}(\pi_u - b_\varphi\rho_u - \beta\pi_p\rho_u - \beta\pi_u\phi_u)u_{t+1} + \pi_u u_{t+1}\right). \quad (45) \end{aligned}$$

Matching coefficients we obtain the sub-system

$$\begin{aligned} -\bar{\imath} - \imath_\pi\bar{\pi} - \frac{\imath_x}{\bar{\kappa}}[(1-\beta)\bar{\pi} - (b_\varphi + \beta\pi_p)\bar{\rho}] &= \log\beta - \bar{\pi} - -\frac{\gamma}{\bar{\kappa}}(\pi_p - b_\varphi\rho_p - \beta\pi_p\rho_p)\bar{\rho} - \pi_p\bar{\rho}\\ &+ \frac{1}{2}\left(\frac{\gamma}{\bar{\kappa}}(\pi_u - b_\varphi\rho_u - \beta\pi_p\rho_u - \beta\pi_u\phi_u) + \pi_u\right)^2\sigma_u^2, \quad (46)\\ -\imath_\pi\pi_p - \frac{\imath_x}{\bar{\kappa}}[\pi_p - (b_\varphi + \beta\pi_p)\rho_p] &= \frac{\gamma}{\bar{\kappa}}(\pi_p - b_\varphi\rho_p - \beta\pi_p\rho_p)(1-\rho_p) - \pi_p\rho_p, \quad (47)\\ -\imath_\pi\pi_u - \frac{\imath_x}{\bar{\kappa}}[(1-\beta\phi_u)\pi_u - (b_\varphi + \beta\pi_p)\rho_u] - 1 &= -\frac{\gamma}{\bar{\kappa}}(\pi_p - b_\varphi\rho_p - \beta\pi_p\rho_p)\rho_u\\ &+ \frac{\gamma}{\bar{\kappa}}(1-\phi_u)(\pi_u - b_\varphi\rho_u - \beta\pi_p\rho_u - \beta\pi_u\phi_u)\\ &- \pi_p\rho_u - \pi_u\phi_u. \quad (48) \end{aligned}$$

The complete system is given by equations (42)-(44) and (46)-(48). This system allows us to obtain the equilibrium parameters $\{\bar{\pi}, \pi_p, \pi_u, \bar{\rho}, \rho_p, \rho_u\}$. Notice that equations (43) and (47) only

depend on π_p and ρ_p. Therefore, we can use these two equations to solve for these two parameters. After some algebra manipulations we obtain

$$\begin{aligned}\gamma\beta^2\rho_p^4 - \beta\left[\bar{\kappa} + \gamma(1+\beta+b_R) + \beta\imath_x\right]\rho_p^3 \\ + \left[\gamma(\beta + b_R + \beta(1+b_R)) + \bar{\kappa}b_R + b_\varphi\underline{\kappa} + \imath_\pi\bar{\kappa}\beta + \imath_x\beta(1+b_R)\right]\rho_p^2 \\ - \left[\bar{\kappa} + \imath_\pi\left(\bar{\kappa}b_R + b_\varphi\underline{\kappa} + \imath_x(b_R+\beta)\right) + \gamma(1+b_R+\beta)\right]\rho_p + \imath_\pi\bar{\kappa} + \gamma + \imath_x = 0.\end{aligned}$$

The coefficient π_p can be obtained from

$$\pi_p = \frac{b_\varphi\rho_p[\gamma(1-\rho_p)+\imath_x]}{[\gamma(1-\rho_p)+\imath_x](1-\beta\rho_p) + \bar{\kappa}(\imath_\pi - \rho_p)}.$$

