Temporary and Permanent Components of Colombia´s Output

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Abstract

Structural time series models, frequency domain analysis, the HP-filter, and the Blanchard-Quah decomposition, are used to observe, some peculiarities of the business cycle. Such properties are those related to the volatility of the temporary component and the duration of the business cycle during both 1925-1994 and 1950-1994. For the longer period we find that cycles between three and six years seem to be the most important for the variability of output; volatility is greater for GDP than for per capita GDP, except when the processes are linearly detrended. For period 1950-1994, although the linear trend plus cycle model does not perform very well, cycles of about eight years seem to be most important for the cycle. The results of the Blanchard and Quah decomposition show that demand shocks have important explanatory attributes for output fluctuations. However, supply shocks, are dominant in the behaviour of output.

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Keywords: cycles, structural time-series analysis, frequency-domain, Blanchard-Quah decomposition.

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1. Introduction

The goal of this paper is to obtain some evidence on the nature of the temporary and permanent component of Colombian output. Since, a priori, there is no preferred method, we use a variety of techniques in a complementary way, in order to capture the peculiarities of the business cycle in Colombia, a small export commodity-dependent economy. The properties in which we are interested, are those related to the volatility of the temporary component and the duration of the business cycle between 1925 and 1994, with special emphasis in the period after 1950. In a *bi-variate* environment, output growth is decomposed to observe the quality of its reaction when facing either a nominal or real shock.

Given the cyclical pattern suggested by the correlogram of the series, the first method we examine is the structural time series representation due to Harvey [1985, 1993] in which, as main feature, the parameters are time varying and have a direct interpretation. Second, we consider the Hodrick-Prescott filter brought to the context of output fluctuations by Kydland and Prescott [1982] and Prescott [1986] and widely used in real business cycles models when computing the stylized facts of the macroeconomic variables. Third, some basic spectral techniques are employed to show the relative importance of different frequencies in the explanation of the variance of output. The results of the spectral analysis relate to the first difference, the linearly-detrended and the Hodrick-Prescott filtered output. Finally, we depart from the univariate framework to consider a decomposition based on the vector autoregressive method proposed by Blanchard and Quah [1989], motivated with a small-macro model used to investigate demand (money) shocks and supply (technology) shocks.

Analysis of the components of Colombia's output has been previously undertaken using a different approach. Clavijo [1992] and Cuddington and Urzua [1989], for example, use the Beveridge and Nelson [1981] decomposition to estimate the permanent and temporary components of GDP. However, given that this decomposition is not unique, it is not used it in this work.

This paper consist of five sections, the first of which is this introduction. In the second section, unobserved components models, are used to examine the structural components in the evolution of output, as well as, the Hodrick and Prescott [1980] filter. In the third section, we use some basics of spectral analysis to find the hidden cycles of output behaviour between 1925 and 1994. In this case we use some frequency domain concepts with clear interpretation in structural time series models. In the fourth section, the temporary and permanent components problem is considered in a *bi*-variate framework, following the method put forward by Blanchard and Quah [1989]. The fifth section presents some conclusions.

2. Decomposing Output Time Series

Suppose that, in a univariate framework, output can be decomposed into two components: trend (permanent) and stationary (temporary) components. Thus, we may write:

$$Y_t = m_t + e_t \tag{1}$$

where Y_t, m_t, and e_t are the observed output, trend and stationary components, respectively. Different assumptions have been used with respect to the statistical properties and the implied representations of the components of Y_t in (1). For example, to derive the properties of forecasts, Muth [1960] assumed a trend component represented by a random walk:

$$m_t = m_{t-1} + u_t \qquad (2)$$

where the white noise u_t is uncorrelated with the temporary component e_t, also assumed white noise; that is $e_t \sim$ $i.d.(0, s_e^2)$, $u_t \sim i.d.(0, s_u^2)$, and $E(e_t, u_{t-i})=0$, for all *i*. Accordingly, the first difference of Y_t is stationary:

$$\Delta Y_t = u_t + e_t - e_{t-1} \qquad (3)$$

In (3) there is only one observable realisation ΔY_t and two intrinsic indivisible or *unobservable* realisations, u_t and e_t, each with identified variances. The reason for this is that the variance of ΔY_t, $s_{\Delta Y}^2$, is $s_u^2 + 2s_e^2$ while the first lag autocovariance is equal to s_e^2, so that we can obtain s_u^2 as $s_{\Delta Y}^2 - 2s_e^2$. With this information the components of (2) are recoverable (see Enders [1995]). This will be the case as long as the trend component - m_t - is a random walk and u_t and e_{t-i} are (for all *i*) not correlated.

Beveridge and Nelson [1981] assume that u_t and e_t are perfectly correlated. They show that a sequence with an ARIMA (*p*,*1*,*q*) representation contains a random walk stochastic trend and that such a sequence can be decomposed into a stochastic trend plus a stationary component. To illustrate the Beveridge-Nelson decomposition for a general ARIMA (*p*,1,*q*) model we may write:

$$f(L)\Delta Y_t = q_0 + q(L)a_t \qquad (4)$$

where, $f(L)=1- f_1 L - ... - f_p L^p$ and $q(L)=1-q_1 L - ... - q_q L^q$. This approach assumes that u_t and e_t from (1) and (2), respectively, are linearly combined into a_t in (4). Solving for ΔY_t, we have:

$$\Delta Y_t = f(L)^{-1} q_0 + f(L)^{-1} q(L)a_t = a + y(L)a_t \qquad (5)$$

while the corresponding expression for Y_t is:

$$Y_t = Y_{t-1} + a + y(L)a_t \qquad (6)$$

If lagged Y_{t-1} is recursively substituted out, and it is assumed that $Y_0=0$ and $a_j=0$, for $j £ 0$, we obtain:

$$Y_t \;=\; at \;+\; \sum_{j=1}^{t} y\;(L)a_j \qquad (7)$$

which may be re-expressed as:

$$Y_t \;=\; at \;+\; y\;(L)a_t + \sum_{j=1}^{t-1} y\;(L)a_j \qquad (8)$$

The first two terms on the right-hand side correspond to the permanent component (m_t, which is a random walk with drift), while the third term identifies the stationary component. Expression (2) can be also written as:

$$Y_t \;= a + m_{t-1} + y\;(L)(1-L)a_t + \sum_{j=1}^{t-1} y\;(L)a_j \qquad (9)$$

which provides the Beveridge-Nelson decomposition of Y_t which is represented by a general ARIMA (*p,1,q*)[‡]. Note that in (9) the sequence a_t involves the white noise sequences u_t and e_t in the case above, which, as a result, appear to be perfectly correlated, since both vary with a_t. However, there is no reason to believe *a-priori* that innovations of the permanent and temporary components are perfectly correlated. The correlation between them can lie between -1 and 1. Without such a knowledge, the decomposition of any sequence into a random walk plus drift and a stationary component is not unique. Cuddington and Urzua [1989] use the computational method suggested by Cuddington and Winters [1987] to perform the Beveridge-Nelson decomposition of the logarithm of Colombian *GDP* between 1930 and 1985. They conclude that under this decomposition, the cycles have shorter duration and less amplitude than in the case of a *linearly detrended* sequence[§].

Different assumptions about m_t and e_t, other than Beveridge-Nelson's perfect correlation have been introduced to model unobserved components[**]. For example, to decompose the logarithm of real US *GDP*, Watson [1986] assumes that the stationary component is AR(2) and the innovations u_t and e_t are uncorrelated, which seems to be more sensible. Harvey [1985, 1989] and Clark [1987], on the other hand, use the Kalman filter, within a state space framework, to set up unobserved components models. These models are called structural time series models, which we consider next.

2.1. Structural Time Series Modelling

[‡] *For a discussion of the Beveridge-Nelson decomposition, see also Stock and Watson [1989], Hamilton [1994, p 504] and Enders [1995, p 186], among others.*

[§] *See also Clavijo [1992] for a Beveridge-Nelson decomposition of Colombia´s output.*

Under structural time series modelling the components have a direct interpretation, explanatory variables are functions of time and the parameters are time varying [Harvey, 1993]. Within this context, the statistical formulation defined by (1) and (2) is referred to as the *local level* (or *signal plus noise*) model from which we can compute the *signal-to-noise ratio* s_u^2 / s_e^2. A *local linear trend* model can be obtained by considering (1) together with:

$$m_t \;=\; m_{t-1} + \; b_{t-1} \;+\; u_t \qquad (10a)$$

$$b_t \;=\; b_{t-1} \;+\; x_t \qquad (10b)$$

where u_t and x_t are white noise disturbances mutually uncorrelated with zero means and variances s_u^2 and s_x^2, respectively. The error u_t affects the level of the stochastic trend, while x_t affects the slope of it. Hence, when $s_u^2 = s_x^2 =0$, m_t reduces to a deterministic linear trend. When $s_u^2 =0$ but $s_x^2 >0$, the trend is an I(2) process, which is relatively smooth. For annual economic time series, as we have here, the traditional formulation extends (1) by introducing a cycle term, y_t. Thus we have:

$$Y_t \;=\; m_t \;+\; y_t \;+\; e_t \qquad (11)$$

where the stochastic linear trend, m_t, remains as in (10), and e_t is white noise uncorrelated with x_t and u_t for all t. A deterministic cycle is a sine-cosine wave with a given *period.* A stochastic cycle results when a deterministic one is shocked with disturbances and the *damping* factor is added. The statistical specification of the stochastic cycle y_t takes the form:

$$\begin{bmatrix} y_t \\ y_t^* \end{bmatrix} = r \begin{bmatrix} \cos w & \sin w \\ -\sin w & \cos w \end{bmatrix} \begin{bmatrix} y_{t-1} \\ y_{t-1}^* \end{bmatrix} + \begin{bmatrix} k_t \\ k_t^* \end{bmatrix} \qquad (12)$$

while 0 £ w £ p, is the *frequency* in radians, κ_t and κ_t^* are white noise uncorrelated disturbances with zero means and variances $s_k^2 (= s_{k^*}^2)$; y_t^* appears by construction in order to form y_t; and $0 \leq r \leq 1$ is a *damping* factor of the amplitude. The disturbance makes the cycle stochastic rather than deterministic, and the cycle will be stationary if $r <1$.

