Conditional Portfolio Choice in the U.S. Bond Market: The Role of Liquidity

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Abstract

In this paper, I estimate the non-parametric optimal bond portfolio choice of a representative agent that acts optimally with respect to his/her expected utility one period forward, provided that he/she observes the ex ante liquidity signal. Using daily observations of zero-coupon Treasury and TIPS bonds yields, I construct equally-weighted returns from 2004-2012. Considering alternative measures of liquidity, I find that the liquidity differential between nominal and TIPS bonds appears to be a significant determinant of the portfolio allocation to U.S. government bonds. In fact, conditional allocations in risky assets decrease as market liquidity conditions worsen, and the effect of market liquidity decreases with the investment horizon. I also find that the bond return predictability translates into improved in-sample and out-of-sample asset allocation and performance.
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1 Introduction

Numerous empirical studies conclude that excess bond returns are predictable in the sense that they depend on the current value of some predictor

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variables. In addition, the term structure slope, the forward spread, the lagged excess returns, the Cochrane and Piazzesi (2005) tent-shaped factor, and macroeconomic fundamentals are some of the variables that have been identified as predictors for Treasury bonds (Fama and Bliss (1987), Campbell and Shiller (1991), Cochrane and Piazzesi (2005), Ludvigson and Ng (2009) and Cooper and Priestley. (2009)). The role of market liquidity as a predictor variable for government bonds has been studied more recently by Fontaine and Garcia (2011), Pflueger and Viceira (2012) and Gomez (2013). They provide empirical evidence for liquidity as a source of predictability for U.S. Treasury bonds, U.S. Treasury Inflation-protected bonds (TIPS), or for both.

The question as to whether or not asset returns are predictable is of significant importance for portfolio choice. In their seminal papers, Merton (1969) and Samuelson (1969) show that if asset returns are independently and identically distributed (IID) over time, then the optimal asset allocation is constant over time. However, Kim and Omberg (1996), Brennan et al. (1997) and Viceira and Campbell (1999) show that if asset returns are predictable, then the optimal asset allocation depends on the investment horizon and the predictive variables.

While some studies provide insight into the role of liquidity as a predictor variable, few studies examine the effect of liquidity risk on optimal portfolio allocation. Ghysels and Pereira (2008) provide empirical evidence that the relevance of liquidity for stock portfolio choice depends on both the asset and the investment horizon. Garleanu (2009) studies portfolio choice and pricing in markets in which trading may take place with considerable delay, and shows that the liquidity level has a strong impact on portfolio choice. This paper focuses on examining how changes in liquidity risk premium influences optimal portfolio allocations in U.S. government nominal and index-linked bonds.

Throughout this paper, I assume that the investor makes decisions in real terms where the investment horizon is one-month, one-quarter and one-year. I only consider a short-term investor in the empirical analysis. The reason for this is related to the fact that for a buy-and-hold long-term investor, whose investment horizon perfectly matches the maturity of the bond, TIPS offer full protection against inflation if held until maturity.[1] Similarly, an investor who adopts a buy-and-hold strategy for TIPS mitigates risk arising from illiquidity,

[1] TIPS are a useful hedge against inflation, but they do not guarantee a real rate of return. This is because the mechanics of adjusting for inflation for TIPS limit the exactness of the inflation adjustment and allow only approximate inflation hedges especially at high inflation levels. In fact, for TIPS, the reference price index is the non-seasonally adjusted CPI-U, and the indexation lag is three months. Therefore, TIPS operate with an indexation lag of three months. In other words, it takes three months from the incidence of price inflation (the month when a reference index reading is recorded) until it is incorporated into the coupon payment of the inflation-linked bond. Consequently, the indexation lag affects how well TIPS compensate for contemporaneous inflation, and prevents TIPS from guaranteeing a specified real return.

given that he/she does not face higher costs of buying or selling the bond before it reaches maturity. However, TIPS are currently issued with only a few specific maturities: 5-year, 10-year and 30-year, therefore the investment horizon over which I consider investors who hold assets does not match the maturity of any outstanding TIPS.[2] Hence, I study a short-term investor who maximizes real wealth but is not able to invest in a risk-less asset in real terms (given that TIPS are a risky asset both in nominal and in real terms), and also faces liquidity risk. Notice, however, that a short-term investor benefits from the availability of TIPS in terms of a wider investment opportunity set that allows an increase in the returns per unit of risk, investing even a small fraction of his wealth in TIPS (Cartea et al. (2012)).

The investor's problem is to choose optimal allocations to the risky asset as a function of predictor variable: the TIPS liquidity premium. As risky assets, I consider equally weighted bond portfolios on short-term bonds (1 to 10 years maturity); and on long-term bonds (11 to 20 years maturity), each of them are computed for Treasury bonds and for TIPS. The existence of a TIPS liquidity premium is well established. In fact, TIPS bonds have been characterized by being less liquid than nominal Treasury bonds.[3] TIPS' lack of liquidity compares with nominal Treasuries results in TIPS yields having a liquidity premium relative to Treasuries.[4] Since this liquidity premium is unobservable, different alternative ways of proxing liquidity have been proposed in literature. In particular, I test two market-based measures for the liquidity differential between inflation-indexed bonds and nominal bonds proposed by Christensen and Gillan (2011) and Gomez (2013). The first one is computed as the spread between synthetic and cash break-even inflation rates, while the second one corresponds to the asset swap spread on similar maturity inflation-linked and Treasury bonds. Both measures allow us to identify the relative liquidity premium between two comparable assets, which in this case arise from the cost derived from TIPS liquidity disadvantage relative to Nominal bonds.

The particular choice of these two measures for liquidity is motivated by the fact that: *i*) even though they are highly correlated (which suggests that all of them are capturing similar information about the liquidity differential between

[2] U.S. Treasury inflation-protected securities were introduced in January 1997. TIPS bonds have been offered in 5-, 10-, 20-, and 30-year denominations. However, TIPS that have less than one year remaining to maturity are not easy to find in the secondary market, given that they have extremely high transaction costs.

[3] The existence of this liquidity premium in TIPS yields has been well documented in the academic literature by Sack and Elsasser (2004), Shen (2006), Hordahl and Tristani (2010), Campbell et al. (2009), Dudley et al. (2009), Christensen and Gillan (2011), Gurkaynak et al. (2010), Pflueger and Viceira (2012), among others.

[4] Liquidity risk premium is defined here as the total cost of all frictions to trade a relative less liquid asset beyond those of the more liquid asset against which it is being compared (Christensen and Gillan (2011)).

nominal and TIPS yields), they are measured using information from different markets, which would allow them to capture different aspects of the liquidity premium, especially in times of financial distress where each market tends to be driven by its specific dynamics, such as funding costs.[5] Next, I am interested in testing if the optimal portfolio choice depends on a particular choice to proxy liquidity premium, *ii*) they are market-based measures of liquidity which is straightforward to compute, and by construction they are also model-free.

