Working papers economics - Stationary Density Estimation for ARCH Models with Applications in Quantile Regression

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AUTHORS AND/OR EDITORS Hernández-Bejarano, Manuel Darío Zhao, Zhibiao
Can more accurate estimates be obtained from complex economic and financial data? This study proposes a new statistical tool that improves upon the accuracy of traditional methods and demonstrates its usefulness through applications to data from the S&P 500 Index and monthly U.S. inflation.

Publication Date: Friday, 20 of November 2026 **Approach**
This paper presents a new, more efficient statistical method for estimating the stationary density of nonparametric autoregressive conditional heteroskedastic models, a method useful for data related to economic and financial phenomena characterized by time-varying volatility. The study shows that this method yields more efficient results than one of the most widely used techniques in practice, the nonparametric kernel estimator. Numerical exercises further confirm its superior performance.

The methodology is applied to data from the S&P 500 stock market index and is also used in quantile regression models, a tool commonly employed to analyze different patterns of economic variables. In particular, the new method enables more accurate estimation of the uncertainty associated with the results of these models. To illustrate its usefulness, it is applied to the analysis of monthly inflation in the United States.

Contribution
The study’s main contribution is the development of a method that improves the estimation of data distributions in complex economic and financial models. Compared with traditional approaches, the proposed method offers several advantages.

First, its accuracy increases as more information becomes available. Second, it produces smoother and more stable estimates, facilitating its use in statistical and economic applications. Finally, the study shows how this tool can be used to obtain more accurate measures of the uncertainty associated with estimated parameters in quantile regression models.

Simulation exercises confirm that the proposed method outperforms the traditional estimator in terms of accuracy.

Results

The results show that the proposed method estimates the data distribution more efficiently than traditional approaches. Moreover, this advantage becomes more pronounced as the number of observations increases. Numerical exercises and simulations support these findings and demonstrate that the new method also provides more accurate estimates of the uncertainty surrounding results obtained from quantile regression models. This represents an improvement over the statistical procedures commonly used in the specialized literature.

Fuente: https://www.banrep.gov.co/en/publications-research/working-papers-economics/stationary-density-estimation-arch