The stochastic process (12) becomes a first-order autoregressive process if w is 0 or p; this arises because $\sin w$ is zero when $w=0$ or $w=p$. As a result, the equation generating y^* is redundant. The first equation of the system described by (12) becomes $y_t = r y_{t-1} + k_t$ when $w =0$ or $y_t =- r y_{t-1} + k_t$ when $w =p$.

** *Stock and Watson [1989] present a survey of methods of decomposing macroeconomic variables and the most relevant assumptions that such methods embody.*

By using the lag operator, *L*, so that $X_{t-d} = L^d X_t$, the cyclical component (12) can be written as:

$$\begin{bmatrix} y_t \\ y_t^* \end{bmatrix} = \begin{bmatrix} 1 - r \cos wL & - r \sin wL \\ r \sin wL & 1 - r \cos wL \end{bmatrix}^{-1} \begin{bmatrix} k_t \\ k_t^* \end{bmatrix} \qquad (13)$$

The solution for y_t is:

$$y_t = \frac{(1 - r \cos w.L)\ k_t + (r \sin w.L)\ k_t^*}{1 - 2r \cos w.L + r^2.L^2} \qquad (14)$$

and after substituting (10) and (14) in (11), the following expression for Y_t is obtained[††]:

$$Y_t = \frac{u_t}{(1-L)} + \frac{(1 - r \cos w.L)\ k_{t-1} + (r \sin w.L)\ k_{t-1}^*}{[1 - 2r \cos w.L + r^2.L^2]} + \frac{x_t}{(1-L)^2} + e_t \qquad (15)$$

The model in (15) is known as the *trend plus cycle model*[‡‡], which is estimated in the time domain framework by using the Kalman Filter[§§]. Consequently, the models are set into state-space form, where the state vector is $a_t = (m_t, b_t, y_t, y_t^*)'$. To initiate the Kalman filter, the mean square errors of m_t and b_t are set equal to large but finite numbers, while the mean squared error matrix of $(y_t, y_t^*)'$ is set equal to the unconditional covariance matrix of $(y_t, y_t^*)'$. The likelihood function is maximised (numerically) with respect to s_e^2, s_u^2, s_x^2, s_k^2, *w*, and *r*.

The *measurement equation* of the *trend plus cycle model* required by the *state space* formulation can be written as:

$$Y_t = [1 \quad 0 \quad 1 \quad 0]\ a_t + e_t \qquad (16)$$

while the *transition matrix* may be expressed as:

[††] *ARIMA models can be interpreted as reduced forms of structural time series model. These involve several disturbance terms which are combined in the ARIMA models which currently have only a single disturbance term [Harvey, 1993]. An example of this occurs in the previous section when we move from Muth [1960] to Beveridge and Nelson [1981] decompositions.*

[‡‡] *An alternative model used by Harvey [1985] is the cyclical trend model. This model is specified by carrying the cyclical component from (13) to (10a). Thus, we would have* $Y_t = m_t + e_t$ *where* $m_t = m_{t-1} + b_{t-1} + y_{t-1} + u_t$.

[§§] *Appendix 1 to this work contains a sketch of the Kalman Filter and the state space form of the particular problem here.*

$$a_t = \begin{bmatrix} m_t \\ b_t \\ y_t \\ y_t^* \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & r\cos w & r\sin w \\ 0 & 0 & -r\sin w & r\cos w \end{bmatrix} \begin{bmatrix} m_{t-1} \\ b_{t-1} \\ y_{t-1} \\ y_{t-1}^* \end{bmatrix} + \begin{bmatrix} u_t \\ x_t \\ k_t \\ k_t^* \end{bmatrix} \qquad (17)$$

The covariance matrix of the vector of disturbances in (17) is a diagonal matrix with elements { s_u^2, s_x^2, s_k^2, s_k^2 }. The model is observable (identifiable) unless r is zero or w is either zero or p. The condition that s_x^2 and s_k^2 be strictly positive is necessary for stability (see Harvey [1989]). Diagnostic checking tests can be carried out by using the *Ljung-Box Q-statistic*, the *H-statistic*, and the R_D^2 measure of goodness of fit. The *Q-statistic* is constructed as:

$$Q \;=\; T^*(T^*+2)\sum_{t=1}^{p}(T^*-t)^{-1}r^2(t) \qquad (18)$$

where T^* is the number of residuals (usually *T-2*) and *r(t)* is the t-th autocorrelation in the residuals. Under the null hypothesis, the *Q-statistic* follows a c^2 distribution with *p-(n-1)* degrees of freedom, where *n* is the number of parameters. The *H(m)-statistic* is a test for heteroscedasticity, which is constructed as:

$$H(m) \;=\; \frac{\sum_{t=T-m+1}^{T} e_t^2}{\sum_{t=k+1}^{m+k} e_t^2} \qquad (19)$$

where $m=T^*/3$ or the nearest integer, $k=T-T^*$. The *H(m)*-statistic is the ratio of the sum of squares of the last *m* residuals to the sum of squares of the first *m* residuals. This statistic is centred around unity and should be treated as having an *F* distribution with *(m,m)* degrees of freedom. A high (low) value indicates an increase (decrease) in the variance over time [Koopman et.al, 1995].

Finally, the goodness of fit is carried out with the coefficient R_D^2 which compares the residual sum of squares with the sum of squares of the first differenced observations about their mean. Hence, we may write:

$$R_D^2 \;=\; 1 - \frac{T^* s_e^2}{\sum_{t=2}^{T}(\Delta Y_t - \overline{\Delta Y})^2} \qquad (20)$$

The sample autocorrelations of *DGDP* and *DGDPP*C between 1950-1994 and 1925-1994, shown in figures 1.a -1.d, do not clearly suggest a white noise process for any series[***]. Instead, because of the wave-shaped behaviour, the correlograms indicate a cyclical pattern: this is the most important criterion employed to decide whether a structural time series decomposition should be used or not. Therefore, it allows us to carry out a structural decomposition of output by using (15) [Harvey, 1985]. The corresponding estimates of the model (15) and the local linear trend model -with no trigonometric cycle at all[†††]- appear in table 1. The results are not, however, auspicious. Except, for the case of *GDP* (1950-1994), the estimates in the model with no trigonometric cycle (local linear trend) present a better goodness of fit (R_D^2) than the estimates of (15). In addition, the estimates of *Y* are, in all cases, close to unity, which indicates that either there is high persistence or the model may be inappropriate since the variable could be still nonstationary. In the case of *GDP* (1950-1994), the trend plus cycle model suggests a period near to eight years. The rest of the results suggest a deterministic cycle between 5 and 9 years, but the diagnostic statistics suggest that these models may not be very reliable. Only in the case of *GDP* (1950-94) the *Q-statistic* is not significant and some evidence of heteroscedasticity is found in the residuals for the longer sample period in both *GDP* and *GDPPC.*

Needless to say that we could not extract an obvious conclusion from the results of structural time series in terms of business cycles since no neat deterministic cycle arises according to the size of r and stochastic movements are important only in the case of *GDP* (1950-1994), according to s_k^2. Among the models in table 1, the linear trend plus cycle model for *GDP* (1950-1994) presents some attractions in R_D^2, the standard error, and the estimated variances of the residuals, specially s_x^2.

2.2. The Hodrick-Prescott Decomposition

There has been increasing use of the Hodrick-Prescott[‡‡‡] (HP) filter for trend removal of the aggregate economic series to the study of *stylized facts* in the context of business cycles. The key feature of the HP filter is in defining the *trend* by the computational procedure used to fit the smooth sequence such that its key business cycles facts are kept. Explicitly, the criteria used by Kydland and Prescott [1990] to select the trend component -that results from the HP-filter are that: *i)* it is that curve which would be drawn by hand to fit the sequence; *ii)* it has to result from a linear transformation of the sequence which has to be standard for all the sequences; *iii)* it has to be

[***] *The data sources are: Easterly [1994] for 1925 - 1929; Cuddington and Urzúa [1989] for 1930-1949; Principales Indicadores Económicos 1923 - 1992. Banco de la República for 1950-1992; and Revista Banco de la República different issues for 1993 1994.*

[†††] *The model set in this way (with no sinusoidal cycle) is similar to that used by Clark [1987] to study the behaviour of US industrial production and gross national product using quarterly data. The cyclical component in Clark [1987] is assumed to be an AR(q) process.*

[‡‡‡] *The method is also called Whittaker-Henderson Type A method (see Hodrick and Prescott, [1980] and Prescott [1986]). Applications of the filter are found throughout, e.g. Kydland and Prescott [1990], Kim, Buckle and Hall [1995].*

simple to compute, free of judgements and reproducible for all the sequences; and *iv)* small changes in the sample size should not produce considerable changes in the values of the components.

The HP filter is a *low-frequencies suppressor* which results from minimizing the squared deviations of a trend component, t_t, from a sequence, $\{ Y_t \}$, subject to the constraint that the sum of the squared first difference of the growth rate should not be too large. That is:

$$\min_{\{t_t\}_{t=1}^{T}} \quad \sum_{t=1}^{T} (Y_t - t_t)^2 \tag{21}$$

subject to

$$\sum_{t=2}^{T-1} [(t_{t+1} - t_t) - (t_t - t_{t-1})]^2 \leq m \tag{22}$$

where Y_t is the logarithm of the (raw) variable. In general, a smooth trend results when the value of m is small.