Finally, I consider the portfolio policy of an investor who is able to invest in only one risky asset, and I differentiate various portfolio allocation problems: first, where the investor chooses between the portfolio of short-term or long-term Treasury bonds and a risk-free asset; and second, where the investor chooses between a portfolio of short-term or long-term TIPS and a risk-free asset. I also study an investor with mean-variance (MV) and constant relative risk aversion (CRRA), with different degrees of risk aversion, in order to test the sensitivity of the optimal portfolio choice to the higher moments.

There are a series of ways in which this study contributes to the literature. First, it incorporates financial information (liquidity premium) in an asset allocation context, and shows how this can be of significance for both a mean-variance and a CRRA investor. Second, it focuses on a bond portfolio choice that is relatively unexplored in the literature, since the majority of the studies on asset allocation examine stock-only portfolios. Brennan et al. (1997) (who were the first to analyze portfolio choice in the presence of time-varying expected returns), point out that the degree of the asset return predictability has a significant effect on the composition of the optimal portfolio. Therefore, the evidence in favor of bond return predictability (by means of variables such as liquidity) imply that a bond portfolio setting provides a robust framework to examine. Additionally, bonds-only portfolios are extremely important for the fund management industry and for central banks, as well as for liquidity, and inflation risk are highly relevant for insurance and the risk management of pension funds. Third, I examine portfolio choice among multiple government bonds with different maturities. More so, I consider both the U.S. Treasury bonds and inflation-linked bonds in the investor's asset menu.

I make use of an econometric framework based on a portfolio choice problem of a single period investor, where the investor's problem is set up as a statistical decision problem, with asset allocations as parameters and the expected utility as the objective. The allocations are estimated by direct maximization of

[5] Theoretically, there exists a close relationship between bond break-evens and inflation swaps rates. In essence, both measure the markets' expectations of future inflation. However, the most recent crisis showed that U.S. cash and swap markets can diverge significantly, with each market driven by its specific dynamics such as funding costs. Asset swapping activity should theoretically hold the two markets together, but the empirical evidence, discussed by Gomez (2013), shows that such activity is not sufficient to offset diverging forces in stressed market conditions.

expected utility proposed by Brandt (1999). A number of key results emerge from this analysis. First, the liquidity premium seems to be a significant determinant of the portfolio allocation of U.S. government bonds. In fact, conditional allocations in risky assets decrease as liquidity conditions worsen. In particular, an increase in the liquidity differential between nominal and TIPS bonds leads to lower optimal portfolio allocations for nominal Treasury bonds, and also to lower optimal portfolio allocations in TIPS, but at different levels of liquidity. Additionally, the effect of liquidity is a decreasing function of investment horizons, in the sense that for the same degree of risk aversion the investor reacts less abruptly to an increase in the liquidity premium when he/she has a longer investment horizon. Furthermore, as the investment horizon becomes longer, the smaller the optimal portfolio weight, and so, the less is invested in the risky asset.

The above conclusions are not determined by the level of risk aversion or the investors preferences. The relation between optimal portfolio weights and the liquidity premium remains the same for different values of risk aversion, and also across investor preferences. These characteristics mainly change the level of the portfolio function, having a small impact on the shape of the function. In addition, results do not depend on a particular choice of the maturity of the liquidity premium (similar results are found when considering 10-year or 20-year liquidity premium), nor on a specific way to proxy liquidity (I have similar results with both liquidity premium measures).

From the standpoint of practical advice to portfolio investors, a final natural question to ask is whether or not the bond return predictability translates into improved out-of-sample asset allocation and performance. To answer this question, I compare the performance of the optimal portfolio choices of two investors: one investor who makes portfolio allocations based on the belief that bond returns are predictable by liquidity (conditional strategy); and the other who believes that bond returns are independent and identically distributed (i.i.d.), and ignores any evidence of bond return predictability in making his/her portfolios allocation choices (unconditional strategy). I conclude that the conditional strategy outperforms the unconditional strategy, improving not only the in-sample, but the also out-of-sample asset allocation and performance.

The rest of the paper is organized as follows. Section 2 defines the conditional portfolio choice problem, provides a description of the liquidity premium measures available in the literature and presents the non-parametric estimation technique used. I describe the data and provide some basic statistics in Section 3. Section 4 presents the empirical results for different bond portfolios, different types of investors and different investment horizons. Section 5 concludes.

2 The conditional optimal portfolio problem

The traditional problem of optimal portfolio choice considers an investor which maximizes the conditional expected utility of next period's wealth under a budget constraint. Merton (1969) provides the solution, where the investor can trade continuously in a finite set of stocks and bank account. However, given that the stocks and bonds differ in many ways, the theory of portfolio management does not apply as it stands to bond portfolios (see Ekeland and Taflin (2005) for a discussion of this point). For the bond market, Schroder and Skiadas (1999), Ekeland and Taflin (2005), Ringer and Tehranchi (2006) and Liu (2007) have studied this problem using a theoretical approach. In particular, Ekeland and Taflin (2005) and Ringer and Tehranchi (2006) set up, and solve the problem of managing a bond portfolio by optimizing (over all self-financing trading strategies for a given initial capital), the expected utility of the final wealth. Thus, optimal portfolio at time t is a linear combination of self-financing instruments, each one with a fixed time to maturity. Under this set up the value of the portfolio changes only because the bond prices change. Price bonds behave like price stocks, that is, it depends only on the risk it carries and not on time to maturity.

The impact of return predictability on optimal portfolio choice have also been considered in literature. Initially, it was studied under the assumption of no parameter or model uncertainty by Viceira and Campbell (1999), Balduzzi and Lynch (1999), Wachter (2002), Munk et al. (2004). Subsequently, Barberis (2000) incorporates parameter uncertainty, but does not allow for dynamic learning. More recently, Brandt et al. (2005) consider learning about other parameters of the return processes in addition to the predictive relation.

Various other papers investigate the effects of an aversion against ambiguity about the return process on portfolio choice (Maenhoud (2006), Liu (2010), Liu (2011), Chen et al. (2011) and Branger et al. (2013)). There is also a growing literature on portfolio selection that incorporates return predictability with transaction costs, started by Lynch and Balduzzi (2000), Brandt et al. (2004) and recently by Lynch and Tan (2011), and Garleanu and Pedersen (2009). Empirical studies also have been undertaken by Brandt (1999), Ait-Sahalia and Brandt (2001), and Brandt and SantaClara (2006) consider different predictive variables, while Ghysels and Pereira (2008) have the only paper that includes liquidity as a predictor variable (except for stock portfolio allocation problem).

On the other hand, the impact of inflation on portfolio choice also has also been considered in the literature. An initial extension of the Markowitz problem was introduced in the 1970s by Biger (1975), Friend et al. (1976), Lintner (1975) and Solnik (1978), among others. Intertemporal portfolio choice problem under inflation risk was studied by Campbell and Viceira (2001) in discrete time, and by and Brennan and Xia (2002) in continuous time. Both works tell us that a long-term, risk-averse investor prefers the indexed bond or a perfect substitution of indexed bond in order to hedge against the inflation risk. However, in these papers all relevant state variables are assumed observable and the probability distributions of all processes are assumed known. Bensoussan et al. (2009) and Chou et al. (2010) relax that restriction by assuming that the expected inflation rate is unobservable to the investor.

