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Abstract

The focus of this study is to build, from the 'bottom-up', a market with artificially intelligent adaptive agents based on the institutional arrangement of the Colombian Foreign Exchange Market (1994-1999) in order to determine simple agents' design, rules and interactions that are *sufficient* to create interesting behaviours at the macroscopic level – emerging patterns that replicate the properties of the time series from the case study.

Tools from artificial intelligence research, such as genetic algorithms and fuzzy logic, are the basis of the agents' mental models, which in turn are used for forecasting, quoting and learning purposes in a double auction market. Sets of fuzzy logic rules yield adequate, approximately continuous risk and utility preferences without the need to fix their mathematical form ex-ante.

Statistical properties of financial time series are generated by the artificial market, as well as some additional non-linearity linked to the existence of a crawling band. Moreover, the behaviour of the simulated exchange rate is consistent with currency band theory.
Agent's learning favours forecasting rules based on regulatory signals against rules based on fundamental information. Also, intra-day volatility is strongly linked to the rate of arrival and size of real sector trades. Intra-day volatility is also a function of the frequency of learning and search specialisation. It is found that when a moderately low frequency of learning is used, volatility increases.

Key words: adaptive agents, artificial markets, constrained generating procedures, fuzzy logic and genetic algorithms.

Clasificación JEL: G1, G12, G39.

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1. Introduction

In the last 20 years, agent-based models have increasingly commanded the attention of researchers in finance and economics. The possibility of integrating the co-adaptive behaviour of autonomous (intelligent) individuals with an analysis of macroeconomic patterns without the need to explicitly assume investors' homogeneous expectations or positing the hypothesis that all markets for all future times exist today, as well as the opportunity to study the mechanisms whereby the economy or the market evolves over time in a controlled setting - a computational laboratory – makes it a compelling topic of research. It is an area of great interest not only for researchers, but also for investors and policymakers.

The primary focus of this work is the construction, from the 'bottom-up', of a market with artificially-intelligent adaptive agents based on the institutional arrangement – monetary policy, regulations and trading conventions – of the Colombian Foreign-Exchange Market that prevailed between December 1994 and September 1999, period for which a currency bands mechanism was in place.

There are two basic motives for using this particular case. First, the regulations and explicit monetary policy established by the Banco de la República (Central Bank) in terms of a monetary base corridor, an interest rate range and a currency band, combined with a transparent set of intervention rules in each of these markets, enables us to construct a complete model of the institutional environment that confronts the agents. Second, as a very complete data set was made available by the Central Bank, the possibility of studying agents' daily net US-dollar positions, forward market contracts, Colombian Peso (COP) intra-day transactions, amounts traded, daily US-dollar real sector supply and demand, etc. offers insights into market micro-structure and market dynamics, and facilitated the study of emergent properties at the macro level.

A subsidiary goal is to study the behaviour of this artificial market from the perspective of a policymaker since (Durlauf, 1997): (1.) interactions amongst agents can result in aggregate behaviour that may lead the system to undesired states; and (2.) the effectiveness (or consequences) of policies is linked to the nature of the interactions and interdependencies amongst agents. If, for example, positive feedbacks, asymmetric interactions, or “mass” thresholds exist and are reinforced, non-linear effects may influence the path followed by the system and make policy evaluation a difficult and complex task.

The level of reality that is achieved by using the Colombian monetary and regulatory arrangements (including intervention procedures), together with feedback mechanisms in the agent’s expectations regarding price determination and quoting procedures, represents a novel approach to artificial market design. To date, given the lack of adequate daily economic data, artificial markets have operated under an assumed environment where fundamentals are supposed either to have a particular mathematical form (usually some autoregressive process) or to follow models drawn from economic theory.

A two-step fuzzy-logic cognitive model for agent forecasting and quoting - which by means of a fuzzy rule-set and a set of membership functions - manages the agent’s inventory and risk without the need for an explicit ex-ante mathematical declaration of the agent’s utility function and risk aversion. This allows for better modelling of real traders characteristics from a symbolic (deductive) perspective without losing the advantage of numerical processing[2].

The environmental constraints arising from the actual interactions between the financial agents with either the real sector (constructed from actual daily supply and demand data) or the Central Bank, which determine the “physical” constraints of the environment in which agents learn and adapt trough a Fuzzy Logic – Genetic Algorithm system (FLGA), combined with the positive feedbacks embedded between the exchange rate produced by the financial market and the modelled economic environment, adds a new dimension to the learning problem. In markets with artificial agents, the impact of the fundamentals manifests itself (directly or indirectly) through the performance measure, which is used in the learning process.

In this artificial market, a broader approach is taken whereby traders perform actual trades with the Central Bank and real-sector agents, which directly affect their inventory

[2] Below it is shown how fuzzy sets represent knowledge symbolically while transforming it numerically.

and risk. A distinct form of dynamics arises as the timing, amount and rate at which these trades are performed, and the identity of the financial traders by whom they are actually performed, permanently affects the topology of the system and traders' performance. This is a big step because, in order for evolutionary learning to perform adequately, the physical constraints on the agent's actions must be well defined and in this market Central Bank and real-sector participation are continuously (at the intra-day level) constraining agents' interactions and behaviour.

2. The Functional Model

We introduce the artificial market from a functional perspective. See Reveiz (2008) for the presentation of the general modelling framework based on Holland´s cgp's (1995, 1998).

2.1. Basic Assumptions

The main assumptions used in the implementation of the model are:

1. Agents can have Heterogeneous Expectations.
2. Information is discounted within relevant [individual] theoretical frameworks
3. Agents can dynamically improve the models they are using (learning).
4. Agents have internal mechanisms for computing the profit and risk associated with the transfers.
5. Trade is a Profit-Improving bilateral barter between agents.
6. Price formation is the result of *actual trades* and not the product of the intervention of the neoclassical so-called Auctioneer.
7. The Quoting process is a function of the agent's forecast, inventory, risk, spread, trading success and his or others' last quotes. It is also a function of an estimation of market supply (SS) and demand (DD), Market's Best Bid/Ask, Average Bid/Ask and Bid/Ask Spread.

2.2. The Belief Model

An agent can be characterized by a belief set and a set of actions. The belief set includes the mental models, the learning mechanisms, a set of parameters and the fitness of the models, which determine which *expectational* model is actively used. Combining the belief set with the appropriate inputs, plausible actions are then taken by the trader. The belief set is summarised by a 7-tuple as follows:

$$B_{i,t} = \{\mathrm{M}_{i,t}, \aleph_{i,t}, R_{i,t}, \mu_i, \chi_i, \Delta_i, \Gamma_i / \varphi_{i,t}\}$$

$B_{i,t}$: Belief Set for agent *i* at time *t*.

$\mathrm{M}_{i,t}$: Fuzzy Sets and corresponding Membership Functions for agent *i* at time *t*.

$\aleph_{i,t}$: Rule sets for agent *i* at time *t*.

$R_{i,t}$: Rule sets performance for agent *i* at time *t*.

μ_i : Recombination rate for agent *i*.

χ_i : Mutation rate for agent *i*.

Δ_i : Delta for learning interval determination for agent *i*.

Γ_i : Degrees of freedom for learning interval determination for agent *i*.

$\varphi_{i,t}$: Information set for agent *i* at time *t*.

The plausible actions are defined as a 4-tuple and action states are particular instances of this action (Quoting) set, i.e. whether the trader is in a {buying}, {selling} or {market making} state is implicitly determined by the quoting actions:

$$\mathrm{A}_{i,t} = \{\Delta\mathrm{Bid}_{i,t}, \Delta\mathrm{BidAmount}_{i,t}, \Delta\mathrm{Ask}_{i,t}, \Delta\mathrm{AskAmount}_{i,t} \mid \varphi_{i,t}\}$$

$\mathrm{A}_{i,t}$:	Plausible actions
$\Delta\mathrm{Bid}_{i,t}$:	Change in the Bid Quote by agent i at time t.
$\Delta\mathrm{BidAmount}_{i,t}$:	Change inf the Bid Quote amount by agent i at time t.
$\Delta\mathrm{Ask}_{i,t}$:	Change in the Ask Quote by agent i at time t.
$\Delta\mathrm{AskAmount}_{i,t}$:	Change in the Ask Quote amount by agent i at time t.
$\varphi_{i,t}$:	Information set for agent i at time t.

In order to study the agent's behaviour, we can modify his belief set either by changing the rate at which he learns (learning interval), the amount of exploration (mutation) or specialisation (recombination), and the initial possibilities he is given (by modifying the number of models the agent is given, we limit the rate at which he finds good solutions). The information available to each agent can also be modified. Varying the number of rules that make up a rule-set can also change the sophistication of the models used by the agent.

2.3. The Goal Model

The microscopic objective of each of the agents is simply to make profits by maximizing the performance of his/her *expectational* model. Clearly, each agent is confronted with a difficult maximization problem because the environment, which determines the fitness landscape for each model, is continuously changing. Consequently, a model that is successful under some conditions may perform poorly under other conditions. Moreover, as performance is an ex-post measure, it may very well be that the best model may not perform particularly well while it is active because of a change in environmental conditions. Such a situation can only be corrected in the next learning state (if conditions do not vary significantly again). Bear in mind that the environment includes all of the other agents' strategies, which may vary more than the economic variables, thereby inducing more *environmental* volatility.