The growth rate of the trend component [$(t_t - t_{t-1})$, for $t=1,..T$] is assumed to vary smoothly over time while the deviations from t_t are assumed to have zero (or near to zero) mean. The term $(Y_t - t_t)$, (a T´1 vector that we label below y ,) can be interpreted as the cyclical component, while the term [$(t_{t+1} - t_t)$-$(t_t - t_{t-1})$] can be interpreted as the change in the growth rate of the trend component[§§§]. The operability of the HP is straightforward. In matrix form, the problem stated in (21)-(22) can be rewritten as $min\{ t_t \}\ y\ 'y\ + l\,(K_t)'K_t$, where l is the Lagrange multiplier, K is a T´T matrix and t is a T´1 vector. The product K_t (=[$(t_{t+1} - t_t)$-$(t_t - t_{t-1})$]) is written as:

$$K\ t = \begin{bmatrix} 1 & -2 & 1 & 0 & 0 \ldots 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 \ldots 0 & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & & \\ \ldots & \ldots & \ldots & \ldots & \ldots & & \\ 0 & 0 & 0 & 0 & 0 \ldots 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \\ . \\ . \\ t_T \end{bmatrix} \tag{23}$$

The solution to the minimization problem is found for $t_t = A^{-1} Y_t$, where $A=I+ lK'K$ and I is the identity matrix (see Danthine and Girardin [1989]). The Lagrange multiplier l relates the variance of the cyclical component to the variance of the (second difference of) trend component. In other words, the parameter l signals the importance attributed to the fit of $\{ t_t \}$ with respect to the smoothness of $\{ t_t \}$. When l is high the HP filter operates as a linear time trend and persistence increases. This occurs since l penalizes the variations in the growth rate of the trend component $\{ t_t \}$. Put another way, when we choose a high l we are choosing a smooth t_t from (21)-(22); when we choose λ to be zero, $Y_t = t_t$ and the cycle is zero for all t.

[§§§] *These terms are interpreted as the fit of* $\{ t_t \}$ *and the smoothness of* $\{ t_t \}$*, respectively.*

The choice of the smoothing parameter (*m*) is the main drawback of the HP filter. If the cyclical component and the change in the growth rate of the trend component were identically and independent normally distributed with mean zero and variances s_1^2 and s_2^2 respectively, the problem: $min\{t_t\}\ s_1^{-2}(Y_t - t_t)^2\ s_2^{-2}$ $[(t_{t+1} - t_t)\text{-}(t_t - t_{t-1})]^2$ has the same solution as that of (21)-(22). Thus, $\sqrt{l} = (\sqrt{s_1} / \sqrt{s_2})$. However, these components are not normally distributed. Hence, Hodrick and Prescott [1980] picked a value of *m* so that the Lagrange multiplier of the constraint is 1,600 by arguing that a 5% of the deviation from the trend per quarter is moderately large as is a one-eighth of one percent change in the growth rate in a quarter. For annual data, the parameter *l* is set to 400[****].

The panels of figure 2 show the trend obtained by applying the HP procedure to *GDP* and *GDPPC*, along with the trend in the limiting case (*l® large or m® 0*): the linear trend. The panels in figure 3 show the HP detrended *GDP* and per capita *GDP* and contrast them with linearly and differenced detrended series. Regardless that fluctuations are less pronounced in the case of the HP filtered series, these follow the linearly detrended series. However, Singleton [1988] shows some evidence according to which HP filtering has similar effects to differencing the data since both detrending methods give essentially zero weight to the very low frequencies while amplifying the power spectrum at high frequencies[††††]. Nevertheless, we could add that such a similarity depends on the values selected for the Lagrange multiplier *l*: the smaller the value of the Lagrange multiplier the higher the similarity between HP and first-difference filtering[‡‡‡‡]. According to Singleton [1988], an implication of filtering the data is that stylized facts, used to characterize business cycles, could be distorted. King and Rebelo [1993] reinforce this view by pointing out that the HP filter could remove important time series components that have traditionally been regarded as representing business cycle phenomena. However, in the univarite context of this work, it is not appealing to verify this hypothesis, since King and Rebelo refer basically to cross-correlations with other variables typically used in business cycles analysis. In the case of *GDP* and *GDPPC* of Colombia, the standard deviations for the HP filtered series are 2.62% and 2.77%, respectively. The same statistics for the linearly detrended and differenced series are: 5.40% and 4.12%; and 2.41% and 2.52%, respectively, for the period 1925-1994. These estimates suggest that some caution should be taken when considering the volatility of output since *GDPPC* appears more volatile than *GDP* but only when the processes are linearly detrended.

Apart from the intuitiveness of the definition of trend, the HP filter has the ability to render stationarity even to series integrated up to the fourth order [Danthine and Donaldson, 1993]. However, King and Rebelo [1993]

[****] *The preceding arguments about l or m are set by Harvey and Jaeger [1993] in terms of the structural time series representation of the previous section. According to them, the HP filtering is equivalent to imposing the restrictions* $s_u^2=0$, $y_t=0$, *and* $s_e^2 / s_x^2 = l$ *in models (10) and (11). Thus, the cycle is explained by* e_t, b_t, *and* x_t.

[††††] *Prescott [1986] interprets the HP decomposition as a high pass linear filter which resolves in a better way, than conventional spectral filters, the problem of the end of the sample produced by the sharp cut-off (i.e. no tapering is required). See next section.*

[‡‡‡‡] *This result is not shown.*

demonstrated that under some conditions the HP filter can be an optimal filter for a second order integrated process. Cogley and Nason [1995a] showed that filtering the data could introduce, spuriously, behaviour of business cycles even to random walks. On the other hand, Harvey and Jaeger [1993] have criticised the HP procedure since only irregulars (e_t, in (11)) are reported as cycles under this filter. Accordingly, a model such as that described by (11) and (10), which also contains y_t should be considered. However, in the previous section, we found no strong support for this model during 1925-1994 and, in the case of *GDP*, only irregular fluctuations of the trend component ($s_u^2 > 0$) were detected by that procedure. Therefore, the cyclical component computed through the HP filtering could be more valid.

3. Some Frequency-Domain Peculiarities of Output

The frequency-domain approach is a technique both complementary and competitive to the time-domain approach used to analyse the behaviour of the variables. Spectral methods provide a natural mathematical approach to the mixture of regularity and nonregularity exhibited by business cycles, in particular to the lack of periodicity of fluctuations [Granger and Hatanaka, 1964]. It is well known that the results of a spectral analysis will be much more reliable the larger the sample size mainly when the analysis in focused on low-frequency variations. Here we shall use a sample of seventy data points (1925-1994) which is not as large as frequency-domain analysts demand, although it surpasses Granger and Hatanaka's recommendation of using data of at least seven times the length of the largest cycle to study when the variable has no trend in mean [Granger and Hatanaka, 1964, p. 17-18]. Regardless that our interest here is in cycles of less than seven years, (since we attribute cycles of greater duration to growth phenomena rather than to business cycles), we shall use the technique to find the more relevant cycles in the behaviour of Colombia's output. In other words, we shall use this approach to trace the 'hidden periodicities' of the sequence of output fluctuations in the last seventy years.

The information we are looking for is contained in the *(power) spectra density function*[§§§§], *S(w)*, which in strict sense shows the variance of Y_t, decomposed into variance attributed to different frequencies (Englund, et.al [1992]). The spectral density function is estimated through the *periodogram*, a diagram which show peaks at points corresponding to the hidden periods of a process. For the mean-subtracted sequence, $(Y_t - \overline{Y})$, the periodogram or *sample spectral density function*, can be computed as:

$$I_{n,Y_t-\bar{Y}}(q) = \frac{1}{2pn} \left| \sum_{t=1}^{n} (Y_t - \overline{Y}) e^{-iqt} \right|^2 \qquad (24)$$

§§§§ *The power spectra density function is the Fourier transform of the autocovariance function, R(k), of the sequence* Y_t*. See Priestley [1981, p. 211] for a formal proof. Sargent [1979, p. 233] deals with this equivalence. The standardised version of the power spectra density function is called the power spectrum. Hence, the area under it is equal to one (see Harvey [1993, p. 167]).*

where *n* is the sample and *q* is a vector of parameters. Note that (24) involves *n* sample *autocovariances.*

The periodogram is asymptotically unbiased; however, in practice we obtain a biased periodogram since the sample size is finite. In addition, it is not a consistent estimate of the spectral density function since *i)* the variance of $I_{n,Yt-\bar{Y}}(q)$ does not tend to zero as the data points increase[*****], and *ii)* the covariance between estimates at different but adjacent frequencies (w_1, w_2), i.e. *Cov [*$I_{n,Yt-\bar{Y}}(q_1), I_{n,Yt-\bar{Y}}(q_2)$*]*, decreases as *n* becomes large (*n* ® ¥), hence the probability of deciding in favour of a spurious cycle is higher. Thus, a smoothing mechanism is implemented to reduce such *erratic* and *wild* fluctuations of $I_{n,Yt-\bar{Y}}(q)$, and to generate a consistent estimator of the *spectrum S(w).* Such an estimator, $S^*(w)$, may be written as:

$$S^*_k(w) \;=\; \frac{1}{2p} \sum_{s=-n+1}^{n-1} k_n(s)\; R_{(s)}\;\cos\, sw \qquad (25)$$

where $k_n(s)$ is the *lag window generator* and $R_{(s)}$ represents the *autocovariance function.* The function $k_n(s)$ is a sequence of "weights" gradually decreasing, originated as a function of *M (< n)*, which is the *window parameter* or *truncation point*[†††††]. However, the smoothing procedure introduces some additional bias which has to be traded off with the gain in consistency. Equivalently, $S^*_k(w)$, can be written as[‡‡‡‡‡]:

$$S^*_k(w) \;=\; \int_{-p}^{p} I^*_{n,y_t-\bar{y}}(q)\, W_n(w-q)\,dq \qquad (26)$$