More recently, motivated by the fact that all these papers disregard model uncertainty (inflation model misspecification), Munk and Rubtsov (2012) solve a stock-bond-cash portfolio choice problem for a risk- and ambiguity-averse investor in a setting where the inflation rate and interest rates are stochastic and the expected inflation rate is unobservable. Also, De Jong and Zhou (2013) investigate the optimal portfolio and consumption policies for a finite horizon investor in a life-cycle model with habit formation and inflation risk.

Most of the existing studies on portfolio choice (with or without inflation risk), focus on stock-only portfolios (Viceira and Campbell (1999), Barberis (2000), Wachter (2002)), or examine the stock-bond mix portfolio choice (Munk et al. (2004)). Given the extensive literature for equity markets, it is surprising to note that no effort has been undertaken to examine the influence of liquidity in government bond portfolio choice. Filling this gap is one contribution of this paper. To follow, I define the investor's maximization problem, describe the conditioning information, and finally, introduce the estimation technique.

2.1 Investor utility maximization

2.1.1 Portfolio choice without inflation

Ekeland and Taflin (2005) and Ringer and Tehranchi (2006) express the solution of optimal portfolio choice as portfolios of self-financing trading strategies which naturally include stocks and bonds. In particular, they fix a utility function u and a planning horizon $T > 0$, and consider the functional $J(\varphi) = \mathbb{E}^{\mathbb{P}}[u(W_T^{\varphi})]$ where W_T^{φ} is the accumulated wealth at time T generated by the self-financing trading strategy φ. The goal is to characterize the strategy that maximizes J.

Following on from this literature, I consider the problem of optimal portfolio choice when the traded instruments are a set of risky bonds and a risk-less bond. In particular, and without loss of generality, I consider a bond market where only zero-coupon bonds are available. Fixing a utility function $u(W_{t+1})$ and a planning horizon $T > 0$, I consider an investor who maximizes the conditional expected utility of next period's wealth, subject to the budget constraint:

$$\begin{aligned}\max_{\alpha_t \in \mathcal{A}(\varphi)} \quad & \mathbb{E}[u(W_{t+1}) \mid Z_t] \\ \text{subject to:} \quad & W_{t+1} = W_t[R_{f,t+1} + \alpha_t(R_{b,t+1} - R_{f,t+1})]\end{aligned} \tag{1}$$

where W_{t+1} is the accumulated wealth at time $t+1$ generated by the self-financing trading strategy φ (which belongs to the set of admissible self-financing strategies denoted by $\mathcal{A}$), α_t represents the proportion of wealth invested in a risky bond with return $R_{b,t+1}$ and the remaining proportion $1-\alpha_t$ is invested in risk-free bond with return $R_{f,t+1}$. The expectation is conditional on a state variable Z_t. The investor can have three different horizons: one-month, one-quarter or one-year (this represents the difference between t and $t+1$).

The weight that maximizes the expected utility function is the solution to the following Euler optimality condition

$$\mathbb{E}\left[\frac{\partial u(W_{t+1})}{\partial W_{t+1}} \frac{\partial W_{t+1}}{\partial \alpha} \mid Z_t\right] = 0. \tag{2}$$

In particular, the solution of the investor's problem is the mapping from the state variable Z_t to the portfolio weights

$$\alpha_t = \alpha(Z_t), \tag{3}$$

and it denotes the portfolio choice of observing a signal $Z_t = z$.

The relation between the portfolio policy and the predictability of individual moments of the returns given the predictor Z_t depends on the specification of the utility function. I consider two types of investor preferences: mean-variance (MV) and power-utility (CRRA) preferences. An investor with mean-variance preferences maximizes

$$\max_{\alpha_t \in \mathcal{A}(\varphi)} \quad \mathbb{E}[W_{t+1} \mid Z_t] - \frac{\gamma}{2}\mathbb{V}[W_{t+1}^2 \mid Z_t], \tag{4}$$

where $\gamma > 0$ represents the coefficient of absolute risk aversion. The investor portfolio policy when the choice includes a risk-free rate is proportional to the conditional mean-variance ratio of the tangency portfolio

$$\alpha_t^{tg} = \frac{1}{\gamma W_t} \frac{\mathbb{E}[R_{t+1}^{tg} \mid Z_t]}{\mathbb{V}[R_{t+1}^{tg}]},$$

where R_{t+1}^{tg} is the return of the tangency portfolio. The reason I consider MV preferences is because it can be stated as a primitive, or can be derived as a special case of expected utility theory. Also, under MV preferences, portfolio weights depend exclusively and analytically on the two first moments of returns, which serve as benchmark case in this study.[6]

I also consider the most popular objective function in the portfolio choice literature, which is an investor with CRRA or power utility. In this case, the investor solves the following problem

$$\max_{\alpha_t \in \mathcal{A}(\omega)} \begin{cases} \mathbb{E}\left[\frac{W_{t+1}^{1-\gamma}}{1-\gamma}\right] & \text{if } \gamma > 1 \\ \mathbb{E}\left[\log(W_{t+1})\right] & \text{if } \gamma = 1 \end{cases} \tag{5}$$

subject to the budget constraint in (1), and where $\gamma > 0$ now measures the coefficient of relative risk aversion. As is well known, unlike mean-variance preferences, CRRA does not permit a closed form solution to the investor's portfolio problem. However, I consider CRRA preferences to be able to test whether or not an investor cares about higher order moments of the return distribution.

2.1.2 Portfolio choice with inflation

In this section, I follow Cartea et al. (2012) who solve the optimal portfolio choice problems for investors concerned with maximizing real wealth. Here, I assume that investors make allocation decisions in real terms, and are worried about the purchasing power of their terminal wealth, and do not suffer from money illusion. As before, I consider the optimal investment allocation of investors who are not worried about what may happen beyond the immediate next period but rather, care about the purchasing power of their wealth.

To avoid exposure to inflation risk, investors can: (*i*) invest in a risk-less asset in real terms; and/or (*ii*) invest in assets that covary with inflation. However, in this empirical analysis I only consider investors who have a maximum investment horizon of 1-year; they cannot find TIPS with this maturity and thus they are not able to invest in a risk-less real asset. Additionally, given that real interest rate changes affect TIPS returns, investors consider TIPS as a risky asset in both nominal and real terms.

An investor with MV or CRRA preferences maximizes the same problem in (4) and (5), respectively, but are now subject to the budget constraint

$$W_{t+1}^R = W_t^R[R_{f,t+1} + \alpha_t(R_{b,t+1} - R_{f,t+1})],$$

where W_{t+1}^R is now the terminal real wealth, and $R_{b,t+1}$ and $R_{f,t+1}$ are real risky and risk-free bond returns, respectively, as already seen.[7] In the absence

[6] Although the limitations of mean-variance analysis are well established in portfolio theory, its relative simplicity and easy intuition contributes to its continued use among investment professionals, in theoretical and empirical studies.