2.4. The Trader's mental model

Following Lyons (2000) a hybrid approach to determine the trader's next quote is used, in which exchange rate macroeconomics and market microstructure are linked and where a change in the nominal exchange rate is a function of the fundamentals $f(i, m, z)$ – including current and past values of domestic and external interest rate (i), the money supply (m) and any other macro determinants, denoted by z - and of some micro factors $g(X, I, Z)$ including the order flows (X), the inventory (I) and other micro determinants, denoted by Z.

$$\Delta P_t = f(i, m, z) + g(X, I, Z) + e_t$$

In order to implement this hybrid approach, forecasting and the consequent quoting are accomplished in two steps using a combination of two fuzzy systems. The first system creates a forecast by looking at the expected supply and demand in the market, the interest rate level, the position of the exchange rate within the exchange rate band, the level of international reserves and a set of moving-average calculations for the exchange rate. Given an expected behaviour for the market price, the trader decides how to modify his quotes. In order to do this, he/she must check other relevant variables.

These come from two sources: market information and the trader's private information. The first includes inter-alia: market spread, the best bid and ask rates in the market, an estimation of market demand/supply from other traders; while the second focuses mainly on risk exposure and inventory indicators.

Rules for the forecasting function are constructed from the expected relationship between variables that arise from economic models. The complete set of rules represent the "fundamentals" of the economy in the sense that, if every trader is given the complete set of rules, they could in equilibrium conditions forecast accurately the behavior of the exchange rate: the efficient market hypothesis holds for this artificial market - under these conditions - if the full information set is given to each of the traders.

If each trader is randomly given a rule-set that contains "good" and "bad" rules, or a limited number of rules, his forecasting model is not fully accurate and we should expect trades to arise (Arrow, 1987) as agents have distinct views on the market. In this context, learning can easily be incorporated by allowing traders to dynamically improve their models.

This is done using a genetic algorithm. Each trader is given various rule-sets and, although forecasts are produced only with the fittest rule-set, all of them are ranked in terms of performance. Consequently, they are periodically used as a population for the genetic algorithm, which by recombination and mutation improves these rule-sets.

2.5. Central Bank Intervention

Central Bank (CB) intervention is represented by a 3-tuple {Price, Amount, Action} where trade intention is determined by the action (bid or ask). In the period under study, Banco de la República's intervention in the foreign-exchange market was accomplished through two mechanisms that corresponded to the two objectives prevailing for the Colombian authorities: (i.) a currency band that provides the private sector with some degree of certainty and limits extreme trends; and (ii.) intra-marginal (dirty) intervention performed to lower intra-day daily volatility. Together with a monetary corridor and an interest rate range, these are the practical formalization of the CB objective function.

The Currency Band was public knowledge as was used in the agent's forecasting models. When exchange rate transactions reach either the top or the bottom of the currency band, the authorities sell or buy foreign currency in quoted amounts of 10,000 units[3] at the band's exchange rate for that specific day. For a detailed description of the issues related to the design, implications and effective and potential complexity of currency bands, the reader is referred to Brooks and Revéiz (2002).

In the artificial market, marginal (inside the band) intervention is performed in batches of 250 units of foreign currency when the percentage change in the exchange rate exceeds the volatility range set by the authorities. When the exchange rate is close to the limits of the currency bands, the intra-marginal range converges to the currency band exchange rates.

$$\begin{aligned}&\mathrm{Int}_t = f(\mathrm{FXCR}, \mathrm{IntraB}, \mathrm{CurrB})\\&\mathrm{FXCR}: \text{Level of the Exchange Rate}\\&\mathrm{IntraB}: \text{Intra - marginal Range}\\&\mathrm{CurrB}: \text{Currency Band Boundaries}\end{aligned}$$

Although intervention rules are defined deterministically, the occurrence (and amount) of intervention cannot be forecasted because the specific moments when the boundaries

[3] Traders' quoted amounts range between zero and 1000 units. Partial trades are also allowed.

are reached cannot be predicted since they depend on the traits of and the interactions amongst agents.

2.6. Real Sector Participation

Real Sector participation in the market is accomplished indirectly. The economic environment provides the daily supply and demand of dollars from the real sector which are derived from historical data. The amount that the real sector will sell to, and/or buy to and from the financial sector for each intra-day time step is allocated randomly. Then, at each of these intra-day time steps, the real sector calls a group of randomly-chosen financial institutions and trades with the trader who wins the "tournament". This procedure is repeated as many times as necessary for amounts of 250 units[4] until the total supply and the total demand for that intra-day time step are traded[5].

2.7. Market Mechanism Function

The market mechanism is organized around three concepts: *Trading Documents* (Messages), the *Order Book* and the *Board*. *Trading Documents* identifies the trader, present his disposition to trade, the prices at which he/she would like to transact, as well as the quantities. The Order Book includes the messages, a time stamp, the time for which this quote is valid and a trader *id-tag*. The *Board* displays the *Trading Documents* of all the traders, as well as the rate and the amount of the last trade that occurred in the market (similar to Colombia's foreign-exchange Citiinfo market). Glosten (1994) has shown that an open and accurate book enhances market efficiency, in a theoretical framework. Moreover, a small bid/offer spread may appear.

Following Miller (1996), messages can be represented as a 3-tuple {*action*,*p*,*q*}, where *action* gives the action to be taken {bid or ask}, *p* gives the price for the bid and *q* the quantity that corresponds to this price. For instance, the message {ask, 1000, 300} represents a market order to sell 300 units at a price of 1000. The order book is made up out of a set of orders – that change over time – and a set of rules – fixed over time – that determine how/when trades are consummated among two or more traders.

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[4] In the inter-bank market, traders can quote 100, 300, 500, 700 or 1000 units at any time.

[5] A balancing transaction for any remainder is also performed.

The Order Book also includes a time-stamp and an id-tag[6], which together provide a unique “key” as to the intentions of each trader at every time step[7]. A trader can send messages to trade simultaneously on both sides of the market (Market Maker).

By assuming that the sign of the amount quoted reflects the desired action (positive/negative for bid/ask respectively), the state of the order book can be represented by the collection of messages and their corresponding time stamps and id-tags: $[\{p_1, q_1, t_1, id-tag_1\}, \{p_2, q_2, t_2, id-tag_2\}, ..., \{p_M, q_M, t_M, id-tag_M\}]$ where M is the number of orders in the book.

The set of rules in the Order Book constitutes a simplified double-auction market. Traders are able to post quantities from the set {100, 300, 500, 700, 1000}, and each order can be partially fulfilled. As mentioned above, traders submit messages, to buy and/or sell. They can also accept posted bids or asks. A transaction occurs when the bid and ask are either equal or cross.

This book cannot contain orders that can be sequentially executed in such a way that positive profits are earned for a zero net position (Miller, 1996). If this situation arises, the order in question is immediately filled with quotes that: (i.) meet the conditions in terms of price and intention; and (ii.) have the highest priority, as specified by the id-tag and time stamp “key”.

It should be noted that the double auction achieves high efficiency in a variety of environments, that convergence occurs rapidly and has been the natural choice for most experimental studies (Williams and Smith, 1984; Chan et al, 1999). For more details on the double auction mechanism, the reader is referred to McCabe (1993) and Williams (1980).

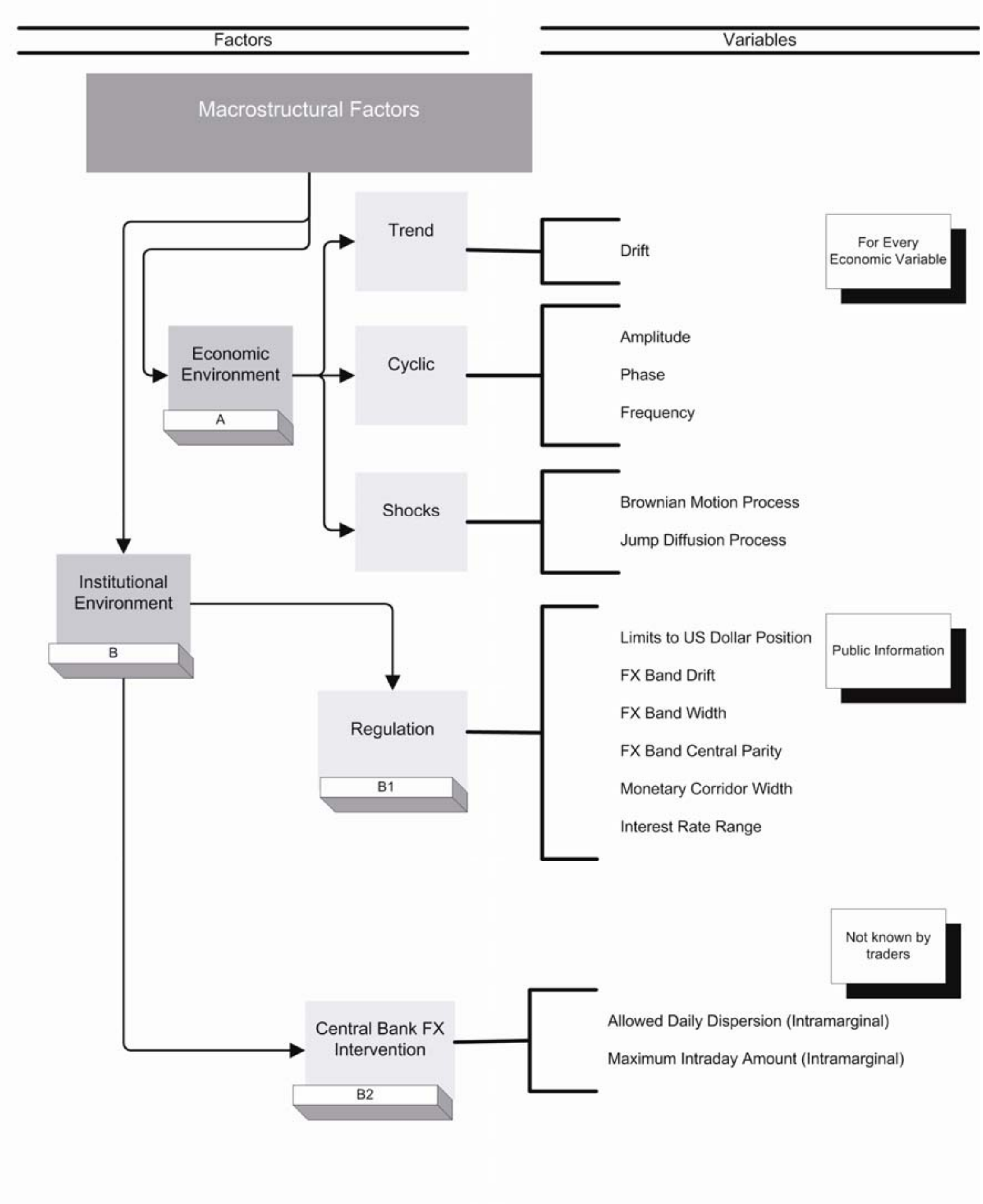
[6] Miller (1996) includes both the time of the message and the trader identification in the time stamp. For clarity, these will be treated separately in this section.