The *spectral window,* $W_n(w-q)$, is the *Fourier transform* of *K(s)* which takes its maximum when *q = 0.* Given *w*, $S^*_k(w)$ can be interpreted as a weighted average of periodogram-type estimates at frequencies centred on *w* [Granger and Newbold, 1986]. Its diagram shows the decomposition of the series's variance into variances attributed to different frequencies of such series. A band of frequencies is considered important if it contributes a high proportion to the variance of the series. The estimates of the spectral density can exhibit bias because of the presence of one or more large peaks (*side lobes or window leakage*) in the underlying spectral density (Granger [1966]). This fact could be explained by the high correlation between neighbour values of the data in levels, mainly when the frequency of the data tends to be higher. However, such bias in spectral estimates can be reduced by

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[*****] *The sample spectral density function is not an ergodic process.*

[†††††], *There is no agreement about which criterion to use to select the truncation point, s: it can be the same (lag) at which there is a cut off in the autocorrelation function or it can be a fixed proportion of n or a window closing process. Different expressions for* $k_n(s)$ *have been proposed. Here we use the Bartlett window [*$k_n(s)=1-|s|/M$ *if M³|s|, or 0 if |s|>M]. For a complete set of windows used see Priestley [1981].*

[‡‡‡‡‡] *In practice, however, the expansion of any process is done by means of the "fast Fourier transform" algorithm that computes the Fourier transforms and permits computation of the periodogram* $I_{n,Y}(w)$ *and the spectrum* $S^*(w)$ *directly from the data and does not need to compute (M-1) autocovariances as in (25), (Priestley [1981,p. 575]).*

prewhitening the data through a mechanism called a *taper*[§§§§§] (*fader* or *data window*). A second source of bias depends on the *bandwith* (the interval or distance between the frequencies) adopted in the estimation of the spectrum. The possibility of a biased estimator is reduced when the bandwith is chosen to be sufficiently small. The result is a higher resolution of the estimate.

3.1. Data Preparation and Results

We regard the natural logarithm of the sequences of *GDP* and *GDPPC* as nonstationary. For the sake of comparison, we assume that stationarity could be reached by detrending the sequences in the two usual ways: by differencing the data set, and by removing a linear trend. In the first case, the mean is subtracted after differencing the series. Hodrick-Prescott (HP) filtered series (with λ=400) are also included into the analysis. Thus, the frequency-domain techniques are used on six time series, each zero-mean. The series are transformed into complex numbers; we use a *trapezoidal taper* to smooth the cut off produced by the end of the series with the aim of reducing the bias produced by the *window leakage*; the series, without trend, are also padded (added with zeros) from 69 (or 70) up to 128 (the closest power-of-two number) to apply the *fast Fourier transform* algorithm more efficiently; finally, the tent-shaped *Bartlett window* of size nine (*M=9*), with a truncation point in the lag window of four (*s=4*), has been implemented to increase the consistency of the estimated spectrum[******]. Frequency is converted from the rank [0,π] to the rank [0,34] expressed into years, so as to have a frequency up to 34 cycles of a period of two years each during the sample period.

According to the results, the estimated power spectrum of *GDP* and *GDPPC* under the three detrending procedures do not represent a random walk generating process (figures 4.a-4.c). If this was the case, the level of the log spectrum would be about -*3,5 [»ln(1/34)]*. The estimated mass spectrum for the linear detrended series is nearer to this value than the mass spectrum of differenced and HP-filtered sequences.

The upper panel of figure 4.a, corresponding to Δ*GDP*, suggests a higher importance for hidden cycles between 3 and 6 years (i.e. 23 and 12 on the horizontal axis). The mass spectrum is hump-shaped in both directions taking cycles about seven years as benchmark (10 on the horizontal axis). However, the fact that lower frequencies have less importance than higher frequencies is still true. When *GDP* is linearly detrended (upper panel of figure 4.b) the estimated spectra has Granger's "typical spectral shape" which is associated with the problem of *leakage* pointed out above (Granger, [1966]). A high correlation between adjacent values of the variable in levels could explain such a shape. Finally, the estimated mass spectrum corresponding to the HP filtered GDP (upper panel of

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[§§§§§] *Consider a factor b_t applied to the mean-subtracted sequence ($Y_t - \bar{Y}$), as $b_t(Y_t - \bar{Y})$, with b_t =1 if t=1,..n and b_t=0 otherwise. The trapezoidal taper operates as t/15, if 1 £t £15; 1 if 16 £t £55; and t/15 if 56 £t £70. Koopmans [1974, p. 301], reports in addition, a sine-type tapering procedure and a method for constructing tapers.*

[******] *A window size equal to the square root of the sample size is recommended. The explicit form lag window operator (the Bartlett window) that we assume is given by 1-s/M, with s=0,..4, and M=9.*

figure 4.c) peaks at very low frequencies: regardless that hidden cycles between three and six years are still important, cycles of a greater period (say, about 8 years) seem dominant.

The high concentration of the power spectrum of sequences in cycles between three and six years seems to be a feature of *GDPPC* as well. In other words, the proportion of the variance attributed to movements of output about 3-6 years is high with respect to other components[††††††]. This result contrasts with the first signal about the period given by the structural time series decomposition of section 2.2.1 above. Recall that the period of the cycle found there was between five and nine years. However, given the diagnostic statistics, the results of these models should be taken with some caution.

4. Permanent and Temporary Components of Aggregate GDP: A Bivarite View

In the study of permanent and temporary components, the link between changes in long run trends and shorter run departures from the *growth path* or *trend rate of change*[‡‡‡‡‡‡] has been attributed a special role. However, as we saw in section 2, in spite of the degree of correlation between the irregular components of the underlying stationary and nonstationary parts of the sequence, an identification problem arises since this correlation cannot be estimated directly from a single sequence. Assumptions about such correlation are that permanent and temporary innovations are perfectly correlated as in Beveridge and Nelson [1981] or not correlated at all as in the unobserved components models posed by Nelson and Plosser [1982], Harvey [1985], Watson [1986], and Clark [1987], among others. As an alternative to that identification problem, multivariate frameworks have been used in order to find a feasible separation of output's components. This view has been adopted, among others, by Sims [1980], Blanchard and Quah [1989] and King, Plosser, Stock and Watson [1991].

The information we have gathered so far about the statistical nature of Colombia's output, shows sequences characterized by cycles of about 5-9 years under a structural view; and important fluctuations of about 3-6 years on GDP and GDPPC from the frequency domain results. However, we cannot say whether the output's behaviour corresponds to a set of responses due to shocks originated either in the demand side or in the supply one or whether the origin of the shock is immaterial to qualify the permanent and temporary components of output. To find answers to such questions in this section we use a *bi-variate* framework to build a small macromodel where prices and output are endogenous. We use the Blanchard and Quah [1989] decomposition which identifies demand shocks as those having short run (temporary) effects on output but not otherwise. Conversely, supply shocks produce long run (permanent) effects on output. This perception is set as the *identifying restriction* on the parameters of a VAR system involving two endogenous (covariance-stationary) variables: *DGDP* and the rate of unemployment; as in the case analysed by Blanchard and Quah [1989] supply and demand shocks are uncorrelated

[††††††] *A common spectral analysis definition of business cycles is that these are frequencies between six and thirty-two quarters. That is between one year and a half and 8 years. This definition derives from the duration of the cycles isolated by NBER [King and Watson, 1996].*

and only the former is allowed to have a long run effect on output[§§§§§§]. In contrast to the result of Nelson and Plosser [1982], who found in the supply (technological) shocks the driver of the US economy, Blanchard and Quah end up with *DGDP* mainly driven by demand shocks in the case of that economy.

4.1. The Blanchard and Quah Decomposition[*******]

The Blanchard and Quah (BQ) method starts with a *bi*-variate system as:

$$Y_t = \sum_{j=0}^{\infty} B(j)e(t-j) \tag{27}$$

where Y_t is a 2×1 vector of endogenous covariance-stationary variables (growth of output and inflation rate, i.e. Y_t'=[DGDP P]), e_t is a 2×1 vector of uncorrelated disturbances (demand, e_t^d, and supply, e_t^s ,disturbances, e_t'=[e_t^d e_t^s]), and *B(j)* is 2×2 matrix of parameters so that:

$$B(j) = \begin{bmatrix} b_{11}(j) & b_{12}(j) \\ b_{21}(j) & b_{22}(j) \end{bmatrix} \tag{28}$$

where $b_{11}(j)$ and $b_{12}(j)$ show the effect of e_t^d and e_t^s on *DGDP j* periods later while $\sum_{j=0}^{k} b_{11}(j)$ shows the total effect of e_t^d on *DGDP* after *k* periods. Consequently, the matrix *B(0)* corresponds to the contemporaneous effect of disturbances on Y_t.