[7] In this case, the real risk-free bond returns is calculated as $R_{f,t+1} - \pi_{t+1}$, where π_{t+1} is the log inflation rate.

of a real risk-free asset investors face inflation risk and deal with this through the covariances between the returns of risky assets and inflation. Securities which are correlated with inflation help to hedge against inflation, reducing the portfolio variance in real terms.

2.2 Liquidity measures

It is generally acknowledged that liquidity is important for asset pricing. At a theoretical level, two main views (not mutually exclusive), have been advanced to explain why liquidity should be priced by financial markets: illiquidity (i) creates trading costs; and (ii) can itself create additional risk. The first view holds that illiquid securities must provide investors with a higher than expected return to compensate for their larger transaction costs, controlling for fundamental risk. This view was first proposed and tested by Amihud and Mendelson (1986) for stock-market data, and by Amihud and Mendelson (1991) for fixed-income security markets. The second view suggests that liquidity is priced not only because it creates trading costs, but also because it is itself a source of risk, since it changes unpredictably over time, as developed by Pastor and Stambaugh (2003). These two views have resulted in considerable literature on the relation between returns and liquidity.

On the other hand, the existence of differences in market liquidity conditions between nominal and inflation-indexed Treasury securities is well known. Different practical approaches have been used to measure this liquidity differential. In general, two approaches have been implemented: market-based measures used by Christensen and Gillan (2011) and Gomez (2013); and a regression procedure used by Pflueger and Viceira (2012).

Christensen and Gillan (2011) identify the liquidity component in TIPS yields using information from the bond market and also from the inflation swap market. An inflation swap is a bilateral contractual agreement. It requires one party (the inflation payer), to make periodic floating-rate payments linked to inflation, in exchange for predetermined fixed-rate payments from a second party (the inflation receiver). The most common contract is the zero-coupon inflation swap, which has the most basic structure with payments exchanged only on maturity.

The rates observed, $IS_{n,t}$, represent the fixed rate paid by the inflation receiver, that is, the rate that fixed rate agents are willing to pay (receive) in order to receive (pay) the cumulative rate of inflation during the life of the swap. Hence the quoted rate can be also viewed as a break-even inflation rate (BEI), which depends on expected inflation over the life of the swap. Thus, it is possible to use the quoted rate to derive market-based measures of expectations for inflation.

In theory, the inflation compensation implicit in the prices of nominal bonds relative to index-linked bonds should be the same as that found in inflation swap rates. The two should be consistent due to arbitrage.[8] Thus, in a frictionless world this equality must hold

$$IS_{n,t} = \pi^e_{n,t} = BEI_{n,t},$$

where $BEI_{n,t}$ denotes the cash break-even inflation rate and $\pi^e_{n,t}$ is the expected average inflation rate for the next n years.

However, in reality the cash BEI and inflation swap rates are not equal. As occurs in the ILB market, the market for inflation swaps are less liquid than the market for nominal Treasury bonds, such that the observed price of each asset should contain a non-negative time-varying liquidity premium that biases its yields upwards (Christensen and Gillan (2011)). That means that inflation swap rates should be adjusted by liquidity risk. The observed inflation swap rate (commonly referred as *synthetic break-even rate*) are given by

$$\widehat{IS}_{n,t} = IS_{n,t} + L^{IS}_{n,t},$$

where $L^{IS}_{n,t}$ is the liquidity premium included in the inflation swap rates.

Christensen and Gillan (2011) argue that the liquidity component in BEI identify from the difference between observed BEI and inflation swap rate

$$\Delta_{n,t} = \widehat{IS}_{n,t} - \widehat{BEI}_{n,t} = L^{IS}_{n,t} + L^{ILB}_{n,t}, \tag{6}$$

They showed that this result hold under two assumptions: *i)* the market for ILBs and inflation swaps are less liquid than the market for nominal Treasury bonds; and *ii)* the nominal Treasury yields we observe are very close to the unobservable nominal yields that would prevail in a frictionless world, that means $\hat{y}^N_{n,t} = y^N_{n,t}$. Under these assumptions, the difference between the two rates is the sum of the liquidity premiums in TIPS and inflation swaps.

In a recent paper, Gomez (2013) measures the market liquidity premium in TIPS by looking at how inflation-linked asset swaps on nominal bonds corresponds to inflation-linked ones. The idea is that this asset swap spread captures the relative financing cost, the special nature and the balance sheet cost of TIPS over nominal Treasuries. These characteristics make some securities easier to liquidate and more attractive to hold than others, so this spread should be a good market-based measure of the market perception of relative liquidity in a bond market.

An asset swap is a derivative transaction that results in a change in the form of future cash flows generated by an asset. In the bond markets, asset

[8] That is because the pay-offs of index-linked bonds can be replicated using inflation swap contracts. Two portfolios with identical future pay-offs should have the same price via arbitrage. Hence, with perfect markets we would expect perfect substitution between break-even rates available in the inflation swap and bond markets

swaps typically take fixed cash flows on a bond and exchanges them for Libor (i.e. floating rate payments) plus asset swap spread (ASW), which can be positive or negative. Thus, an asset swap is equivalent to buying a bond and entering into an interest rate swap with maturity matching the bond.[9]

The *z-asw* spread between a nominal and inflation-linked asset swaps, is given by:

$$L_{n,t}^{z\text{-}asw} = z\text{-}asw_{n,t}^{ILB} - z\text{-}asw_{n,t}^{N} = L_{n,t}^{ILB}. \quad (7)$$

This spread should be non-negative, $L_{n,t}^{z\text{-}asw} \geq 0$, and equal to the liquidity premium in the inflation linked bond.

Pflueger and Viceira (2012) estimate the TIPS liquidity premium explicitly using a model. They regress the break-even inflation rate on a set of three measures of liquidity in bond markets: the nominal off-the-run spread, relative TIPS transaction volumes and the difference between TIPS asset-swap-spreads and nominal U.S. Treasury asset-swap spreads. They also control for inflation expectation using the survey of long-term inflation expectations (π^{SPF}) and the Chicago Fed National Activity Index (CFNAI). They estimate

$$\widehat{BEI}_{n,t} - \pi^{SPF} = a_1 + a_2 X_t + a_3 CFNAI_t + \varepsilon_t,$$

where X_t is a vector containing our three liquidity proxies. They obtain the TIPS liquidity premium as the negative of the variation in $\widehat{BEI}_{n,t} - \pi^{SPF}$ explained by the liquidity variables, while controlling for the CFNAI as a proxy of short-term inflation expectations. Hence, the estimated relative liquidity premium in TIPS yields equals

$$\hat{L}_{n,t}^{PV} = -\hat{a}_2 X_t. \quad (8)$$

An increase in $\hat{L}_{n,t}^{PV}$ reflects a reduction in the liquidity of TIPS relative to nominal Treasury bonds. Given that their liquidity estimate most likely reflects liquidity fluctuations in both nominal bonds and in TIPS, they assume that the liquidity premium $\hat{L}_{n,t}^{PV}$ is entirely attributable to time-varying liquidity in TIPS rather than in nominal bonds.