[7] This implies that for orders that arrive simultaneously, some mechanism is used to order them in time. In this case, the best quote is given precedence if two quotes arrive simultaneously.

3. Description of Experimental Setting and Preliminary Discussion

Modifications to the parameters in the simulation permit the study of changes in market dynamics as the result of variations in micro-structural factors, either intrinsic to the object or the interactions, or macro-structural factors, such as the institutional and economic environment. An agent's intrinsic micro-structural factors can be part of his/her "genetic traits", such as risk aversion and capital, or can be related to the "cognitive model" such as learning frequency or forecasting model's sophistication and complexity. Changes in the interactions are limited to variations in the auction mechanism and distinct quoting procedures (adaptive vs. non-adaptive quoting), although interactions respond indirectly to the co-adaptation and coupling of the learning processes through the performance measure. Macro-structural factors can be further divided into the drift, cyclic and shock parameters of the variables for that describe the economic factors and into regulatory requirements, public monetary policy mechanisms (FX band, monetary corridor and interest-rate range) and specific intra-marginal intervention rules such as the maximum targeted volatility and the maximum intervention amount (intra-marginal) for any given day. Figures 2 and 3 illustrate these macro-structural and micro-structural factors, respectively. The description of each specific variable or factor appears in the second column (*Variables*) in the diagrams. Factors have been indexed with letters and numbers. For instance, B2 refers to parameters that affect the way the authorities perform their intra-marginal intervention: greater dispersion would result in lower Central Bank activity in the market when the exchange-rate is away from the limits of its band and the maximum amount allowed for a given day limits the impact of the authorities in the market in order to avoid affecting the general trend (as posited by the objectives of the Colombian Authorities). In discussing the experiments, this structure will be maintained. Each parameter is described below.

Figure 1. Macro-structural Simulation Factors



The economic environment (A) is modelled with a set of stochastic differential equations where the deterministic part is composed of drift and cyclical components and stochastic behaviour is generated with a combination of Brownian and Poisson diffusion processes to capture volatility and non-normal behaviour (jumps). Modifications to the cyclic factors must be handled carefully as different processes - e.g. the interest-rate and the monetary base – are correlated as implicitly captured by the genetic algorithm fit, mainly in the phase factor. This is also the case for the volatility and jump-diffusion parameters because economic variables may share the same sources of uncertainty. In macroscopic terms, although modification of the economic parameters may yield interesting results, the scope of the simulations was limited to a study of the impact of the institutional (B) factors. Simulations replicate the Colombian (historical) case, using either the observed data or the fitted parameters.

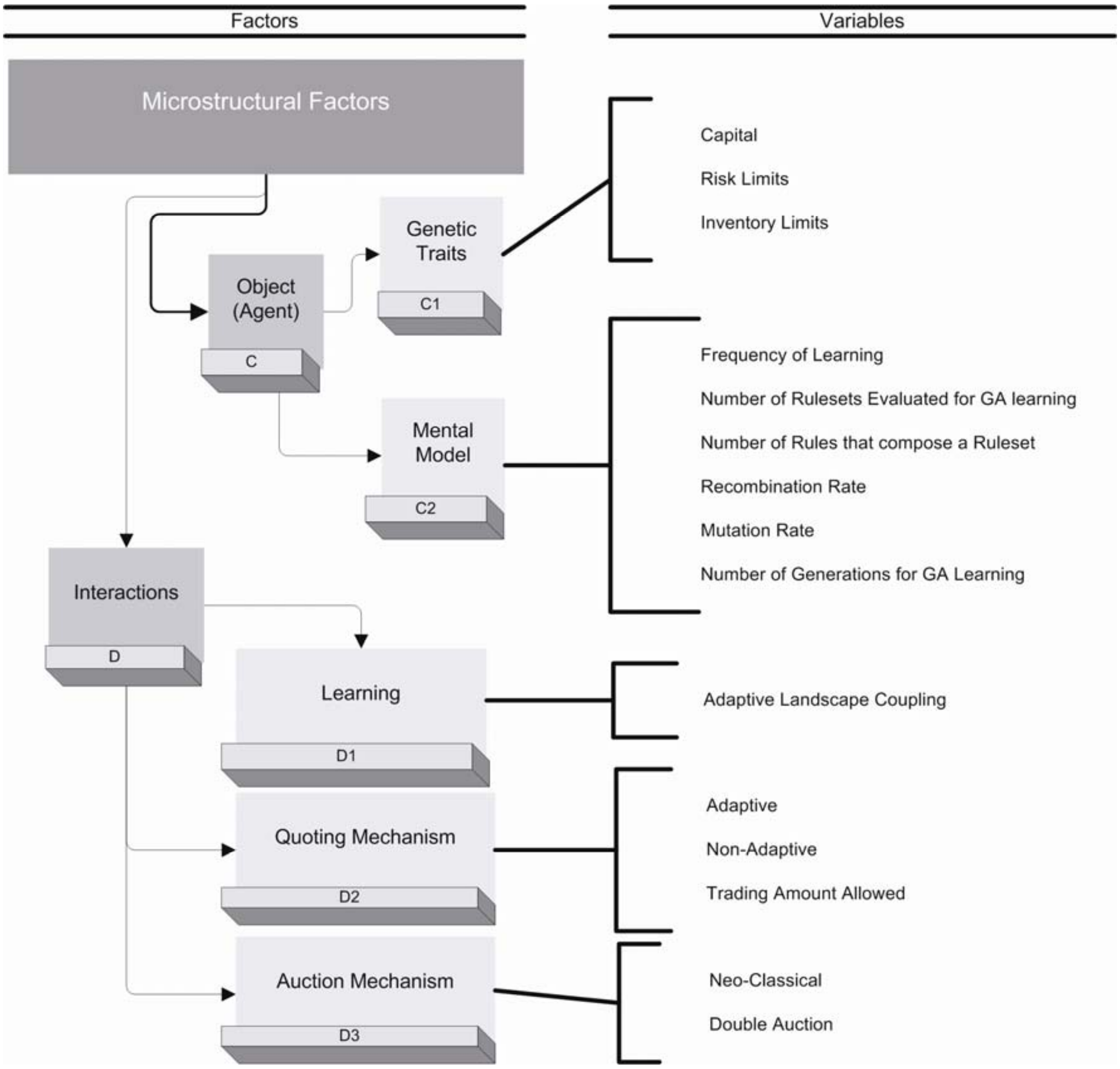
The analysis of changes to the institutional environment (B) is of the utmost importance in emerging countries, and in the Colombian case in particular, because regulatory changes occur often and an understanding of the potential consequences of these changes can increase the effectiveness of policy implementation (from the authorities' perspective) or provide significant returns to investors. For example, we could ask what would happen if short net positions were allowed in the FX market or if the band's width were to be decreased.

In accordance with complexity theory, microscopic factors are approached from the perspective of the interactions between the objects (D), and the objects (C) themselves, see figure 1 above. First, "genetic" fixed traits (C1) can be modified[8], resulting in changes in the market's topology. Second, agent's exploration or specialization in terms of choice of cognitive alternatives (models) can be modified with the recombination and mutation parameters of the GA.

Moreover, exploration or specialization in a given environment is a function of the number of generations that the GA runs for every call to the learning mechanism, and the frequency of this call (C2). For instance, if the number of generations is high for every call, the algorithm will repeatedly recombine from a limited pool of rules, containing the active rule-set plus the other rule-sets evaluated by the agents in the Fuzzy Logic Genetic Algorithm (defined by the parameter *Number of Rules Evaluated for GA Learning*).

[8] They remain fixed for the whole simulation.

Figure 2. Micro-structural Simulation Factors



This results in an over-specialization of the cognitive model for the specific conditions prevailing in the market, so that if, say, the exchange rate was near the upper limit of its band, rules relevant for this condition will be favored by the GA, and when the exchange rate moves away from its band, the agent's model may be inadequate for the new conditions and the agent's relative fitness will decrease (factor D1). This phenomenon will also occur if the GA is called too frequently although this will partially be offset by the fact that every call occurs under distinct environments.

The sophistication of any "cognitive" model is linked to the number of rules that make up a rule-set. When traders are allowed to choose rules (including erroneous rules) from the whole universe of discourse, the level of sophistication will be a function of the combination of the number of right and wrong rules and the mechanism of aggregation of the Fuzzy Logic system.

Therefore, when erroneous rules are included in the alternatives available to the agent, co-adaptation of fitness landscapes (comparative fitness of the agent's models) becomes volatile as a single erroneous rule may sharply decrease an agent's performance in a given environment, leading in turn to modification of most of the rules in the rule-set even though they may be adequate in most of the environments. The same applies to the absence of a given rule (or subset of rules) in a very specific environment, e.g. missing the proper rule near the limits of the band.