Because of the absence of correlation between disturbances, after normalization, we can write $Var(e_t)=I_2$. The restriction about the effects of demand shocks on output implies:

$$0 = \sum_{j=0}^{\infty} b_{11}(j) \tag{29}$$

which closes the BQ specification. To explain the operability of the BQ decomposition consider a VAR system which involves the two endogenous variables (*DGDP* and *P*):

$$Y_t = \sum_{j=1}^{\infty} A(j)Y_{t-j} + e_t \tag{30}$$

where *A(j)* is a 2×2 matrix of parameters and e_t is a 2×1 vector of innovations. The moving average representation of (30) can be written as:

[‡‡‡‡‡‡] *Brunner and Meltzer [1986], use this equivalence.*
[§§§§§§] *The scheme of Blanchard and Quah [1989] is essentially the same as King et al. [1991]. However, the latter is more general since it considers a tri-variate system instead of a bi-variate one.*
[*******] *See also Quah [1988] for more theoretical issues about the decomposition.*

$$Y_t \;=\; [I - \sum_{j=1}^{\infty} A(j) L^j]^{-1} \; e_t \;=\; \sum_{j=0}^{\infty} C(j) e(t-j) \tag{31}$$

where *C(0)=I.* The elements of *C(j)* will be:

$$C(j) = \begin{bmatrix} 1 - a_{11}(j) & -a_{12}(j) \\ -a_{21}(j) & 1 - a_{22}(j) \end{bmatrix}^{-1} \tag{32}$$

The variance-covariance matrix of innovations is defined as $Var(e_t) = L$. Given (27) and (31), we can write:

$$B(0)e(t) \;=\; e(t) \tag{33}$$

hence,

$$B(j) \;=\; C(j) \times B(0) \tag{34}$$

Now, to identify *B(0)*, note first, that:

$$E(e_t e_t') \;=\; Var(e_t) \;=\; \Lambda \;=\; B(0) \times B'(0) \tag{35}$$

which imposes three restrictions on the components of *B(0)*; and, second, allowing for (29), we have:

$$0 = [1 - \sum_{j=0}^{\infty} a_{22}(j)] \times b_{11}(0) + \sum_{j=0}^{\infty} a_{12}(j) \times b_{21}(0) \tag{36}$$

which is a fourth identifying restriction. That is, to identify *B(0)*, Blanchard and Quah have imposed : *i)* orthogonality of the elements of e_t; *ii)* unit variance of the elements of e_t; and, *iii)* no long-run effect on output growth of one of the elements of e_t. Once *B(0)* is identified, the original disturbances (demand and supply shocks) are identified as well. To solve the *4´4* system that contains the restrictions, we adopted the positive root when dealing with the quadratic expression which results in the solving process since the results seem to be more sensible.

4.2. Motivation and Results

To motivate the BQ decomposition of GDP between permanent and temporary components, consider a model close to that originally used by Blanchard and Quah [1989]. The model, in logarithms, is:

$$Y_t^d = M_t - P_t \tag{37}$$

$$Y_t^s = a\;(P_t - E_{t-1}P_t) + l_t \tag{38}$$

$$l_t = l_{t-1} + e_t^s \quad (39)$$

$$M_t = M_{t-1} + e_t^d \quad (40)$$

where Y_t^d is aggregate demand, Y_t^s is aggregate supply, M_t is money, P_t is price level, l_t is technology, e_t^d and e_t^s are the demand and supply original disturbances, E is the expectations operator and $E_{t-1}P_t$ is the expectation formed at the end of *t-1* about the price level in period *t*. Equation (37) represents the (quantity theory type) aggregate demand in terms of real balances; (38) is aggregate supply explained by differences between actual and expected level of prices[†††††††] and shifted by technology; equation (39) represents the motion of technology; and, finally, equation (40) shows the behaviour of money supply. By construction, the model associates technology shocks to aggregate supply and money shocks to aggregate demand.

After some algebra the solution of the model can be written in terms of the original disturbances as:

$$\Delta Y_t = \frac{a}{1+a}(e_t^d - e_{t-1}^d) + \frac{a}{1+a}(e_t^s - e_{t-1}^s) + e_t^s \quad (41)$$

$$\Delta P_t = \frac{1}{1+a}(e_t^d - a\, e_{t-1}^d) - \frac{a}{1+a}(e_t^s - e_{t-1}^s) - e_t^s \quad (42)$$

which matches the system described in (27). However, it still cannot elude the point of Lippi and Reichlin [1993] in the sense that an infinite order system, as in equation (27), is being represented by a first order system of equations (41) and (42). But, this solution incorporates more lags in the disturbances, of the equation other than of output, than the model used by Blanchard and Quah [1989][‡‡‡‡‡‡‡]. The restrictions imply that technology shocks have both short and long run effects on output and inflation; they also imply that money shocks have short and long run effects on inflation but only short run effects on output. The BQ decomposition is implemented by using the first difference of the logarithm of GDP and the rate of inflation, computed as the log-difference of Consumer Price Index (CPI) in the period 1950-1994 (figure 5)[§§§§§§§]. Following Gaviria and Posada [1994] we have selected an order-two VAR system to match the equation described by (30)[********].

Figures 6.a to 7.b. show the impulse response and the accumulated response functions after a shock received by the aggregates. Accordingly, the short and medium run dynamic motion of *GDP* growth in Colombia is explained by both technology shocks or supply shocks and demand shocks. In the very long run, however, the motion of the sequence of aggregate *GDP* is explained only by technological innovations. In figure 6.a. we observe that a money supply shock of one standard deviation has effects on *GDP* that last for about 10 years while the effects on prices remain for about 20 years. The initial effect on inflation is about 4.0% while it is only about 0.8%

[†††††††] *This specification might be associated with both Lucas's model of price misperceptions and nominal rigidity such as Fisher's long-term labour contracts model all signed at the same moment.*

[‡‡‡‡‡‡‡] *See also Blanchard y Quah [1993] and Quah [1995].*

[§§§§§§§] *The Dickey-Fuller test does not fail to reject the null that these variables are I(1).*

[********] *In the rank given by SBC criteria this order is second top. The conclusions are not affected by working with this order instead of with one.*

on Δ*GDP*. However, there are two clear differences in the reactions of the variables: whereas the effect on inflation has the same sign as the demand shock, the effect on Δ*GDP* is positive up to the third period after the shock but negative from there on. The message is that monetary shocks have a positive effect on Δ*GDP* in the short run but not in the medium and long-run. In figure 6.b. the result is different since positive technological shocks have unambiguous proportional effects on Δ*GDP* but negative in the case of inflation: a positive supply shock of one standard deviation increases Δ*GDP* by 1.0% and reduces inflation by about 4.5%. From these initial points, the effects start to die out, a process that lasts for 10-15 years.

The accumulated response functions of *GDP* (figure 7.a.) show that the effect of supply shocks on *GDP* is permanent as expected from the BQ decomposition, whereas the effect of demand shocks is only temporary. In the first case, the trend of *GDP* shifts by about 2.5% in the long run. Inflation (figure 7.b.) increases permanently with demand shocks (by about 20%) in the long run and reduces permanently with productivity shocks (by about 10%). The fact that the effects of any demand innovation take such a long time to disappear contributes to the persistence of *GDP*. However, the effects of supply shocks dominate the variability of *GDP*, while demand shocks are the most important explanation in the variability of inflation mainly in the long run. This assertion is supported by the variance decomposition (table 2).

5. Conclusions

Given the results, the final considerations about this work can be split into two sets depending on the period to which we are referring. First for period 1925-1994, the frequency-domain analysis suggests that cycles between three and six years make a high contribution to the formation of variability. That is, cycles between 3 and 6 years are important in the formation of the power spectrum of *GDP*. With respect to the volatility of *GDP* and *GDPPC*, the standard deviations for the HP filtered series are 2.62% and 2.77%, respectively. The same statistics for the linearly detrended and differenced series are: 5.40% and 4.12%; and 2.41% and 2.52%, respectively. These results suggest some caution before considering the volatility of output since *GDPPC* appears more volatile than *GDP* but only when the processes are linearly detrended. Structural time series decomposition does not suggest the existence of any other cycle during this period. Conversely, the local linear trend seems to be the most appropriate model for the series.

Second, for period 1950 - 1994, given that no neat deterministic cycle arises, the cyclical component of output analysed through the *linear trend plus cycle model*, does not seem to be a characteristic of the time series used here since, except for *GDP*, when the model presents some interesting results: only cycles of about eight years are important in the behaviour of output. In the same way, stochastic movements are important only in the case of *GDP*. Finally, the results of the Blanchard and Quah decomposition show that demand shocks have important explanatory attributes for output fluctuations. However, supply shocks are dominant in the behaviour of output. In particular, nominal shocks of one standard deviation have effects on *GDP* that last for about 10 years while the effects on prices doubles it. The initial effect on inflation is about 4.0% while it is only about 0.8% on ΔGDP. The message is that money shocks have a positive effect on ΔGDP in the short run but not in the medium and long-run. Positive technological shocks, on the other hand, have unambiguous proportional effects on ΔGDP but negative in the case of inflation: a positive supply shock of one standard deviation increases ΔGDP by 1.0% and reduces inflation by about 4.5%. From these initial points, the effects start to die out, a process that lasts for 10-15 years. The accumulated responses of *GDP* show that the effect of supply shocks is permanent, whereas the effect of demand shocks is only temporary. In the first case, the trend of *GDP* shifts by about 2.5% in the long run. Inflation increases permanently with demand shocks (by about 20%) in the long run and reduces permanently with productivity shocks (by about 10%). The effects of supply shocks dominate the variability of *GDP*, while demand shocks are the most important explanation in the variability of inflation.