Using equations (44) and (48) we find ρ_u and π_u. The sensitivity of inflation to the policy shock solves

$$\begin{aligned}\pi_u &= \left[\bar{\kappa}^2(\phi_u - \imath_\pi) - (\gamma(1-\phi_u)+\imath_x)\bar{\kappa}(1-\beta\phi_u) + \frac{1-\beta\phi_u}{b_\varphi - \beta\phi_u}\underline{\kappa}\right. \\ &\times \left.\left(\pi_p(\gamma+\bar{\kappa}) + (b_\varphi + \beta\pi_p)(\gamma(1-\rho_p-\phi_u)+\imath_x)\right)\right]^{-1}\bar{\kappa}^2\end{aligned}$$

and the sensitivity of the relative price to policy shocks is

$$\rho_u = \frac{\mathbb{K}}{b_\pi - \beta\phi_u}(1-\beta\phi_u)\pi_u.$$

From equations (42) and (46) we find $\bar{\rho}$ and $\bar{\pi}$. The constants are

$$\begin{aligned}\bar{\pi} &= \left[1 - \imath_\pi - \frac{\imath_x}{\bar{\kappa}}(1-\beta) + \frac{\mathbb{K}}{b_\pi - \beta}(1-\beta)\left(\frac{\gamma}{\bar{\kappa}}(\pi_p - b_\varphi\rho_p - \beta\pi_p\rho_p) + \pi_p + (b_\varphi + \beta\pi_p)\frac{\imath_x}{\bar{\kappa}}\right)\right]^{-1} \\ &\times \left[\bar{\imath} + \log\beta + \frac{1}{2}\left(\frac{\gamma}{\bar{\kappa}}(\pi_u - b_\varphi\rho_u - \beta\pi_p\rho_u - \beta\pi_u\phi_u) + \pi_u\right)^2\sigma_u^2\right]\end{aligned}$$

and

$$\bar{\rho} = \frac{\mathbb{K}}{b_\pi - \beta}(1-\beta)\bar{\pi}.$$

C Inflation in Individual Industries

We can write the inflation within industry I as a function of the state variables:

$$\pi_{I,t} = \bar{\pi}_I + \pi_{I,p} p_{R,t-1} + \pi_{I,u} u_t \,. \tag{49}$$

We know that the first order Taylor expansion of the relative price relation is

$$p_t - p_{I,t} = \varphi_{-I} p_{R,t}$$

and the inflation in sector I is

$$\pi_{I,t} = \kappa_I x_t + \frac{\kappa_I b_I}{\zeta} p_{R,t} + \beta \mathbb{E}_t \left[\pi_{I,t+1}\right] .$$

Combined with the equilibrium conditions in Section 3, we find the coefficients for industry inflations as

$$\begin{aligned}
\bar{\pi}_I &= \frac{\kappa_I}{1-\beta}\left[\bar{x} - \frac{\varphi_{-I}\bar{\rho}}{\zeta(1-\beta\rho_p)} + \frac{\beta\bar{\rho}x_p}{1-\beta\rho_p}\right] ; \\
\pi_{I,p} &= \frac{\kappa_I}{1-\beta\rho_p}\left[x_p - \frac{\varphi_{-I}\rho_p}{\zeta}\right] , \\
\pi_{I,u} &= \frac{\kappa_I}{1-\beta\phi_u}\left[x_u - \frac{\varphi_{-I}\rho_u}{\zeta(1-\beta\rho_p)} + \frac{\beta\rho_u x_p}{1-\beta\rho_p}\right] .
\end{aligned}$$

D Labor Income

Denote by $LI_{I,t}$ the real labor income at time t in industry I, given by

$$LI_{I,t} = \int_0^1 \frac{w_{I,t}(i)}{P_t} N_{I,t}(i) di.$$

Using equation (9),(32), and (33), real labor income can be written as

$$LI_{I,t} = \frac{\varphi_I Y_t^{1+\gamma+\omega}}{A^{1+\omega}} \left(\frac{P_{I,t}}{P_t}\right)^{-\theta(1+\omega)} \int_0^1 \left(\frac{P_{I,t}(i)}{P_{I,t}}\right)^{-\theta(1+\omega)} di \,.$$

Substitute the output under flexible price

$$\left(Y_t^f\right)^{\omega+\gamma} = \mu^{-1} A^{1+w} \,,$$

we obtain

$$LI_{I,t} = \frac{\varphi_I}{\mu}\frac{Y_t^{1+\omega+\gamma}}{(Y^f)^{\omega+\gamma}}\left(\frac{P_{I,t}}{P_t}\right)^{-\theta(1+\omega)}\int_0^1\left(\frac{P_{I,t}(i)}{P_{I,t}}\right)^{-\theta(1+\omega)}di = LI^f X_t^{1+\omega+\gamma}\int_0^1\left(\frac{P_{I,t}(i)}{P_{I,t}}\right)^{-\theta(1+\omega)}di\,.$$

where $LI_t^f = \frac{\varphi_I}{\mu}Y^f$ is the real labor income under flexible prices.