Auctions mechanism (D3) are defined in exactly the way the Citiinfo[9] information trading system worked in Colombia during the period under study whereas distinct quoting mechanisms (D2) were explored mainly during the calibration of the model. Trading amounts allowed are also taken from the Colombian market conventions.

Given the intrinsic computational power limitations to agent-based modelling, two sets of simulation exercises were conducted. In the first one (set 1), the market ran from 900 days to 1750 days with 15 to 30 intra-day transactions (5 experiments), while in the second one (set 2) 150 days and 100 transactions, respectively, were used (3 experiments). In the former, the focus is on long-term (emergent) behaviour whereas in the latter, shorter-term properties are studied. Statistical results and the observed emergent properties are robust across all simulations. Table 1 lists the parameters or the range of values used for the simulations.

Table 1 - Simulation parameter ranges

Variable	Range
Limits US Dollar Positions (USP)	USP $\in$ [0,0.2*Capital] or USP $\in$ [-0.2*Capital, 0.2*Capital]
FX Band Drift (FX_D)	FX_D= 15% all runs
FX Band Width (FX_W)	FX_W= ± 7% all runs
FX Band Central Parity (FX_CP)	FX_CP= Mid-Point or

[9] Foreign Exchange trading system.

Variable	Range
	FX_CP∈[MinBand,MaxBand]
Allowed Daily Dispersion (ADD, Intra. Int.)	ADD ∈ {0.5%, 0.8%, 1%}
Capital (CAP)	CAP ∈ [0-USD100,000,000]
Risk Limits (VAR)	VaR ∈ [0,10%] depending on horizon
Inventory Limits (INV)	INV ∈ [-0.2*Capital, 0.2*Capital]
Frequency of Learning (FLG)	FLG ∈ [1,30]
Number of Rulesets evaluated for GA Learning (NRE)	NRE= 10 all runs
Number of Rules that make up a Ruleset (NRC)	NRC ∈ {30, 40, 50}
Recombination Rate (XR)	XR ∈ {0.4, 0.6, 0.8}
Mutation Rate (MR)	MR ∈ {0.05, 0.1, 0.2}
Number of Generations for GA Learning (NG)	NG ∈ {7, 10, 20}
Trading Amounts Banks (TAB)	TAB= {100, 300, 500, 700, 1000 } all runs
Trading Amounts Real Sector (TAR)	TAR= 250 all runs
Trading Amounts Central Bank (TAC)	TAC= 250 all runs
Maximum Intraday Intervention CB (AMC)	AMC ∈ {10000, 25000}

The results obtained are presented in the next section. The analysis is divided into statistical properties, emergent patterns and learning evolution for a long-term set, and into statistical properties and learning evolution for a short-term set.

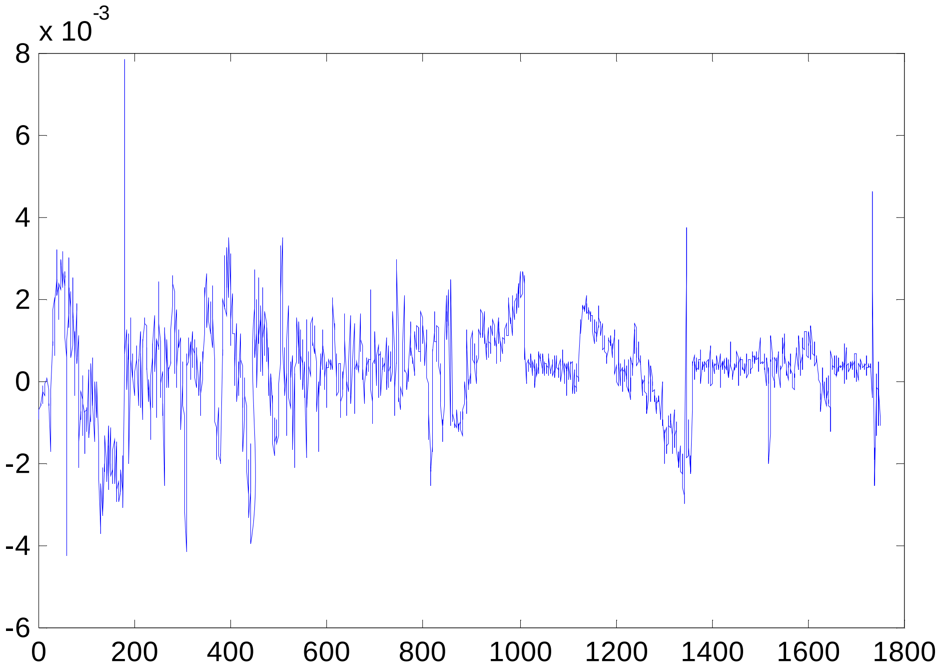
4. Artificial Market Simulations

4.1. Set 1: Long-term behaviour analysis

4.1.1. Statistical Properties

Figure 3 shows a sample of the (daily) returns series obtained for the long-term runs.

Figure 3 - Returns (Set 1)



Statistical analysis show that the series has a small trend and is leptokurtic - see Table 2. The histogram plot presented in Figure 4 supports this. As with the Colombian peso average returns, some autocorrelation is present. It is partially the result of the exchange-rate sticking to the band limits[10] but it also seems to be caused by the concentration of the real-sector supply and demand in only a few transactions as only around 15 intra-day transactions are allowed in this first exercise.

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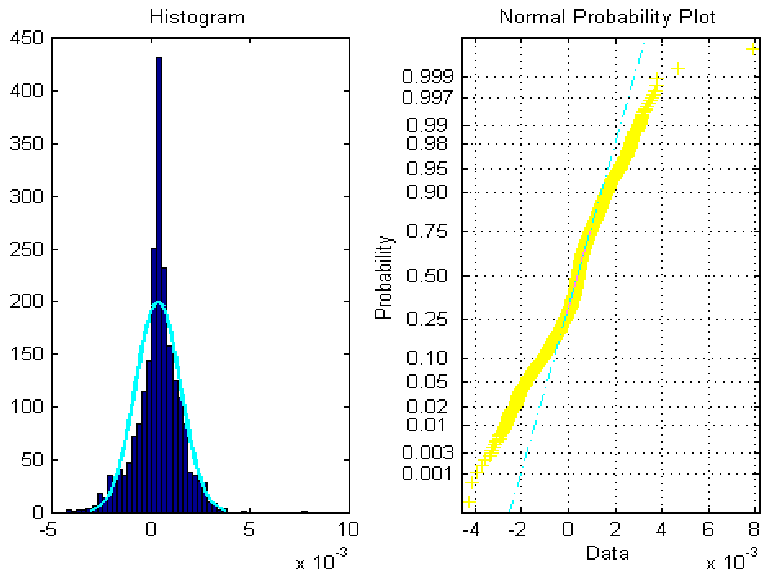
[10] As no realignment occurred, the exchange rate from the simulations stays near the band limits more time that the actual exchange rate from the Colombian market.

Table 2 - Moments for Set 1 Sample

Mean	Median	Standard Deviation	Variance	Skewness	Kurtosis
0.0003528	.00041063	0.0011167	1.247e-006	-0.25869	4.9996

Gaussianity is rejected by the Hinich test. Also, the normality plot presented in Figure 4 shows that returns are not Normal. These findings are supported by the Normal Gaussianity test. Also, the null hypothesis of a unit root is rejected in favor of a stationary alternative.

Figure 4 - Histogram and Normality plot for Returns (Set 1)

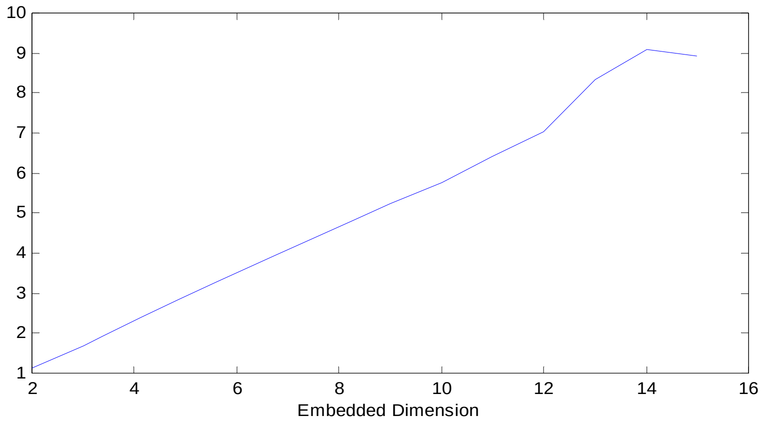


More interestingly, the ARCH test supports the hypothesis of ARCH effects for the returns series. This hypothesis is also supported by the Ljung-Box test on the squared-returns autocorrelation. Moreover, when the BDS test is applied to the estimated residuals of a GARCH(1,1) model, the null hypothesis is rejected as some further degree of non-linearity seems to be present in the returns series.

Additional study of non-linearity shows that the correlation dimension converges when the embedded dimension reaches a high value of 15 – see figure 5. The white noise test supports the hypothesis of deterministic chaos.