Table 1. Structural Decomposition of Output
Maximum Likelihood Estimates of Parameters

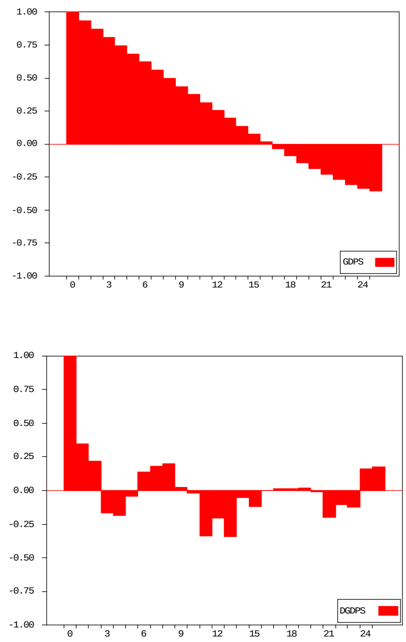
Statistic	*GDP 1950-1994*		*GDPPC 1950-1994*		*GDP 1925-1994*		*GDPPC 1925-1994*	
	No Cycle	*Cycle*	*No Cycle*	*Cycle*	*No Cycle*	*Cycle*	*No Cycle*	*Cycle*
R_D^2	0.072	0.261	0.017	0.0002	0.011	0.011	0.11	0.025
H (*m*)	0.69 (14)	0.49 (14)	1.09 (14)	1.10 (14)	0.23^{**} (22)	0.23^{**} (22)	0.22^{**} (22)	0.22^{**} (22)
Q	18.95^{***}	11.45	9.61^{**}	21.02^{***}	14.15^{***}	15.35^{**}	9.50^{**}	10.79
(*p,d*)	(8,3)	(11,6)	(8,3)	(11,6)	(8,3)	(11,6)	(8,3)	(11,6)
Standard Error	0.017	0.014	0.018	0.018	0.023	0.023	0.024	0.024
Log L	174.1	181.2	169.7	169.7	250.9	250.9	248.5	249.0
2p/w (in years)	-	7.95	-	4.9	-	5.00	-	9.3
w	-	0.79	-	1.27	-	1.25	-	0.67
r	-	0.96	-	0.95	-	0.90	-	0.92
s_u^2 (´10^5)	0.007	0.	3.4	5.8	5.8	5.8	6.2	6.1
s_k^2 (´10^5)	-	0.017	-	0.	-	0.	-	0.31
s_x^2 (´10^5)	1.14	0.022	0.	0.	0.	0.	0.	0.
s_e^2 (´10^5)	0.001	0.03	0.	0.	0.	0.	0.	0.

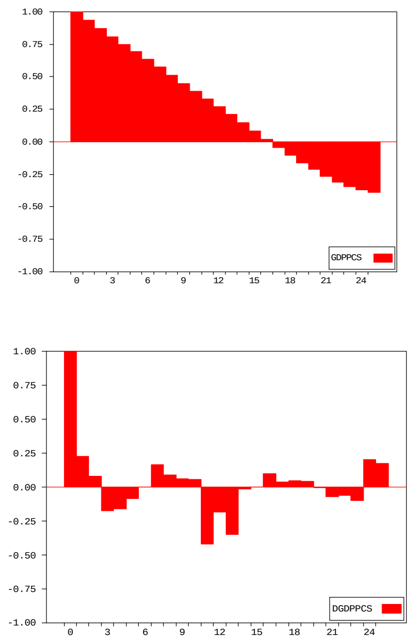
NOTE: Two asterisks mean significant at 5% level of significance, and three asterisks mean significant at the 1% level. "Cycle" corresponds to equation (15) which in the text is called the trend plus cycle model while "No Cycle" is the same model without the sinusoidal cycle component which in the text is called the local linear trend model: equations (1) and (10). Q(p,d) is the value of the statistic of Ljung-Box, where p is the number of lags in (18) and d identifies degrees of freedom (d=p-(n-1)).

Table 2. Blanchard-Quah Decomposition
Variance Decompositions

Periods Ahead	*GDP Growth*		*Inflation*	
	Demand Shock	*Supply Shock*	*Demand Shock*	*Supply Shock*
1	28,7	71,2	47,8	52,2
2	27,5	72,5	54,8	45,2
3	26,3	73,7	58,1	41,9
5	26,6	73,4	61,7	38,3
10	27,6	72,3	63,6	36,4
20	27,9	72,1	63,9	36,1
30	27,9	72,1	63,9	36,1

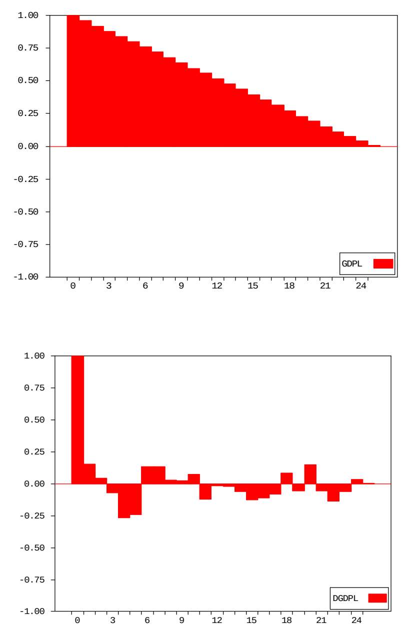
Figure 1. Correlogram of GDP DGDP

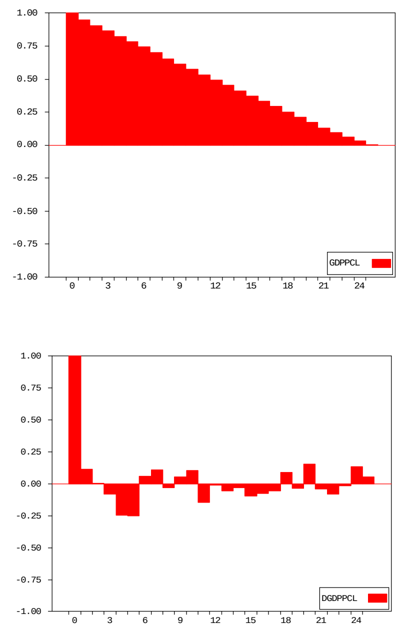




a. GDP and DGDP. 1950 - 1994

b. GDPPC and DGDPPC. 1950 - 1994





c. GDP and DGDP. 1925 - 1994

d. GDPPC and DGDPPC. 1925 - 1994

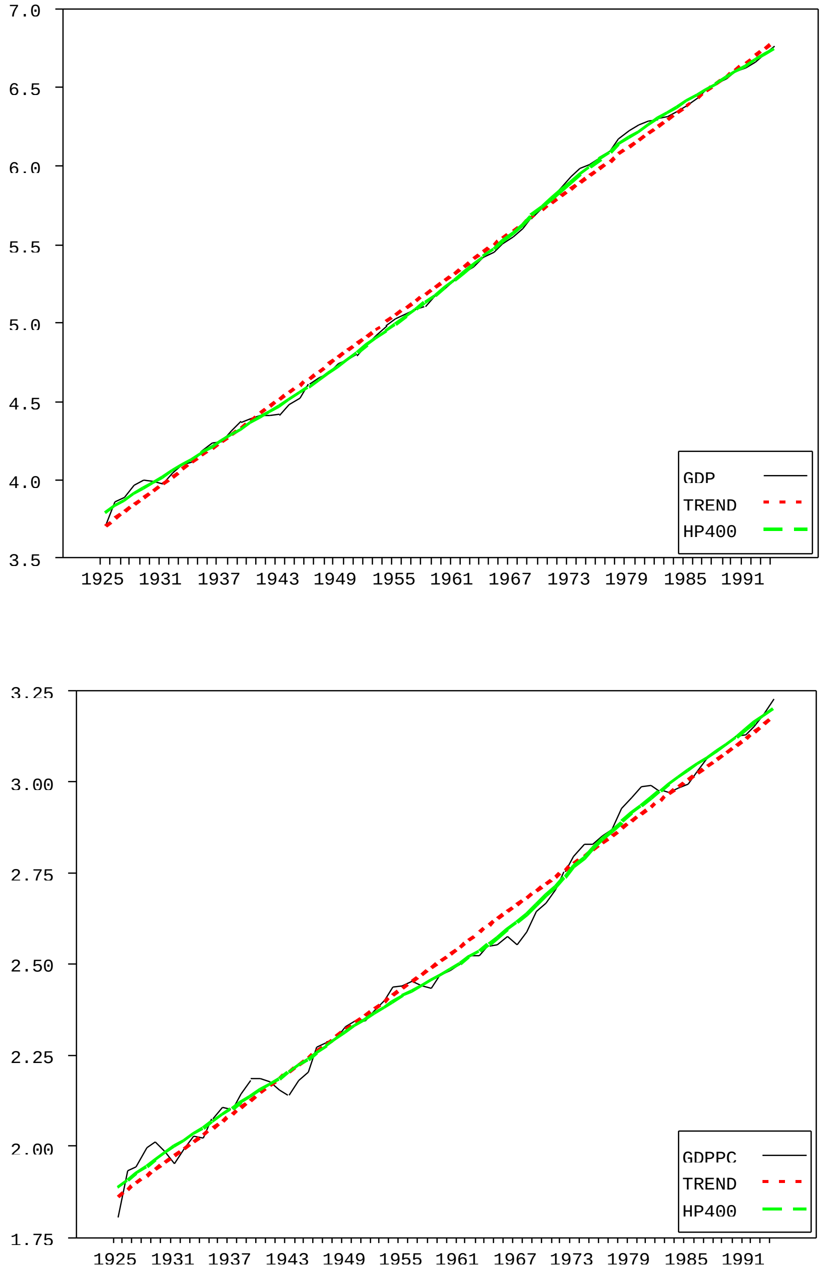


Figure 2. Trends of GDP and GDPPC 1925 - 1994.

NOTE: GDP (GDPPC) is the actual value of the variable. Trend is the linear trend and HP400 is the trend component of the HP filter with l=400.