Decomposing the last term in the labor income equation we obtain

$$\begin{aligned}\int_0^1\left(\frac{P_{I,t}(i)}{P_{I,t}}\right)^{-\theta(1+\omega)}di &= \int_{i\in(1-\alpha_I)}\left(\frac{P^*_{I,t}}{P_{I,t}}\right)^{-\theta(1+\omega)}di + \int_{i\in\alpha_I}\left(\frac{P_{I,t-1}(i)}{P_{I,t}}\right)^{-\theta(1+\omega)}di \\ &= (1-\alpha_I)e^{-\theta(1+\omega)(p^*_{I,t}-p_{I,t})} + \alpha_I e^{-\theta(1+\omega)(p_{I,t-1}-p_{I,t})}.\end{aligned}$$

A first order Taylor approximation results in

$$\begin{aligned}\int_0^1\left(\frac{P_{I,t}(i)}{P_{I,t}}\right)^{-\theta(1+\omega)}di &\approx 1-(1-\alpha_I)\theta(1+\omega)(p^*_{I,t}-p_{I,t}) - \alpha_I\theta(1+\omega)(p_{I,t-1}-p_{I,t}) \\ &= 1-\theta(1+\omega)\left[(1-\alpha_I)p^*_{I,t} + \alpha_I p_{I,t-1} - p_{I,t}\right] = 1\end{aligned}$$

where the second equality comes from the approximations

$$p_{I,t} = (1-\alpha)p^*_{I,t} + \alpha p_{I,t-1}$$

.

Therefore, a first order approximation to labor income is

$$LI_{I,t} = LI^f\left(\frac{Y_t}{Y_t^f}\right)^{1+\omega+\gamma}\left(\frac{P_{I,t}}{P_t}\right)^{-\theta(1+\omega)} = LI^f e^{(1+\omega+\gamma)x_t + \theta(1+\omega)\varphi_{-I}p_{R,t}}.$$

Define the real output in section I as

$$Y^{real}_{I,t} = \frac{Y_{I,t}P_{I,t}}{P_t} = \varphi_I Y_t\left(\frac{P_{I,t}}{P_t}\right)^{1-\theta}$$

where the second equality is implied by the demand function for section I. Using the first order Taylor expansion, we get

$$Y^{real}_{I,t} = \varphi_I Y^f e^{x_t + (\theta-1)\varphi_{-I}p_{R,t}}\,.$$

Therefore, the real profit in sector I at time t is given by

$$\Psi_{I,t} = Y_{I,t}^{real} - LI_{I,t} = \left(1 - \frac{1}{\mu_{I,t}}\right) Y_{I,t}^{real},$$

where

$$\mu_{I,t} = \mu_t e^{-(1+\theta\omega)\varphi_{-I} p_{R,t}}$$

is the time-varying markup for sector I.

E Affine Framework

In order to understand the implications of the countercyclical markup on stock returns, we can use the affine framework to price claims on consumption, real labor income and real profits (stocks). In particular, we can analyze "one-period" claims which only payoff at some future time $t+n$. Therefore, claims on all future aggregate consumption, labor income and profits can be considered as portfolios of the one-period claims for all n. Let's first look at the case with homogenous price rigidities.