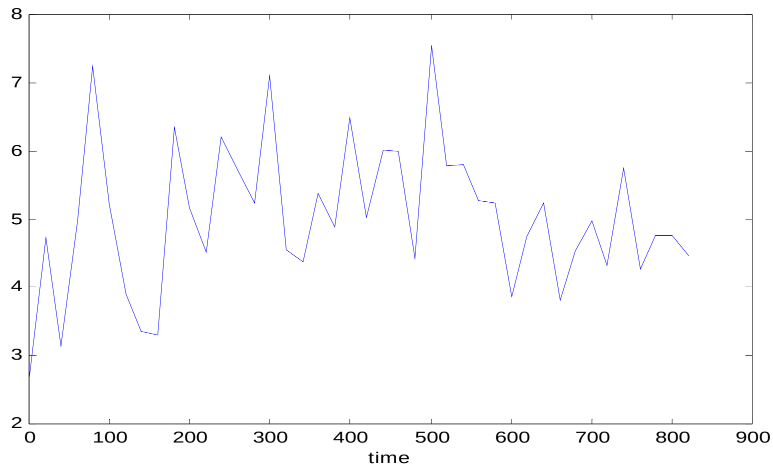
The Lyapunov spectrum shows that the movement is unstable while the momentary largest Lyapunov spectrum is inconclusive. Thus, like other artificial market “agent-based” models, the market gives rise to time series with leptokurtic returns and heteroscedasticity. It also exhibits other sources of non-linearity although its form cannot be determined and deterministic chaos cannot be rejected.

Figure 5 - Correlation Dimension for Returns (Set 1)



As shown in Figure 6, the complexity of the system, as measured by the pointwise correlation dimension, varies significantly. This is to be expected in a system of autonomous agents as their mutually reinforcing behaviors yield a geometry-dynamics duality.

Figure 6 - Pointwise Correlation Dimension for Returns (Set 1)

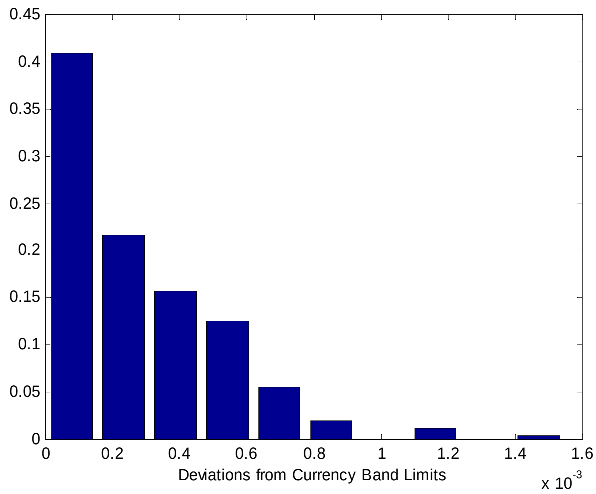


4.2.1.2. Emergent Patterns

As with the actual Colombian peso, although not explicitly encoded, a small trend emerges from the agent’s interactions – see Table 6 above. We now turn our attention to the emergent patterns that the artificial market was able to replicate. It must be stressed that these behaviours arise while the market evolves: they are not encoded in the program. Only the mere possibility of their existence is allowed. For example, the definition of the currency band by the authorities does not ensure that the exchange rate will not cross this threshold. There is no specific rule that forbids agents from trading over this limit or compels them to modify their signal. Central Bank intervention, through actual sales or purchases, must "protect" these limits.

If the intervention is successful, in time agents should include adequate rules in their ruleset. In this context, an event such as "exchange rate not overshooting the bands" would be an emergent property of the system. In terms of the currency bands, for the data series from set 1, around 7.7% of the trades occur slightly over the limits[11] with an average absolute deviation of 0.024% and a maximum absolute deviation of 0.155% of the mid-band – see Figure 7.

Figure 7 - Deviations from Currency Bands (Set 1)

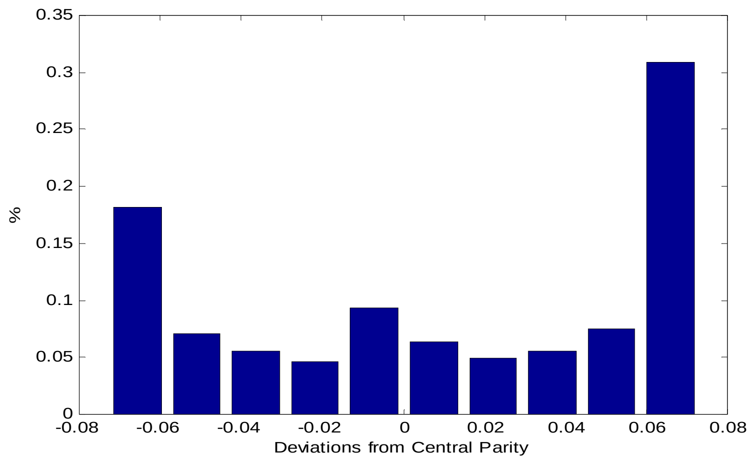


Intervention by the Central Bank, implemented through actual trading with market participants, limits the trend in the market and successfully "defends" the limits stipulated by the authorities. Currency realignment, although possible, does not occur. From it to happen, the international reserves would have to fall by more than 500 million dollars in less than 90 days. Also, mimetic contagion, which could trigger it,

[11] In order for the Central Bank to quote in the market, the market rate must exceed the band limit in the previous time step.

may occur if reserves exceed a given threshold and traders prefer to buy foreign currency[12]. In this situation, however, the only seller will be the authorities and, as we mentioned before, only 15 transactions are allowed. As a result, the rate of decrease of the reserves will not be high enough with an average daily intervention of around US$4.9 million[13], including intra-day intervention. For this reason, the exchange rate tends to remain more time pegged to the upper limit than the observed data. The U-shaped curve for the histogram of deviations, consistent with currency bands theory, is observed – see Figure 8.

Figure 8 - Deviations from Central Parity (Set 1)



Exchange-rate volatility increases in the centre of the band, as the rate changes rapidly and decreases near the limits as expected by currency bands theory. Table 3 lists the standard deviation, the average returns and the time spent in each of the regimes defined. For these regimes, the currency band is divided into four intervals of equal size: the intervals that include the limits are defined as regime 1 and 3 (top and bottom, respectively) while the two inner intervals together constitute regime 2.

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[12] In a currency crisis in an emergent country, traders carefully follow the level of the international reserves and a threshold - either to determine when to take or reverse a position - is usually determined implicitly by participants in the inter-bank market. In the Colombian currency crisis in 1998, a level of reserves of US$7.5 billion seems to have triggered many attacks to the upper band.

[13] The average amount on the days when intervention occurs.

Table 3. Moments for Set 1 Sample

Set 1	Standard Deviation	Daily Average Returns	Time Spent
Regime1	0.0933%	0.0492%	39.26%
Regime2	0.1576%	0.0427%	33.94%
Regime3	0.1161%	0.0211%	26.80%

When the intra-day dispersion allowed by the Central Bank is decreased from 1% to 0.8%, intra-marginal intervention increases slightly, as well as local volatility – more trend reversals occur at the local level although overall volatility is decreased. As Central Bank intra-marginal intervention is generally small, increasing the maximum allowed daily amount of intervention does not change the results of the simulation.

A decrease in the currency band width results in an increase in overall Central Bank intervention but exchange-rate behaviour does not change significantly in terms of its statistical properties, such as mean return, volatility or time spent in each regime. Interestingly, allowing traders to take short positions (up to 20% of their capital) does not significantly affect the market's trend towards the upper limit of the band although reversals are more pronounced – cyclic behaviour has higher amplitude.

An additional, and surprising, emergent property is that, during the learning process (see below), artificial agents include into their rule-sets (and retain) rules that refer to the level of international reserves, just as traders in the Colombian market did during the 1997-1998 currency crisis.

4.2.1.3. Evolutionary Learning[14]

The results of our simulations enable us to assert that a general pattern arises when the artificial agents try to improve their rule-sets. In time, the importance of rules related to fundamentals diminishes, with the exception of the level of the international reserves. Rules that refer to the US dollar supply and demand are gradually discarded from the pool of rules, while agents keep around 10% of rules that refer to the net supply and demand.

[14] For the analysis of evolutionary learning, agents draw rules only from the pool of correct rules.

The role of fundamental rules is conditioned on the position of the exchange rate within its currency band. For instance, when the exchange rate is near one of its limits, fundamental rules are unsuccessful. Since the market rate stays near the limits almost 70% of the time, rules referring to fundamental information are not favoured.

In addition, because no realignment occurs in the simulations, the exchange rate spends more time in regimes 1 and 3 and this may bias agents' learning against economic rules. Hence, in accordance with currency bands theory, additional factors are included in the process whereby agents form their expectations. In this artificial market context, these can either be technical or regulatory factors.

In the long-term simulations, rules based on information about exchange-rate moving averages (technical signals) are eliminated while rules related to regulatory factors, such as the distance to the limits of the band, are incorporated. Tables 4, 5 and 6 report the results of the learning process for the longest simulation of 1750 market days.

In the first 500 days, the importance of economic rules increases because the simulated exchange rate is set equal to the mid-point of the currency band and the rate stays mostly in regime 2. Thus, as expected, while the exchange rate is far from the limits, the GA favours fundamental factors. Interestingly, during this period technical trading also increases – participation goes from 5.5% to 9%. It seems reasonable to conclude that this occurs because profitable trend-following strategies only arise away from the bands.

Table 4. Average Rules Participation in Agents' rulesets by Type of Rule Evolution

Set 1	Fundamentals	Regulation	Technical
250 Days	49.19%	45.34%	5.47%
500 Days	55.84%	35.09%	9.07%
750 Days	52.03%	41.90%	6.08%
1000 Days	44.97%	52.82%	2.21%
1750 Days	41.73%	57.32%	0.96%

After 750 days, when the exchange rate hits the top of its band (regime 1), the weight of the regulatory rules in the agents' cognitive model increases from 35% to 42%, while the use of fundamental and technical signals decreases from 56% to 52% and from 9% to 6% respectively. The importance of the level of the reserves increases sharply from 17.8% to 21.6% - see Table 5-6.