The measures described above allow us to identify the relative liquidity premium between two comparable assets, in this case the cost derived from TIPS liquidity disadvantage relative to nominal bonds.[10] As a result, the

[9] As for a nominal asset swap, the proceeds of a bond are exchanged against a floating rate interest payment, however the proceeds are not fixed but inflation-linked. Thus, a dealer might buy an indexed bond via a repo, provide an inflation-indexed cash flow to the market via an inflation swap and hedge its position with a standard interest rate swap.

[10] Absolute liquidity premium is defined as the price difference between the observed and the unobservable frictionless market outcome of a given asset. However, we work with the relative concept since it is extremely difficult to identify the unobservable frictionless price of an asset directly.

liquidity measures described above meet the same definition of liquidity premium. Specifically, liquidity refers to the total cost of all frictions (wider bid-ask spreads, lower trading volume, etc.) to trade off the less liquid asset beyond that of the more liquid asset against which it is being compared. However, I will use the two model-independent measures of liquidity premium to examine whether or not the liquidity differential between inflation-indexed bonds and nominal bonds (liquidity premium), represented by Z_t, constitute relevant conditioning information in the portfolio choice problem.

The reason for that particular choice is twofold. I use the market-based measures of liquidity because they are model-free and can be readily calculated using daily data, while Pflueger and Viceira (2012) liquidity premium is model-dependent by construction and it is only available on a monthly frequency.[11] Second, there exists a close relationship between bond break-evens and inflation swap rates, because theoretically, both rates measure the markets' expectations of future inflation. However, the most recent crisis showed that U.S. cash and swap markets can diverge significantly, with each market driven by its specific dynamics. Asset swapping activity should theoretically hold the two markets together, but the empirical evidence suggests that such activity was not sufficient to offset diverging forces in stressed market conditions (see Gomez (2013) for a further discussion). Consequently, even though the Christensen and Gillan (2011) and Gomez (2013) measures are highly correlated (which suggests that all of them are capturing similar information about the liquidity differential between nominal and TIPS yields), they are measured using information from different markets. Thus, it would make them capture different aspects of liquidity premium, especially in times of financial distress where each market tends to be driven by its specific dynamics, such as funding costs.

2.3 Non-parametric estimation

I use the methodology proposed by Brandt (1999) and Ait-Sahalia and Brandt (2001). They apply a standard generalized method of moments (GMM) technique to the conditional Euler equation that characterizes the investor's portfolio choice problem. In particular, it consists of replacing the conditional expectation with sample analogues, computed only with returns realized in a given state of nature where the forecasting variable level is $Z_t = \bar{z}$. Brandt (1999) suggests estimating the conditional expectation with a standard non-parametric regression. Ait-Sahalia and Brandt (2001) suggest a semiparametric approach to address the issue of which predictors are important for the portfolio choice when a large number of them are available.

[11] Pflueger and Viceira (2012) estimated liquidity premium from January 1999 to September 2010, only for 10-year TIPS and in a monthly frequency. Consequently, it does not have enough sample points to be considered in this study (74 observations).

Let a neighborhood of Z be $Z \pm h$ for some bandwidth $h > 0$. When the investor is characterized by the power utility, a simple non-parametric estimator of the conditional Euler equation is given by the Nadaraya-Watson estimator, where the moment condition is given by:

$$\hat{\mathbb{E}}\left[\frac{\partial u(W_{t+1})}{\partial W_{t+1}}\frac{\partial W_{t+1}}{\partial \alpha} \mid Z_t = \bar{z}\right] = \frac{1}{Th}\frac{\Sigma_{t=1}^{T}\left(\frac{\partial u(W_{t+1})}{\partial W_{t+1}}\frac{\partial W_{t+1}}{\partial \alpha}\right)k(Z_t, \bar{z}, h)}{\Sigma_{t+1}^{T}k(Z_t, \bar{z}, h)} = 0, \tag{9}$$

where $k(Z_t, \bar{z}, h)$ is the kernel function which is assumed to be Gaussian. I apply exactly identified GMM to equation (2) to obtain $\hat{\alpha}(Z)$ which is a consistent estimate for the unknown optimal portfolio choice $\alpha(Z)$ (See Ait-Sahalia and Brandt (2001) for asymptotic properties of this estimators). The conventional solution to optimize the classical trade-off between variance and bias is to choose a bandwidth of the form: $h = \lambda\sigma_z T^{-1/K+4}$, where λ is a constant, K is the number of predictor variables and σ_z is the standard deviation of the predictor Z (see Hardle and Marron (1985)).

Finally, the optimal unconditional portfolio weight is compute by applying a standard GMM procedure to the unconditional Euler equation. In this case the moment condition is:

$$\hat{\mathbb{E}}\left[\frac{\partial u(W_{t+1})}{\partial W_{t+1}}\frac{\partial W_{t+1}}{\partial \alpha}\right] = \frac{1}{T}\Sigma_{t=1}^{T}\left(\frac{\partial u(W_{t+1})}{\partial W_{t+1}}\frac{\partial W_{t+1}}{\partial \alpha}\right) = 0, \tag{10}$$

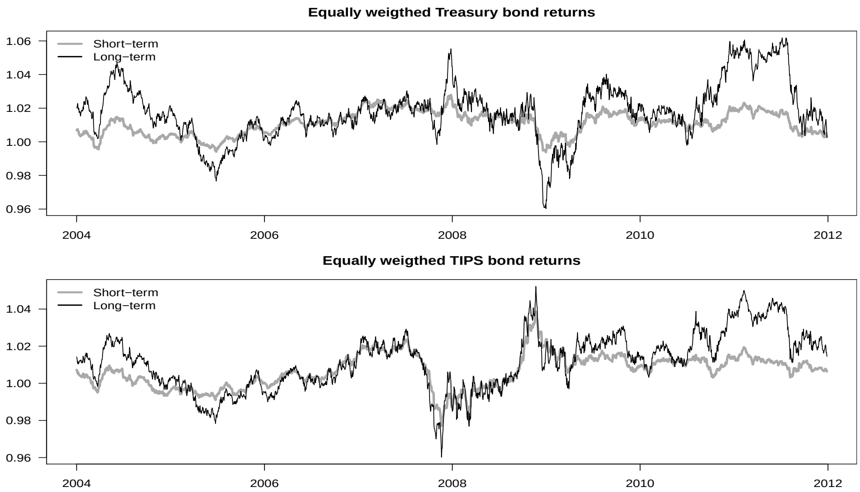
which yields the same results that directly compute weights from equation (3).