Let $r_{C,t+1}^{(n)}$ be the one-period return of a claim on aggregate consumption at time $t+n$. The expected excess return of this claim over the risk-free rate r_t is

$$\mathbb{E}_t\left[r_{C,t+1}^{(n)} - r_t\right] = -\frac{1}{2}\text{var}_t\left(\Delta x_{t+1} + d_{C,t+1}^{(n-1)}\right) - \text{cov}_t\left(m_{t,t+1}, \Delta x_{t+1} + d_{C,t+1}^{(n-1)}\right),$$

where $d_{C,t+1}^{(n)}$ is the price- consumption ratio associated to the claim with payoff at time n. It can be shown that the covariance term is

$$-\text{cov}_t\left(m_{t,t+1}, \Delta x_{t+1} + d_{C,t+1}^{(n-1)}\right) = \gamma\left[\gamma + (1-\gamma)\phi_u^{n-1}\right]\text{var}_t\left(\Delta x_{t+1}\right).$$

A similar analysis for returns on one-period labor income and profits, $r_{N,t+1}^{(n)}$ and $r_{\Psi,t+1}^{(n)}$, respectively, imply

$$-\text{cov}_t\left(m_{t,t+1}, \Delta li_{t+1} + d_{N,t+1}^{(n-1)}\right) = \gamma\left[\gamma + (1+\omega)\phi_u^{n-1}\right]\text{var}_t\left(\Delta x_{t+1}\right),$$

and

$$-\text{cov}_t\left(m_{t,t+1}, \Delta\psi_{t+1} + d_{\Psi,t+1}^{(n-1)}\right) = \gamma\left[\gamma + (1+\omega-\theta(\omega+\gamma))\phi_u^{n-1}\right]\text{var}_t\left(\Delta x_{t+1}\right).$$

It can be seen from these two equations that, for all maturities n, the expected return on labor income claims is higher than the expected return on profits. The differences in the two expected returns increase as the intertemporal elasticities of consumption and labor increase. This is the result of a countercyclical markup. Stocks are less risky than claims on labor income since a higher fraction of production is paid off as labor income during bad times. In addition, more persistent policy shocks imply higher differences between the two claims.

When the price rigidities are different across industries, let $r^{(1)}_{C,I,t+1}$ be the one-period return of a claim on real consumption at time $t+1$ of the good produced in industry I. The expected excess return of this claim on industry H over a claim on industry L is (up to the Jensen's inequality terms)

$$\begin{aligned}\mathbb{E}_t[r^{(1)}_{C,H,t+1} - r^{(1)}_{C,L,t+1}] &\approx -(1-\theta)\text{cov}_t(m_{t,t+1}, \Delta p_{R,t+1}) \\ &= \gamma(1-\theta)x_u\rho_u\text{var}_t\left(\Delta x_{t+1}\right),\end{aligned}$$

which is positive given the negative correlation between the output gap and the relative price. A claim on consumption in industry H is more risky because during bad times, the high product price in H, in comparison to the product price in L, hurts the demand of H in comparison to L.

The growth in real labor income for industry I can be written in terms of growth in aggregate labor income, Δli_t, and changes in the relative price, as

$$\Delta li_{I,t} = (1+\omega+\gamma)\Delta x_t + \theta(1+\omega)\varphi_{-I}\Delta p_{R,t} = \Delta li_t + \theta(1+\omega)\varphi_{-I}\Delta p_{R,t}.$$

When the product price in industry H is higher than the product price in industry L, the value of labor income in that industry declines. It can be shown that the difference in expected excess returns for claims on one-period real labor income in the two industries is

$$\begin{aligned}\mathbb{E}_t[r^{(1)}_{N,H,t+1} - r^{(1)}_{N,L,t+1}] &\approx -\text{cov}_t(m_{t,t+1}, \Delta li_{H,t+1} - \Delta li_{L,t+1}) \\ &= -\gamma\theta(1+\omega)x_u\rho_u\text{var}_t\left(\Delta x_{t+1}\right).\end{aligned}$$

This expected excess return is positive. Workers in industry H demand a higher return in their labor income because, during bad times, markups are higher in this industry and the fraction of production that they obtain is lower than the fraction obtained by workers in L.