Table 5. Average Rules Participation in Agents' rulesets by Signal

Set 1	USD SS	USD DD	Domestic Int. Rate	FX MA	Net SS/DD	FXTop	FXBot	Reserves
250 Days	5.85%	5.90%	5.49%	5.47%	9.86%	24.34%	21.00%	22.09%
500 Days	9.38%	9.26%	8.89%	9.07%	10.45%	19.33%	15.76%	17.85%
750 Days	6.34%	6.47%	6.76%	6.08%	10.81%	21.60%	20.29%	21.65%
1000 Days	3.03%	2.56%	2.43%	2.21%	10.33%	25.80%	27.02%	26.63%
1750 Days	0.97%	0.82%	0.96%	0.96%	10.87%	28.77%	28.55%	28.10%

This trend is maintained until the end of the simulation, with international reserves accounting for most of the fundamental rules' participation. Most traders eliminate technical trading rules and the learning algorithm favours regulatory rules.

Dispersion amongst agents' rule-sets is relatively small, and it decreases for the economic-based and technical-analysis-related rules. In terms of the regulatory-based rules, dispersion amongst agents' choices of rules fluctuates. Moreover, the standard deviation of the rules related to environment regulatory and to international reserves – information that seems to be critical in a currency crisis environment - is several orders of magnitude higher.

Table 6. Standard Deviation of Agents' rulesets by Signal

Set 1	USD SS	USD DD	Dom. Int. Rate	FX MA	Net SS/DD	FX Top	FX Bot	Reserves
250 Days	0.0221%	0.0216%	0.0247%	0.0182%	0.0579%	0.5628%	0.4923%	0.3229%
500 Days	0.0364%	0.0335%	0.0365%	0.0388%	0.1843%	1.1172%	1.1542%	0.9446%
750 Days	0.0316%	0.0239%	0.0325%	0.0221%	0.0795%	0.2231%	0.5534%	0.4339%
1000 Days	0.0025%	0.0019%	0.0018%	0.0020%	0.0199%	0.1661%	0.1988%	0.1031%
1750 Days	0.0018%	0.0029%	0.0024%	0.0022%	0.0732%	0.8687%	1.1629%	1.4034%

On the other hand, evolutionary learning and its repercussions for market dynamics was moderately dependant on recombination and mutation parameters in terms of the rule-set choice made by agents when rules were chosen from the fundamental set of rules - for parameter range used. Volatility and heteroscedasticity, although favoured by a low rate of mutation (around 5%) and relatively high rate of recombination (around 65%), is more affected by the frequency of learning and the number of generations used by the GA in every call to the algorithm. Representative values of these parameters are a moderately high frequency of learning - 5 to 10 days - and between 5 and 7 generations for every GA call.

If the learning mechanism is invoked every day with a moderately high number of generations, volatility decreases as agents change their models too often and have no means of validating their rule-sets. On the other hand, if the learning algorithm is called infrequently, initially agents increase their inventory rapidly with a consequent increase in traded volume. Then, once the inventory and risk limits are reached, traded volume decreases and transactions are settled instead with the real sector, as in the homogeneous expectations case.

In contrast to the Santa Fe artificial market (Arthur et al. 1997) - where volatility arises from the vast number of possible models[15] from which agents can choose - variability in this market is created by the parallel association between the agents' rules and the entire ruleset with the environment (including other agents' rule-sets).

For a given rule-set that the agent may have at any given time step, his success is not only dependant on the intrinsic properties of his set of rules but also is conditioned upon the *generating conditions* – the physical environment (position in the currency band, economic trends) and the propensities created through feedback between the agents' rule-sets and the constraining physical environment (currency bands, real-sector participation, etc) prevailing at that particular point in time. This in turn is a function of the correlation amongst rule-sets.

4.2.2. Set 2: Short-term behaviour analysis

Interestingly, some statistical properties of the simulated exchange rate and some features of the agents' behavior differ from the results presented in the previous section for the long-term runs. For example, the arrival of real sector trades in 100 transactions every day (versus around 10 in the long-term runs) seems to increase the frequency of trend reversals and the exchange rate spends less time near the limits of its bands. The statistical properties and learning behavior of the agents are presented in the following sections.

For comparative purposes, for the 100 intra-day transactions, returns every 10 transactions were computed for running the tests. This allows us to check if the market

[15] Those arise from the multiplicity of signals that, through the Classifier system mechanism, gives the agents countless possibilities.

is generating different patterns when running with more or less intra-day transactions or if its behavior is just similar and the timing and amount of real-sector and Central Bank intervention does not affect market dynamics.

4.2.2.1. Statistical Properties

Table 7 lists the moments of returns for the time series obtained from a sample simulation from Set 2. The histogram and the Normality plots in Figure 9 show that returns are leptokurtic and have fat tails.

Table 7. Moments of Returns (Set 2)

Mean	Median	Standard Deviation	Variance	Skewness	Kurtosis
4.699e-005	5.9332e-005	0.00073495	5.4015e-007	-1.3354	34.8272

The standard deviation is slightly smaller and kurtosis is significantly higher than the values obtained from Set 1.

Figure 9. Simulated Returns Histogram and Probability Plot (Set 2)

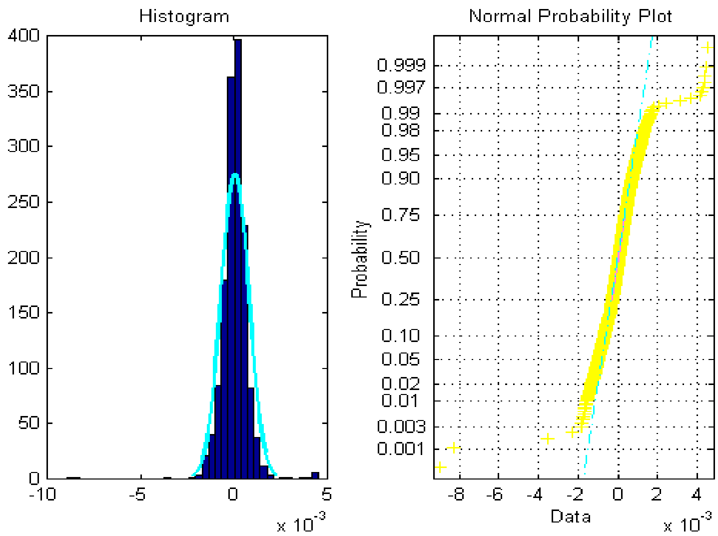


Figure 10 illustrates the simulated exchange rate and the amounts traded during the simulation. The Hurst Exponent of 0.468 supports the null hypothesis of a mean-reverting process as in the case of the COP. The amount traded varies significantly and from the plot we see that market reversals are linked to sharp changes, generally increases, in traded volume. The exchange rate started at the mid-point of its band and during this period (150 days) never touched the limits although the Central Bank intervened intra-marginally on 22 days with, on average, less than 1% of the average amount traded daily in the market.

Figure 10 - Simulated Rate and Amount Traded (Set 2)

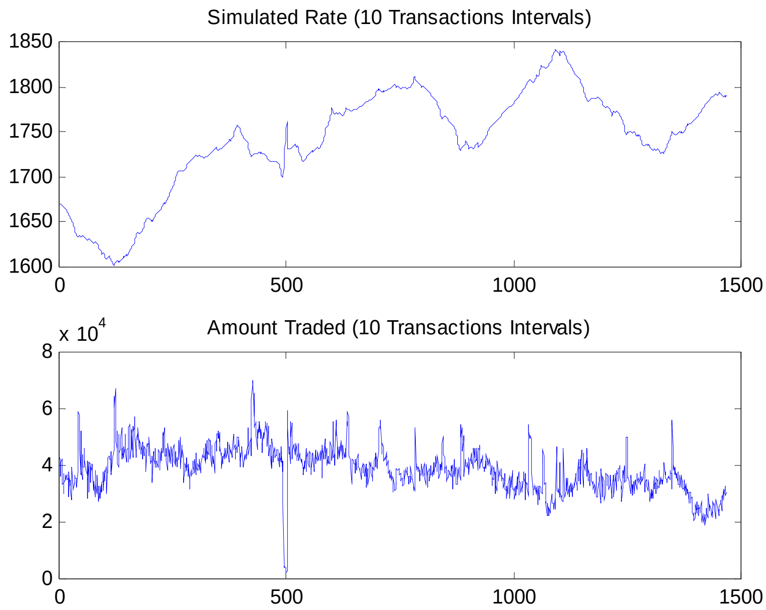
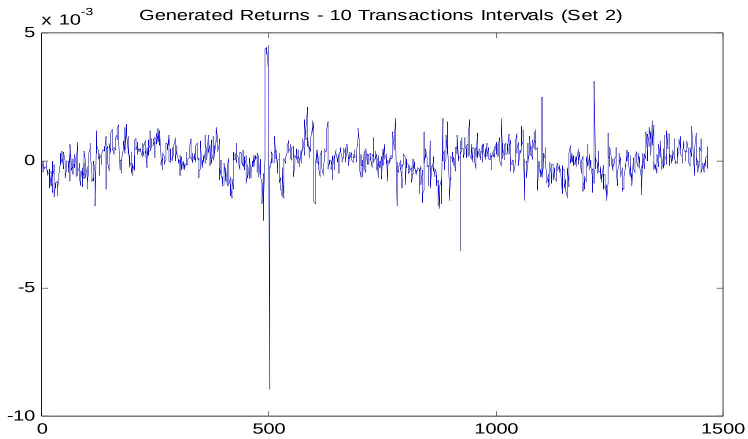