3 The Data and basic statistics

I am interested in the analysis of the empirical time-series relationship between optimal bond portfolio allocations and alternative measures of liquidity. To that end, I calculate monthly, quarterly and annual holding period returns from daily observations of zero-coupon nominal and real Treasury bond yields constructed by Gurkaynak et al. (2007) and Gurkaynak et al. (2010) for observed bond yields, respectively, available through the Federal Reserve web site. This data set contains constant maturity yields for maturities of 2 to 20 years. I construct equally weighted bond portfolios on short-term bonds (1 to 10 years maturity) and on long-term bonds (11 to 20 years maturity), each of them computed for Treasury bonds and for TIPS, ending up with four risky assets presented in Figure 1. The sample period is from January 2, 2004 to December 31, 2012.

For the same period, I also collect information on one-year Treasury bills from the Federal Reserve Board statistical releases. Following Ait-Sahalia and Brandt (2001) and Ghysels and Pereira (2008) I assume Treasury bill is risky-free, and I fix the risk-free rate at its historical average. They argue that the

Figure 1: Yearly return portfolios



Equally-weighted U.S. Government bond return portfolios calculated using daily data from January 2, 2004 to December 30, 2011.

constant risk-free rate assumption guarantees that any difference in the optimal portfolio functions across frequencies is solely due to the relation between returns and liquidity. In summary, the asset universe consists of the short-term Treasury bonds (weight α_{NS}), the long-term Treasury bonds (weight α_{NL}), the short-term TIPS (weight α_{RS}), the long-term Treasury bonds (weight α_{RL}) and the risk-free assets (weight α_{rf}).

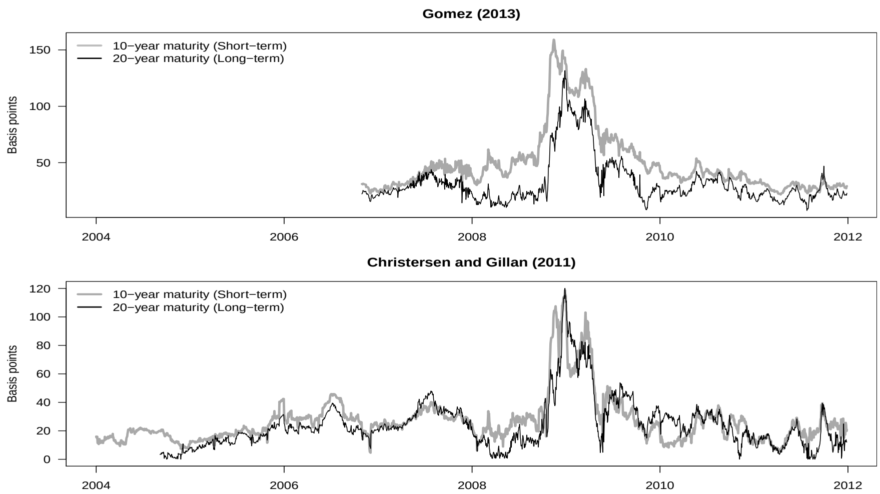
For liquidity, I use two-market based measures available on a daily frequency. The first measure is the liquidity measure proposed by Christensen and Gillan (2011). The data used to construct the liquidity premium proposed by Christensen and Gillan (2011) corresponds to daily estimates of zero-coupon nominal and real Treasury bond yields constructed by Gurkaynak et al. (2007) and Gurkaynak et al. (2010) for observed bond yields. For zero-coupon inflation swap rates, I use U.S. daily quotes from Barclays Live, which I have converted into continuously compounded rates to make them comparable to the other interest rates. I compute their liquidity measure, denoted by $\Delta_{n,t}$, for 10- and 20- years to maturity from January 2004 to December 2011.

The second measure is the asset swap liquidity premium used by Gomez (2013). I obtain daily nominal and TIPS z-spread asset swaps data from Barclays Live, starting in November 2006 until December 2011 for 10-years maturity (short-term portfolios liquidity) and 20-years maturity (long-term portfolios liquidity).[12] The residual spread between different TIPS and nominal

[12] Asset swaps on bonds with less than 12 months to maturity are dropped from the

z-spread asset swaps with the same maturity was calculated. Next, the average spread across different assets for each maturity was computed, and this corresponds to my liquidity premium measure, $L_{n,t}^{z\text{-}asw}$ for $n = 10, 20$ years maturity.

Figure 2: Liquidity measures



The *z-asw* liquidity corresponds to the residual spread between TIPS and nominal bonds asset swaps calculated using daily data from November 1, 2006 to December 30, 2011. The maximum range liquidity corresponds to the difference between cash and synthetic break-even inflation rates proposed by Christensen and Gillan (2011) calculated using daily data from January 1, 2004 to December 30, 2012.

In Figure 2, I plot the evolution of liquidity premium measures $L_{n,t}^{z\text{-}asw}$ and $\Delta_{n,t}$ for short-term portfolios and long-term portfolios at a daily frequency. One can see that the values for both measures are strictly positive. Furthermore, $L_{n,t}^{z\text{-}asw}$ liquidity premiums tend to be downward sloping with maturity, indicating that the shorter-term liquidity premium is greater than the longer-term, especially during the crisis time. However, it seems not to be the case when liquidity is measured using the Christensen and Gillan (2011) measure. Additionally, the magnitude of the liquidity premium varies across measures. In fact, over the whole sample the mean short-term liquidity has been about 49 basis points for $L_{n,t}^{z\text{-}asw}$ compared with 29 basic points for $\Delta_{n,t}$. What is clear in both measures is that the liquidity premium grew substantially during the financial crises of 2008 and 2009. In fact, liquidity shows a peak in late 2008 during the financial crisis. In summary, although they are very similar and seem to be consistent (in the sense that they are able to capture the same events observed in the considered sample period), they show differences

estimation of the liquidity, because the effect of the indexation lag makes the prices of these securities erratic as was noted by Gurkaynak et al. (2010). All other asset swaps are included in the calculation.

in the magnitude of statistics calculated from the two liquidity measures. For this reason, I am interested in testing whether or not the optimal portfolio choice depends on a particular way to measure liquidity premium.

Table 1 shows descriptive statistics of the liquidity predictors and holding period government bond portfolio returns, for the three investment horizons: one-month, one-quarter and one-year. The first lines in each panel show the mean, standard deviation, skewness and kurtosis for each liquidity measure and returns. By construction, and to facilitate the interpretation of the results, liquidity measures have a mean zero and standard-deviation equal to one (i.e. they have been standardized). The correlation coefficients between liquidity measures are more than 0.90. This suggests that all measures are capturing similar variations in the market bond yields. Also, there is evidence of fat tails in returns, especially at the shorter investment horizon. This tail risk suggests that the distribution is not normal, but skewed, and has fatter tails. The fatter tails increase the probability that an investment will move beyond three standard deviations. Nominal returns are negatively correlated with liquidity while TIPS returns are positively correlated. This means that as liquidity conditions worsen (higher liquidity premium), TIPS returns rise in order to compensate for the higher risk in bad times.