Finally, growth in real profits in industry I can be written in terms of growth in aggregate

profits, $\Delta\psi_t$ and changes in the relative price, as

$$\Delta\psi_{I,t} = \Delta\psi_t + \varphi_{-I}(1-\theta)\theta\omega\Delta p_{R,t}.$$

When the relative price increases, the growth in real profits in the industry with more rigid product price is larger than in the industry with the more flexible price. Expected excess returns between real profits in the two industries are

$$\begin{aligned}\mathbb{E}_t[r^{(1)}_{\Psi,H,t+1} - r^{(1)}_{\Psi,L,t+1}] &\approx -\text{cov}_t(m_{t,t+1}, \Delta\psi_{H,t+1} - \Delta\psi_{L,t+1}) \\ &= -\gamma(1-\theta)\theta\omega\text{cov}_t(\Delta x_{t+1}, \Delta p_{R,t+1}),\end{aligned}$$

which is negative, given the negative correlation between output gap and relative prices in equilibrium. The expected excess returns on real profits in L are higher than those in H because the markup in L is lower than the markup in H during bad times, that is, profits in industry L tend to decline more than profits in industry H during bad times.

Notice that the changes in the relative price can also be written in terms of industry inflations as

$$\Delta p_{R,t} = \pi_{H,t} - \pi_{L,t}.$$

It follows that compensations for risk in one industry are higher than in the other one as long as inflation in that industry covaries more with aggregate consumption than inflation in the other industry. It can be shown using equation (12) that inflation in the industry with low price rigidity is more sensitive to the aggregate output gap than inflation in the industry with high price rigidity.[5] Intuitively, inflationary shocks have larger negative effects on the profits of the industry with low price rigidities and, as a result, economic agents demand high compensations for claims on these profits.

F Numerical Solutions

We solve equation (29) for the two industries on a set of grid points of state variables $(p_{R,t}, u_t)$ using value function iteration. The unconditional distributions of relative price $p_{R,t}$ and policy

[5] Appendix C shows the equilibrium process for inflation in the two industries.

shock u_t are normal with

$$\begin{aligned}
\mathbb{E}(u_t) &= 0 \\
var(u_t) &= \frac{\sigma_u^2}{1-\phi_u^2} \\
\mathbb{E}(p_{R,t}) &= \frac{\bar{\rho}}{1-\rho_p} \\
var(p_{R,t}) &= \frac{\rho_u^2(1+\rho_p\phi_u)}{(1-\rho_p^2)(1-\rho_p\phi_u)} var(u_t)\,.
\end{aligned}$$

We choose Np grid points in the range of $[-3var(p_{R,t})^{1/2},\, 3var(p_{R,t})^{1/2}]$ and choose Nu grid points in the range of $[-3var(u_t)^{1/2},\, 3var(u_t)^{1/2}]$. Let's name the grid points as $\{p_i\}_{i=1,\ldots,Np}$ and $\{p_j\}_{j=1,\ldots,Nu}$. We then calculate the real values of high and low rigidity industries at these grid points as follows.

1. Make an initial guess for the value of high rigidity industry, $V_H^0(p_i, u_j)$.

2. Given the equilibrium processes for the relative price and the policy shock, we know the possible values of next period state variables (p', u') with the corresponding probabilities. Therefore, we can calculate the right hand side of equation (29) and update the value function as follows:

$$V_H^1(p_i, u_j) = \frac{\varphi Y^f \exp(x(p_i, u_j))}{\varphi + (1-\varphi)\exp[(\theta-1)p_i]} + \mathbb{E}_t\left[M(p', u')V_H^0(p', u')\,.\right]$$

3. Calculate the difference between V_H^0 and V_H^1 at every grid point. If the maximum of the differences is larger than a pre-decided criterion, then go back to step 2 to get the next iterated value V_H^2 using V_H^1; if not, we have just found the value for the high rigidity industry.

4. Repeat step 1-3 for the value of low rigidity industry V_L at the same set of grid points. The real value of the industry with low price rigidity as

$$V_L(p_i, u_j) = \frac{\varphi Y^f \exp(x(p_i, u_j))}{\varphi \exp[(1-\theta)p_{R,t}] + (1-\varphi)} + \mathbb{E}_t\left[M_(p', u')V_L^0(p', u')\right]\,.$$