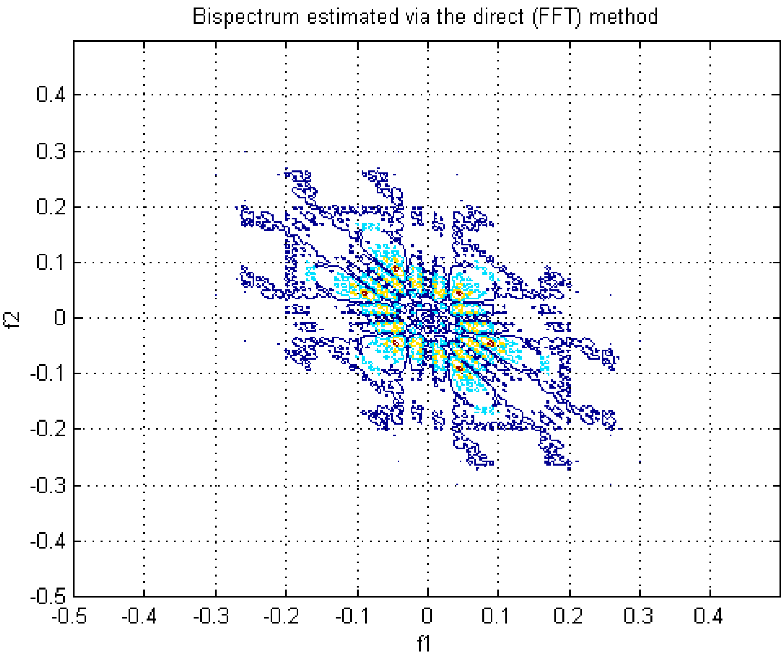
Figure 11 shows the returns generated by the simulation:

Figure 11 - Simulated Rate and Amount Traded (Set 2)



On day 50 (500 transactions), an external shock of excess demand of US$90 million initially stops the market (no amount traded) and then generates a pronounced trend reversal, which is followed by a fall in the exchange rate as many traders violate their inventory and risk limits. Approximately 15% of the total Central Bank intervention is executed in those seven days. Gaussianity is rejected for the returns series when the Hinich test is applied. The bi-spectrum plot and the Gaussianity test support this result. The former is shown in Figure 12. The unit-root hypothesis is rejected at the 1% significance level.

Figure 12 - Bi-spectrum Plot (Set 2)

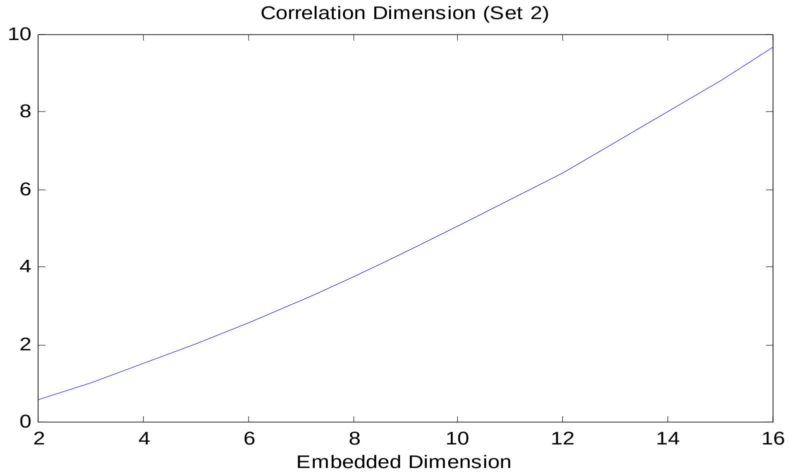


Squared returns differ statistically from zero for all lags tested using the Ljung-Box statistic. There is strong evidence for the presence of ARCH effects in the residuals of the linear model for the returns series. The null hypothesis is rejected when the BDS test is applied to the residuals of a GARCH(1,1). As with the COP hourly returns, rejection is less strong for intra-day returns than for daily returns.

Unlike the previous exercise, non-linearity tests yield a non-converging correlation dimension – see Figure 13. This is supported by the white-noise test. The biggest Lyapunov exponent from the Lyapunov spectrum is mainly positive for various combinations of embedded dimension, number of neighbours and polynomes. On the other hand, the momentary largest Lyapunov exponent does not have a definite bias. Again, although evidence against deterministic chaos is important (correlation dimension, momentary largest Lyapunov Exponent and white-noise Test), it cannot be rejected with confidence.

Complexity, as measured by the Pointwise Correlation Dimension, is high mostly when the demand shock occurs. As in the case of the long-run simulations, the market with artificial agents generates a time series with leptokurtic returns, fat tails and autoregressive heteroscedasticity.

Figure 13 - Correlation Dimension (Set 2)



Increased frequency of market trend changes – local volatility - seems to be linked to the fact that US Dollar flows arrive in batches. Also, there is evidence of non-linearity although the exchange rate never actually reaches the limits of its currency band. This result is consistent with currency bands theory as agents include in their expectations the possibility of central bank intervention.

In the next section, we show that in this set of exercises, agents increasingly include regulatory-related rules in their rule-sets although the limits are never reached.

4.2.1.2. Evolutionary Learning

As Table 8 shows, in this set of experiments the weights given to fundamental rules and the ones related with regulatory rules vary more, although there is a general tendency for the former to decrease and for the latter to increase. Technical trading gradually decreases.

Table 8. Average Rules Participation in Agents' rulesets by Type of Rule Evolution (Set 2)

Set 2	Fundamentals	Regulation	Technical
0 Days	65.758%	19.883%	14.359%
25 Days	64.706%	22.287%	13.006%
75 Days	58.168%	31.781%	10.051%
100 Days	55.106%	35.828%	9.067%
125 Days	55.763%	35.161%	9.076%
150 Days	57.568%	33.494%	8.938%

Analysis of the specific signals shows that the weight given to each of the fundamental related signals, except International Reserves, decreases rapidly[16]. The weight given to the level of the reserves increases from 9% to 17%. Similar increases occur for the variables related to monetary regulation.

Table 9. Average Rules Participation in Agents' rulesets by Signal

Set 2	USD Supply	USD Demand	Dom. Int. Rate	FXMA	Net SS/DD	FXTop	FXBot	Reserves
0 Days	14.930%	14.686%	14.002%	14.359%	12.999%	9.533%	10.350%	9.141%
25 Days	13.127%	13.076%	12.379%	13.006%	12.460%	10.912%	11.376%	13.665%
75 Days	10.854%	10.723%	10.678%	10.051%	9.588%	12.050%	19.730%	16.327%
100 Days	9.762%	9.869%	9.644%	9.067%	10.647%	15.458%	20.369%	15.183%
125 Days	9.959%	9.761%	9.621%	9.076%	11.554%	15.718%	19.443%	14.868%
150 Days	9.853%	9.564%	9.352%	8.938%	11.431%	15.961%	17.533%	17.368%

The variance of the weights given to each of these three variables is greater to the dispersion of the importance given to the other signals. Agents seem to carry a similar proportion of fundamental rules while more dispersion arises with signals related to regulation and international reserves.

Table 10. Standard Deviation of Agents' rulesets by Signal

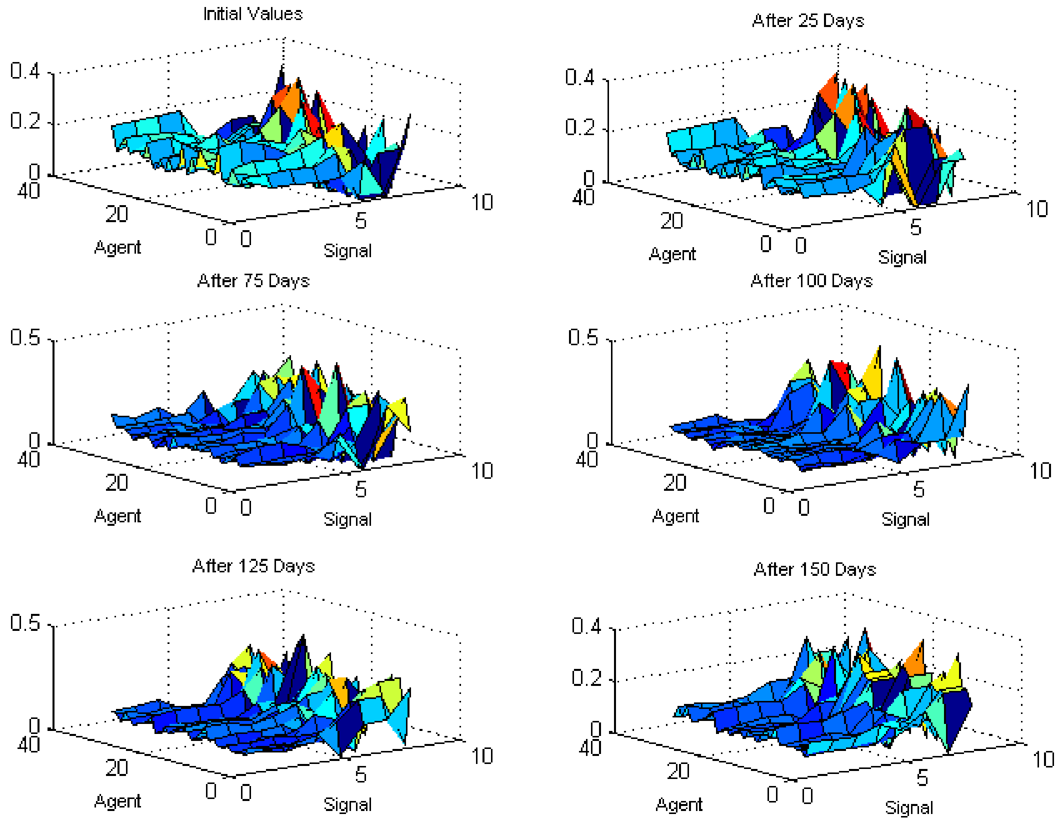
Variance	USD Supply	USD Demand	Dom. Int. Rate	FXMA	Net SS/DD	FXTop	FXBot	Reserves
0 Days	0.156%	0.138%	0.134%	0.123%	0.330%	1.346%	1.525%	1.272%
25 Days	0.101%	0.088%	0.089%	0.100%	0.441%	1.550%	1.669%	1.346%
75 Days	0.051%	0.040%	0.073%	0.040%	0.486%	1.480%	1.374%	1.031%
100 Days	0.025%	0.026%	0.050%	0.030%	0.415%	0.805%	1.286%	1.325%
125 Days	0.052%	0.052%	0.060%	0.056%	0.397%	1.223%	1.496%	1.097%
150 Days	0.048%	0.049%	0.054%	0.074%	0.256%	0.845%	1.066%	1.061%

This phenomenon can readily be seen in Figure 14. Rules 6, 7 and 8 are the distance to the top of the band, the distance to the bottom of the band, and the level of international reserves, respectively. Observe that as the days elapse, the surface flattens for indicators

[16] Agents learn in average every 5 to 10 days.