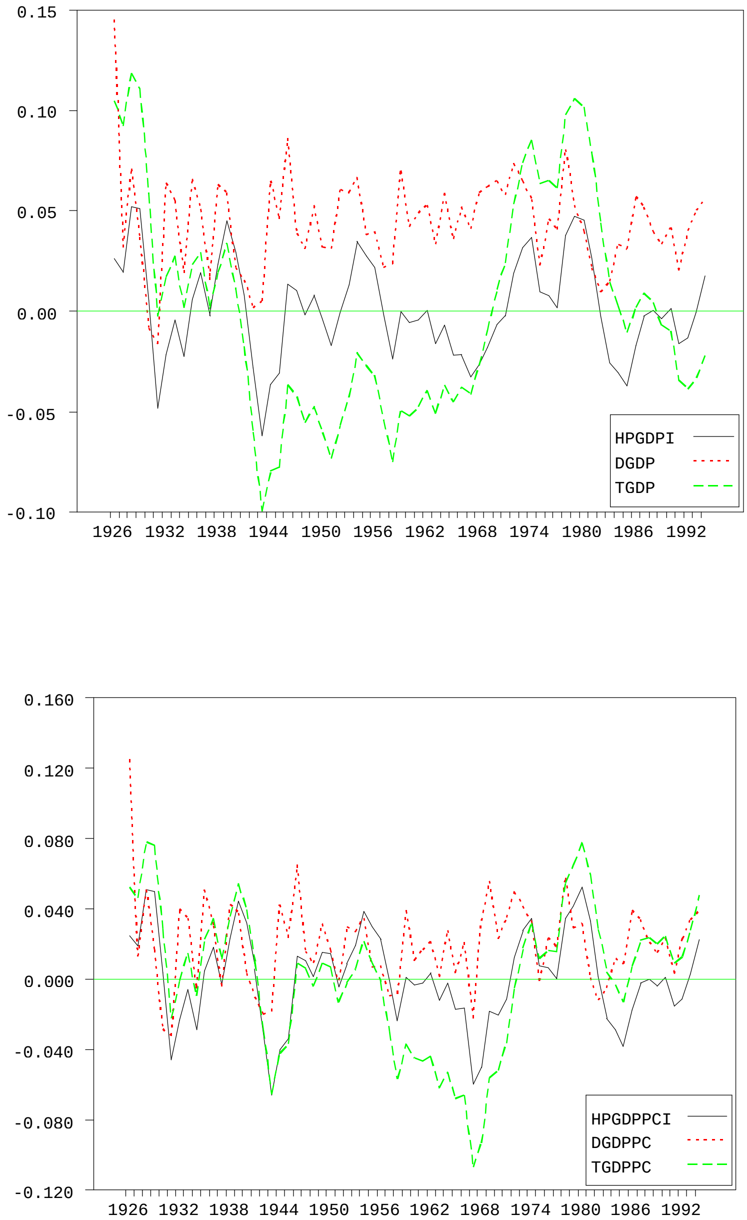


Figure 3. Cyclical Components of GDP and GDPPC. 1925 - 1994

NOTE: HPGDP (HPGDPPC), DGDP (DGDPPC) and TGDP (TGDPPC) are the HP, the first differenced the trend cyclical components of output respectively.

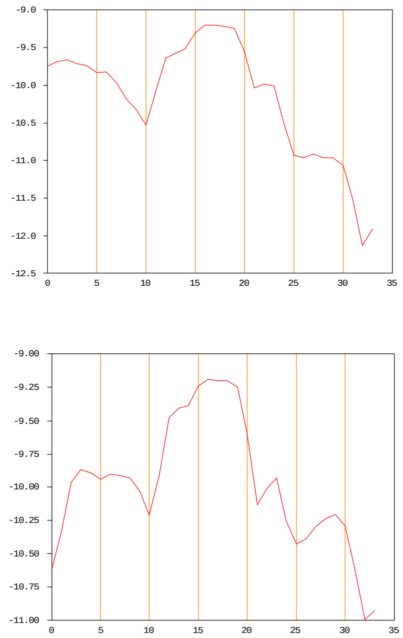


Figure 4.a. Spectrum of DGDP and DGDPPC. 1925 - 1994

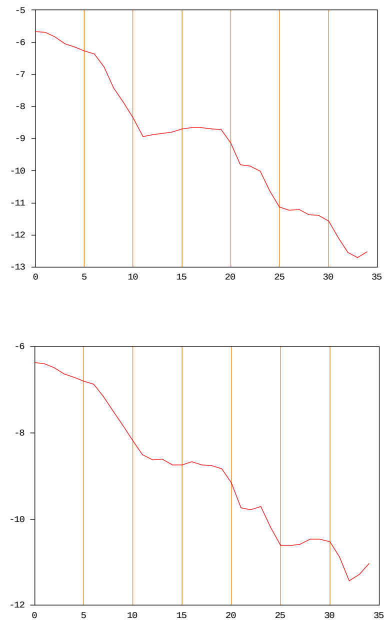


Figure 4.b Spectrum of Linearly Detrended GDP and GDPPC. 1925 - 1994

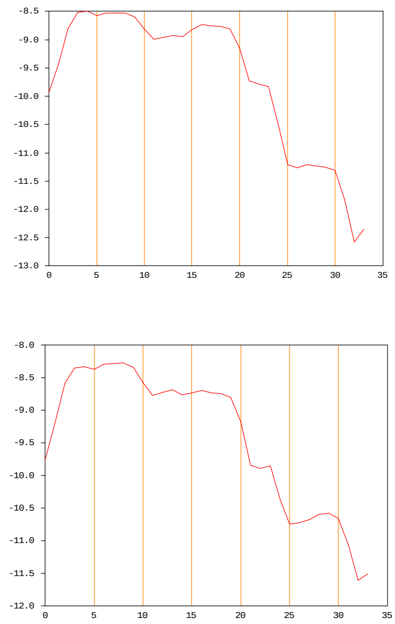


Figure 4.c. Spectrum of HP Filtered GDP and GDPPC. 1925 - 1994

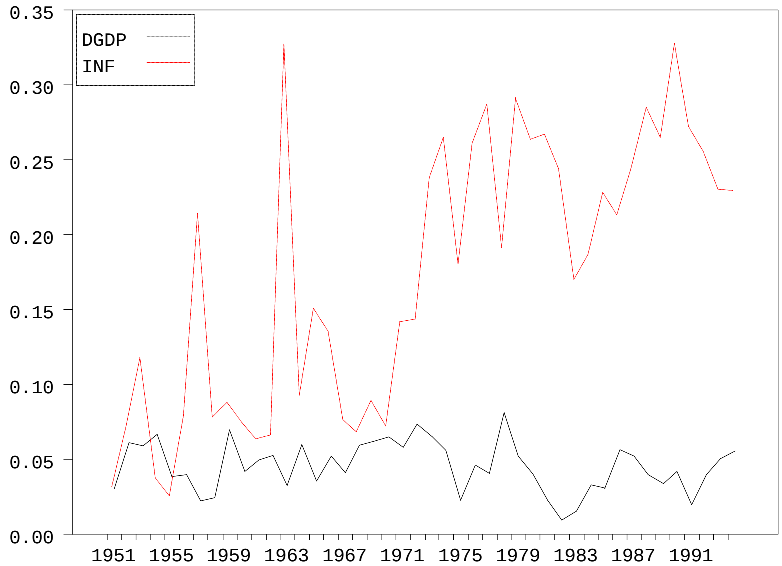


Figure 5. Growth Rate of GDP and Inflation.

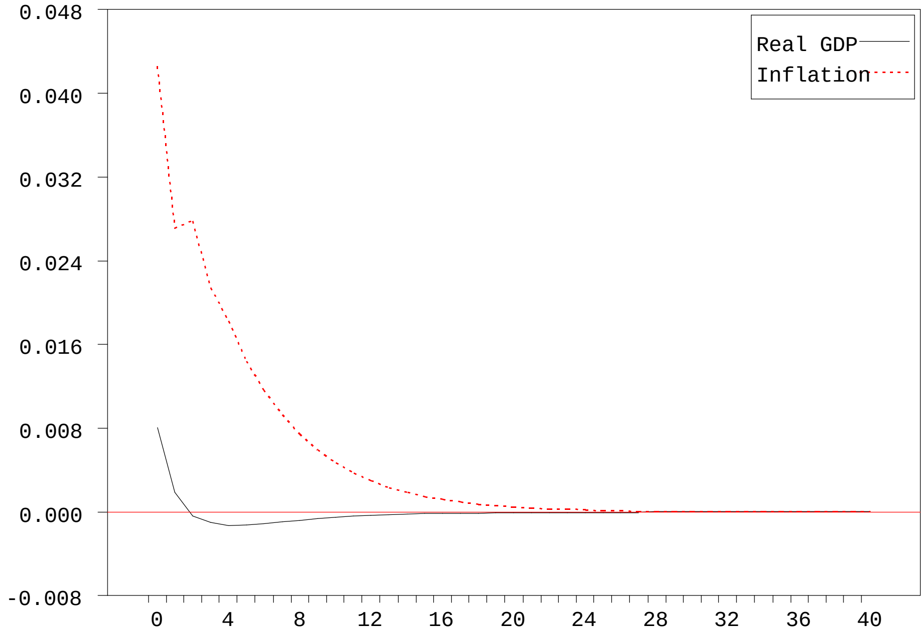


Figure 6.a. Impulse Response Function: Demand Socks.