The following lines show the autocorrelation coefficients for different lags, which do not suggest persistence in most of the variables, especially at any frequency. The last line shows the p-value for the Dickey and Fuller test. The *p*-value for the Dickey and Fuller tests suggest the rejection of the null of a unit root for both short-term and long-term returns, and Christensen and Gillan (2011) 10-years liquidity. However, $L_{n,t}^{z\text{-}asw}$ seems not to be stationary. Given that the non-parametric approach requires stationary data, I would need transform each of those variables in order to make them a stationary series. However, it is not clear in Figure 2 that liquidity is not stationary. First, they are not moving along a decreasing or increasing time trend, and second, there are upward peaks related to the financial crisis, but before and after that they seems stationary. Consequently, I decided to work with the original series.

4 Empirical results

4.1 Unconditional portfolio weights

The goal in this section is to characterize the unconditional portfolio choice which serves as a benchmark for the conditional problem. Table 2 presents estimates of unconditional portfolio choices of investors with MV and CRRA preferences with different risk aversion degrees of γ = 2, 5, 10 and 20, and for three investment horizons. The entries in each column correspond to a portfolio choice between Treasury bills (assumed as risk-free) and one of

Table 1: Descriptive Statistics for the portfolio measures of liquidity and bond returns

	Short-term				Long-term			
	$\Delta_{10,t}$	$L^{z\text{-}asw}_{10,t}$	R^{N}_{t+1}	R^{TIPS}_{t+1}	$\Delta_{20,t}$	$L^{z\text{-}asw}_{20,t}$	R^{N}_{t+1}	R^{TIPS}_{t+1}
Panel A: Monthly frequency								
Mean	0.00	0.00	1.04	1.02	0.00	0.00	1.06	1.03
Stdev	1.00	1.00	0.02	0.02	1.00	1.00	0.04	0.03
Skewness	2.58	1.98	0.03	-0.34	1.91	2.27	0.49	0.10
Kurtosis	11.15	6.33	3.70	6.30	8.36	7.97	5.60	5.91
Percentiles								
5%	-0.95	-0.85	1.01	0.99	-1.17	-0.86	0.99	0.98
50%	-0.19	-0.31	1.05	1.02	-0.18	-0.32	1.06	1.03
95%	2.15	2.54	1.07	1.05	2.06	2.75	1.11	1.07
Cross correlations								
$\Delta_{n,t}$	1.00				1.00			
$L^{z\text{-}asw}_{n,t}$	0.91	1.00			0.93	1.00		
R^{N}_{t+1}	0.05	0.07	1.00		-0.13	-0.11	1.00	
R^{TIPS}_{t+1}	0.33	0.28	0.46	1.00	0.18	0.15	0.59	1.00
Auto correlations								
1-day	0.99	1.00	0.95	0.96	0.99	0.99	0.95	0.94
2-day	0.98	0.99	0.91	0.92	0.98	0.98	0.90	0.89
5-day	0.95	0.98	0.80	0.78	0.95	0.96	0.77	0.72
22-day	0.76	0.89	0.07	0.06	0.77	0.81	-0.06	-0.11
Unit root test								
DF p-value	0.02	0.53	0.01	0.01	0.14	0.36	0.01	0.01
Panel B: Quarterly frequency								
Mean	0.00	0.00	1.05	1.02	0.00	0.00	1.06	1.03
Stdev	1.00	1.00	0.03	0.03	1.00	1.00	0.07	0.05
Skewness	2.58	1.98	0.04	-0.55	1.91	2.27	0.28	-0.26
Kurtosis	11.15	6.33	2.80	6.78	8.36	7.97	3.32	4.27
Percentiles								
5%	-0.95	-0.85	1.00	0.98	-1.17	-0.86	0.95	0.95
50%	-0.19	-0.31	1.04	1.02	-0.18	-0.32	1.06	1.04
95%	2.15	2.54	1.09	1.07	2.06	2.75	1.17	1.11
Cross correlations								
$\Delta_{n,t}$	1.00				1.00			
$L^{z\text{-}asw}_{n,t}$	0.91	1.00			0.93	1.00		
R^{N}_{t+1}	-0.14	-0.12	1.00		-0.23	-0.30	1.00	
R^{TIPS}_{t+1}	0.37	0.31	0.28	1.00	0.23	0.16	0.59	1.00
Auto correlations								
1-day	0.99	1.00	0.98	0.99	0.99	0.99	0.98	0.98
2-day	0.98	0.99	0.96	0.97	0.98	0.98	0.96	0.95
5-day	0.95	0.98	0.92	0.93	0.95	0.96	0.91	0.89
22-day	0.90	0.96	0.86	0.86	0.90	0.93	0.85	0.81
Unit root test								
DF p-value	0.02	0.53	0.01	0.01	0.14	0.36	0.01	0.01

the four different equally-weighted portfolio bonds: short-term nominal bonds (NS), long-term nominal bonds (NL), short-term TIPS (RS) or long-term TIPS (RL). That they do not impose short-sell constraints suggests a less realistic environment, mainly because the Markowitz portfolio tends to have very large quantities of individual assets (sometimes unreasonably so), I do not impose this restriction to make my results comparable with previous papers.

Continuation: Descriptive Statistics

	Short-term				Long-term			
	$\Delta_{10,t}$	$L_{10,t}^{z\text{-}asw}$	R_{t+1}^{N}	R_{t+1}^{TIPS}	$\Delta_{20,t}$	$L_{20,t}^{z\text{-}asw}$	R_{t+1}^{N}	R_{t+1}^{TIPS}
Panel A: Annual frequency								
Mean	0.00	0.00	1.06	1.04	0.00	0.00	1.10	1.06
Stdev	1.00	1.00	0.04	0.05	1.00	1.00	0.09	0.08
Skewness	2.58	1.98	-0.14	0.02	1.91	2.27	0.16	0.06
Kurtosis	11.15	6.33	2.33	3.06	8.36	7.97	3.70	2.76
Percentiles								
5%	-0.95	-0.85	0.99	0.96	-1.17	-0.86	0.94	0.93
50%	-0.19	-0.31	1.06	1.04	-0.18	-0.32	1.09	1.07
95%	2.15	2.54	1.12	1.11	2.06	2.75	1.29	1.21
Cross correlations								
$\Delta_{n,t}$	1.00				1.00			
$L_{n,t}^{z\text{-}asw}$	0.91	1.00			0.93	1.00		
R_{t+1}^{N}	-0.50	-0.53	1.00		-0.60	-0.62	1.00	
R_{t+1}^{TIPS}	0.36	0.30	-0.04	1.00	0.00	0.03	0.46	1.00
Auto correlations								
1-day	0.99	1.00	0.99	1.00	0.99	0.99	0.99	0.99
2-day	0.98	0.99	0.98	0.99	0.98	0.98	0.98	0.98
5-day	0.95	0.98	0.96	0.97	0.95	0.96	0.95	0.95
22-day	0.76	0.89	0.83	0.86	0.77	0.81	0.78	0.82
Unit root test								
DF p-value	0.02	0.53	0.23	0.09	0.14	0.36	0.05	0.02

The *z-asw* liquidity premium corresponds to the residual spread between TIPS and nominal bonds asset swaps calculated using nominal and TIPS z-spread asset swaps rates. The other liquidity measure corresponds to the TIPS Liquidity proposed by Christensen and Gillan (2011). U.S. daily data from January 1, 2004 to December 30 2012 in basis points.