1 to 5. Dispersion for indicators 6 to 8 decreases slightly but clearly agents seem to have heterogeneous views regarding the importance of regulatory rules – some give more weight to the distance to the top while others give more weight to the distance to the bottom. The disagreement is more significant for the distance to the top of the band and the level of reserves.

Figure 14 - Signal Weights in Learning Fuzzy System (Set 2)

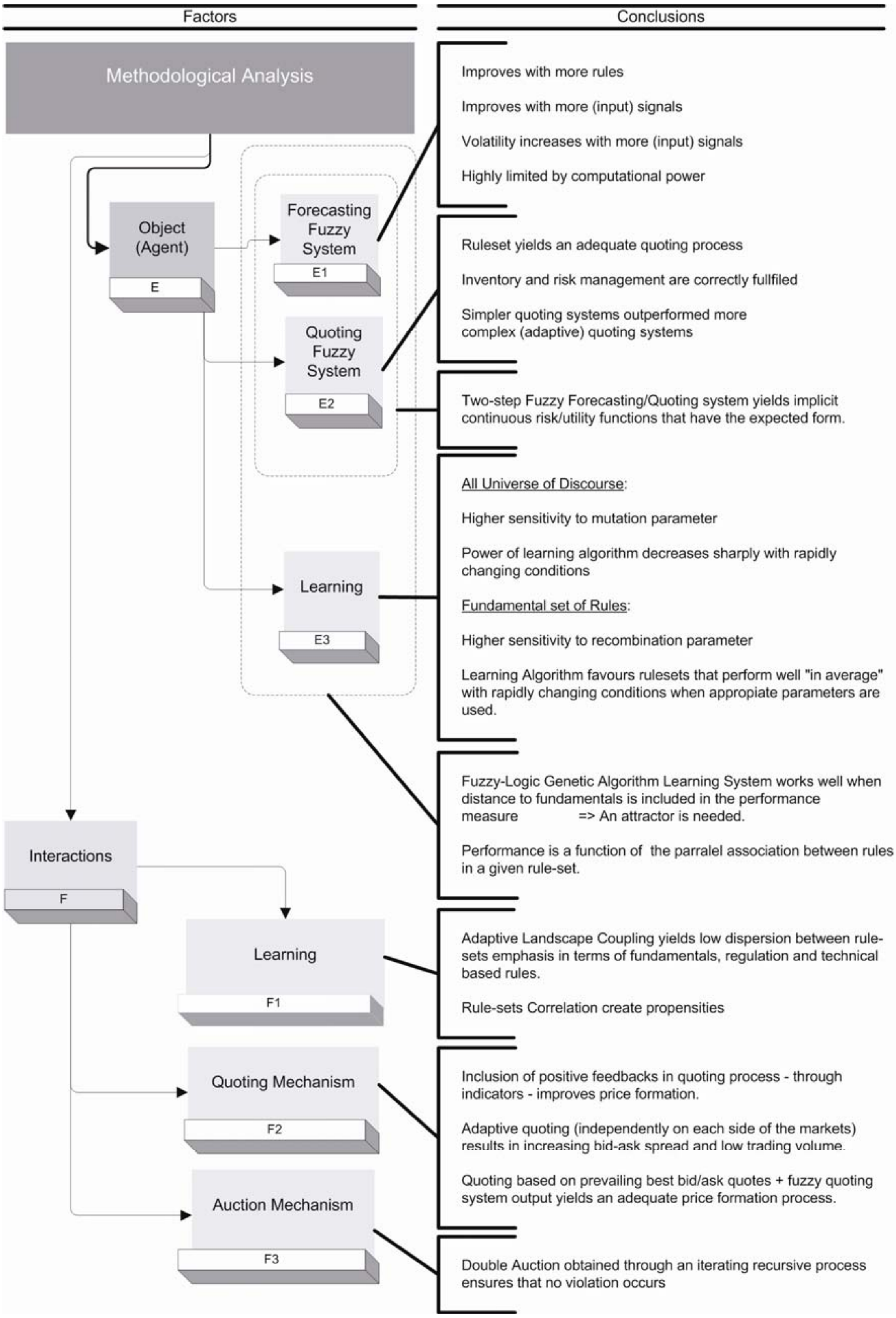


The dynamics of learning are well illustrated in these plots as agents permanently change the weights given to each signal but co-adaptation seems to keep them relatively close. The ‘*red queen*’ effect yields some evolutionary coupling. The fast rate at which rule-sets are modified is also apparent in the plots.

4.3. Methodological Discussion

Figure 15 below summarizes the methodological conclusions that are addressed in this section. As a first step, the discussion of the cognitive model – forecasting, quoting and learning – will be approached from the perspectives of the object (E1, E2 and E3 in the figure above) and the interactions (F1, F2). The forecasting model is able to generate heterogeneous forecasts through the various outputs created from the parallel association of distinct rules in the different rule-sets. This is an advantage of the forecasting subsystem as novel behaviour may arise when a few rules are changed in a rule-set in a given environment.

Figure 15 - Summary of Methodological Conclusions



In addition, although the system is sensitive to changes in the relationship between rulesets and environment, it still retains the averaging properties of fuzzy-logic weighted aggregation and de-fuzzification which provide an approximately continuous space for the relationships between inputs and outputs at each time step.

An increase in input signals in the forecasting mechanism seems to result in increased exchange rate volatility but it is very expensive in terms of computational power. With the set of 9 inputs/outputs used in most of the simulations (and their respective membership functions), around 750,000 rules can be constructed overall. The addition of just one input with three membership functions would increase the number of possible rules to more than 2 million. When more rules are included in an agent's rule set, the GA approximates better the *correct* rule-set – i.e. the set that includes the rules taken from economic models and the appropriate regulatory rules. Marginal improvement, however, decreases when more than around 35% of the rules are given to the trader. In fact, because environmental conditions are continuously changing, the GA tends to find a rule-set that "on average" performs well or that adapts well to the prevailing environment when conditions do not vary rapidly.

When the entire simulation is run, the number of rules that make up a rule-set radically affects computational complexity. For this reason, and bearing in mind that each agent continually evaluates in parallel 10 distinct rule-sets (for GA performance measures), the number of rules is limited to the set {30, 40, 50} for the simulations. This is a form of incomplete or asymmetric information but where favoured or affected traders are not defined ex-ante (informed vs. uninformed) and their status is co-dependent on the environmental constraints prevailing at each time step. This forces the GA algorithm to specialize in order to find the adequate set of rules for the current environment.

Like real traders, artificial agents have to select from amongst the various alternative models. It is not a question of finding the "correct" model but of identifying the appropriate model for a given context; and switching to another model rapidly when the situation changes. It is this search that generates higher volatility when rules are drawn from the *correct* or accepted set of rules only. It also accounts for the lower volatility that arises when rules are drawn from the whole universe of discourse (including erroneous rules) as it has a similar impact to greatly increasing the mutation rate, which in turn polarizes the market to the point where half the market is bearish and the other half is bullish[17] if the market is initialised by giving each trader a ruleset chosen randomly from the whole universe of discourse. Allowing traders to draw rules from the *correct* pool of rules is the best methodological choice because it is clear that an investor who also learns deductively would not use erroneous, illogical rules.

A great advantage of using a fuzzy logic system is that it allows the researcher implicitly to supply the agents with a set of rules that are *deductively* and *rationally* chosen and ensures that the heterogeneity generated in the market does not arise from the use of erroneous models (that human traders would never use anyway).

GA convergence improves significantly when a factor that measures how well the ruleset is forecasting compared to the “fundamental” rule set – the distance to the fundamentals – is included in the performance measure. Although its weight is less than 1/3 of the total indicator, it results in an increase in traded volume and an increase in volatility and heteroscedasticity. It seems that a small attractor is needed in the formation of expectations – just as banks modify their expectations (models) when new information arrives – which increases the level of co-adaptation between rule-sets. Some coherence is needed amongst agents at the microscopic level of the models (rule-sets) for volatility to emerge – this reminds us of Prigogine’s (1998) requirement for coherence at the microscopic level for non-equilibrium to exist.

An adequate quoting process that at the same time manages inventory and risk exposures has been achieved. The inclusion of feedbacks in the quoting system through indicators that refer to best market quotes, average market quotes, bid/ask spreads, market intention and the agent’s own information (his spread, inventory and risk) provide the basis for market dynamics in a setting similar to the way a trader actually works. Again, deductive logic is implicitly (partially) included through the symbolic interface of fuzzy logic systems. Moreover, it yields an approximately continuous set of functions for the risk and expected return of the trader that are constructed from the quoting rule set.

[17] The number of erroneous rules is much greater than the number of correct rules.

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