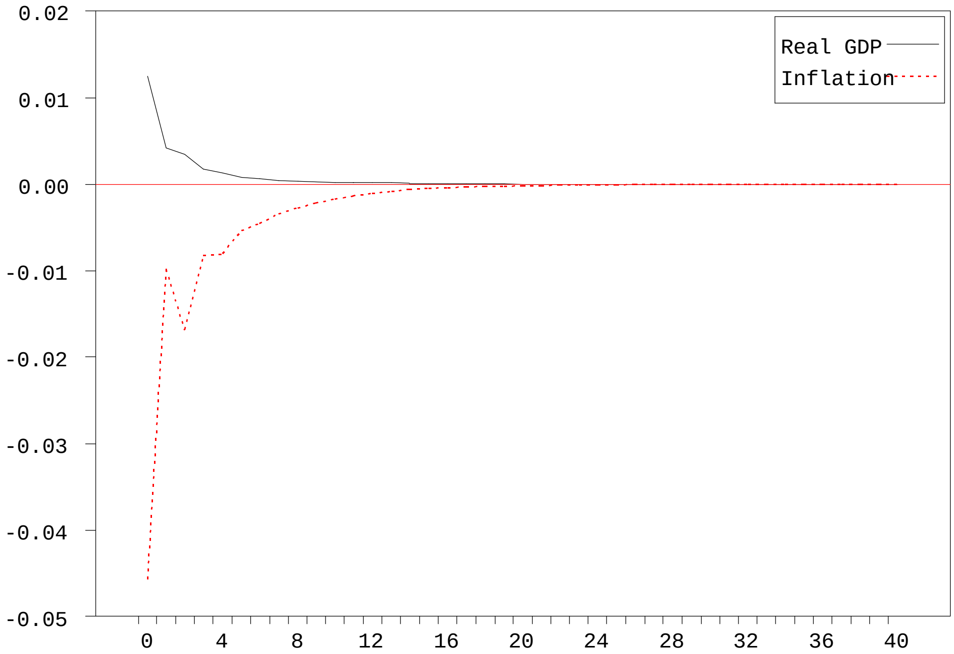


Figure 6.b. Impulse Response Function: Supply Socks.

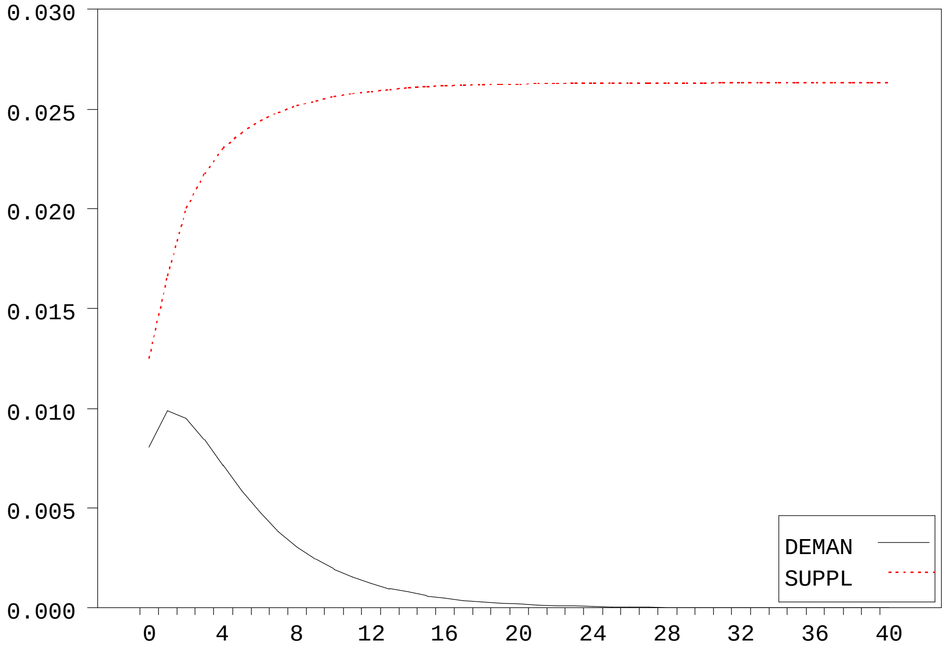


Figure 7.a. Accumulated Responses of GDP to Supply and Demand Shocks

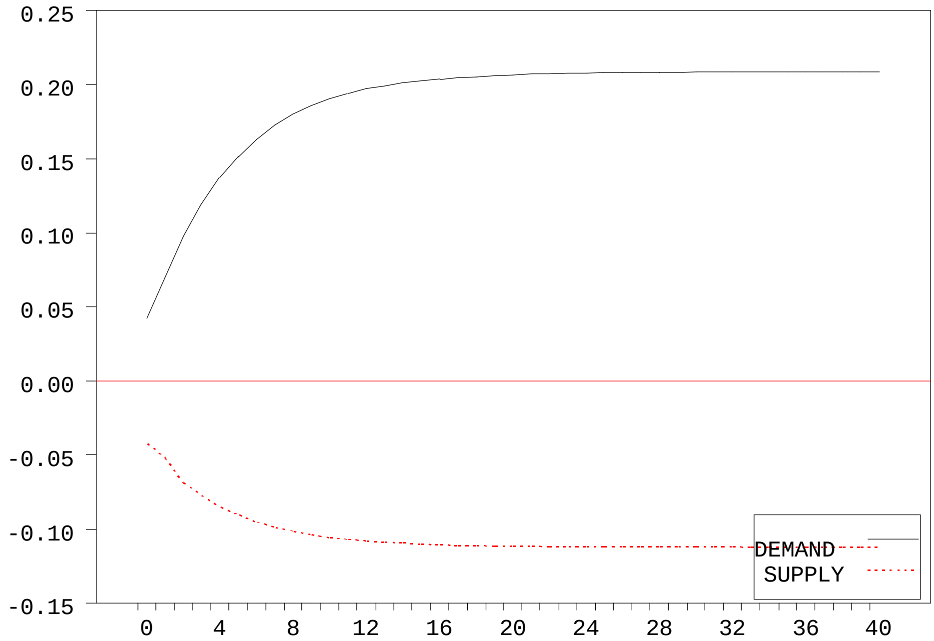


Figure 7.b. Accumulated Response of Inflation to Supply and Demand Shocks

Appendix 1

The State Space Form and the Kalman Filter[††††††††]

The observable sequence $\{ Y_t \}$ can be represented by the *measurement equation*: $Y_t = z_t a_t + e_t$, where z_t is a *1 ´ m* vector, a_t is a *m ´1* vector, known as the *state vector,* and e_t are zero-mean, [$E(e_t)=0$], serially uncorrelated disturbances with $Var(e_t)= h_t$. The unobservable elements of the state vector are generated by a first-order Markov process, known as the *transition equation*: $a_t = T_t a_{t-1} + R_t h_t$, where T_t is a *m ´m,* R_t is a *m ´g* matrix and h_t is a *g ´1* vector of serially uncorrelated disturbances with $E(h_t)=0$ and $Var(h_t)= Q_t$. The *measurement* and the *transition equation* are the core of the *state space* form. However, to complete the state space representation it is necessary to assume that, on the one hand, the first two moments of the initial state vector a_0 are $E(a_0)= a_0$ and $Var(a_0)= P_0$ and that, on the other, the disturbances e_t and h_t are uncorrelated with each other in all periods $E(e_t\ h_t^{'})=0$ and uncorrelated with the initial state $E(e_t\ a_0^{'})=0$ and $E(h_t\ a_0^{'})=0$ in all periods. The *system of matrices* is composed of z_t, T_t, and R_t, which are assumed to be non-stochastic. Under the conditions we have set so far the model is *time-variant*. It would be *time-invariant* or *time-homogeneous* if the subscripts of the system of matrices and h_t and Q_t could be dropped.

The Kalman filter is a procedure of recursions used for computing the optimal estimator of the state vector at time *t*. It is the device used by state space form as just the least squares computations are used by the regression model. The use of the Kalman filter requires that the disturbances of the initial state vector be normally distributed. Observations of *Y* up to period *t-1* are used to compute a_{t-1}, the optimal estimator of a_{t-1}. The covariance matrix of the estimation error is P_{t-1} = $E[(a_{t-1} - a_{t-1}\ )\ (a_{t-1} - a_{t-1})']$. The *prediction equations* are composed of: *i)* the optimal estimator of a_t (it minimizes the Mean Square Error-MSE) which is constructed as: $a_t|_{t-1} = T_t\, a_{t-1}$, and *ii)* the covariance matrix of the estimation error: $P_t|_{t-1} = T_t P_{t-1} T_t' + R_t Q_t R_t'$. The *updating equations* work once a new observation of Y_t is known. These equations are: $a_t = a_t|_{t-1} + P_t|_{t-1} z_t' f_t^{-1}\ (Y_t - z_t\, a_t|_{t-1})$ and $P_t = P_t|_{t-1} - P_t|_{t-1}\ z_t' f_t^{-1} z_t\ P_t|_{t-1}$, where $f_t = z_t P_t|_{t-1} z_t' + h_t$. The set of *prediction and updating equations* conform the Kalman filter. That set of equations can be written as a single set of recursions from $a_t|_{t-1}$ to $a_{t+1}|_t$. In this case, we may write $a_{t+1}|_t = (T_{t+1} - K_t z_t)\, a_t|_{t-1} + K_t Y_t$, where K_t is the *gain matrix*, computed as $K_t = T_{t+1} P_t|_{t-1} z_t' f_t^{-1}$, and

[††††††††] *These paragraphs are based on Harvey [1989], Harvey [1993] and Koopman et al. [1995], where more elaborated presentations can be found.*

the recursive equation for the error covariance matrix is $P_{t+1}|_t =$ $T_{t+1}(P_{t-1}|_t - P_t|_{t-1} z_t' f_t^{-1} z_t P_t|_{t-1}) T_{t+1}' + R_{t+1} Q_{t+1} R_{t+1}'$.