Several well-known features of optimal portfolio choice emerge. Consider the mean-variance portfolio choice weights. First, risk aversion affects how much wealth the investor allocates to risky securities instead of to the risk-free Treasury bill. The more risk-averse the investor, the less they will invest in the risky bond, so that long positions in risky bonds goes down with a higher degree of risk aversion. Second, given that this investor is forming his portfolio using only bonds and the risk-free Treasury bill, he/she will not want to short-sell the risky asset but rather will want to buy it on the margin (i.e. $\alpha > 1$). That means investors borrow money at risk-free rates and go long in risky bonds. For instance, an investor with an annual investment horizon and $\gamma = 20$ borrows 39% of wealth at the risk-free rate to invest a total of 139% in short-term nominal bonds portfolios. Finally, we see less large quantities of short-sales $(1 - \alpha)$ or, in some cases, no short-sales for the risk-free Treasury bill, for the same degree of risk aversion as the investment horizon increases. For example, an investor with $\gamma = 20$ goes short in the risk-free bond at the monthly frequency but goes long in both long-term nominal bonds and the risk-free bond at longer investment horizons. The same situation occurs with long-term bonds with respect to short-term ones in the sense that we see less large quantities for a portfolio of long-term vs short-term bonds. This indicates that a smaller portion of the portfolio is devoted to risky assets as investment horizons increase or when long-run assets are available.

Results for CRRA preferences are very similar to those for MV. In theory, what differentiates a Mean-variance investor from a CRRA investor is that the latter has a preference for higher order moments and not only for the expected return and its variance, thus their risky position depends on relative risk aversion. However, empirical results in Table 2, show that investors seem not to be primarily affected in their decisions by the first two return moments. So, the effect of higher order moments of CRRA investors seem not to be strong enough, especially for TIPS. The biggest holding difference is for short-term nominal bonds at the monthly frequency, where CRRA investors with different levels of risk aversion tend to hold larger quantities.

There are important differences in the optimal portfolio weights between short-term and long-term nominal bonds with both types of preferences. In fact, equally risk-averse investors tend to hold bigger positions on short-term bonds relative to long-term ones, i.e. the short-term bond weight typically exceeds the long-term weight for the same kind of bond. However, these differences become smaller when the investment horizon become longer. Bonds with a longer maturity will usually pay a higher interest rate than shorter-term bonds. However, long-term bonds have greater duration than short-term bonds, so interest rate changes will have a greater effect on long-term bonds than on short-term bonds. As a result, investors are more conservative holding smaller positions in long-term bonds relative to short-term bonds, given that they would offer greater stability and lower risk.

Investors also hold bigger positions in nominal bonds relative to TIPS bonds. These differences could be attributed, at least in the case of CRRA investor, to the negative skewness in short-term TIPS bond returns for monthly and quarterly frequency, as Table 1 shows. Investors prefer positive skewness, because it implies a low probability of obtaining a large negative return. Then, investors tend to the extreme portfolios (Sharpe ratio driven, skewness driven or kurtosis driven) and avoid being stuck in the middle.

4.2 Conditional portfolio weights

4.2.1 Non-parametric optimal portfolio function

In this section I present the optimal portfolio weights as function of the liquidity differential between inflation-indexed bonds and nominal bonds (liquidity premium), represented by Z_t. I apply the utility maximization framework presented above with respect to Z_t. For each kernel grid point,[13] I optimize

[13] I define fifteen not evenly spaced realizations of the liquidity ranging from its mean minus one standard deviation to its means plus three standard deviations, which correspond to

Table 2: Unconditional Portfolio Weights

	Mean-Variance investor				Power Utility investor			
	Treasury		TIPS		Treasury		TIPS	
γ	Short-term	Long-term	Short-term	Long-term	Short-term	Long-term	Short-term	Long-term
				Monthly frequency				
2	79.75	15.58	33.53	12.32	139.32	10.15	7.26	6.57
	[29.70]	[3.48]	[5.73]	[1.80]	[5.91]	[0.77]	[1.16]	[0.45]
5	31.90	6.23	13.41	4.93	64.58	5.11	7.26	3.63
	[11.88]	[1.39]	[2.29]	[0.72]	[10.65]	[0.65]	[1.16]	[0.59]
10	15.95	3.12	6.71	2.46	30.15	2.79	3.97	1.95
	[5.94]	[0.70]	[1.15]	[0.36]	[6.50]	[0.41]	[0.72]	[0.35]
20	7.98	1.56	3.35	1.23	14.51	1.43	2.06	1.00
	[2.97]	[0.35]	[0.57]	[0.18]	[3.60]	[0.22]	[0.39]	[0.19]
				Quarterly frequency				
2	29.45	6.54	10.11	5.31	29.35	6.94	6.00	4.00
	[6.30]	[0.81]	[1.93]	[0.80]	[1.57]	[0.64]	[0.92]	[0.58]
5	11.78	2.61	4.05	2.12	14.04	3.01	2.88	1.82
	[2.52]	[0.33]	[0.77]	[0.32]	[1.45]	[0.36]	[0.60]	[0.35]
10	5.89	1.31	2.02	1.06	6.98	1.50	1.51	0.94
	[1.26]	[0.16]	[0.39]	[0.16]	[0.77]	[0.18]	[0.33]	[0.19]
20	2.95	0.65	1.01	0.53	3.44	0.75	0.77	0.47
	[0.63]	[0.08]	[0.19]	[0.08]	[0.39]	[0.09]	[0.17]	[0.09]
				Annual frequency				
2	13.91	4.71	3.45	3.23	14.78	3.62	3.45	3.20
	[1.60]	[0.64]	[0.73]	[0.37]	[1.34]	[0.39]	[0.84]	[0.49]
5	5.57	1.88	1.38	1.29	6.30	1.77	1.41	1.35
	[0.64]	[0.26]	[0.29]	[0.15]	[0.73]	[0.28]	[0.36]	[0.23]
10	2.78	0.94	0.69	0.65	3.11	0.92	0.71	0.68
	[0.32]	[0.13]	[0.15]	[0.07]	[0.37]	[0.15]	[0.18]	[0.12]
20	1.39	0.47	0.35	0.32	1.54	0.46	0.35	0.34
	[0.16]	[0.06]	[0.07]	[0.04]	[0.18]	[0.08]	[0.09]	[0.06]

This table shows estimates of the optimal unconditional portfolio choice of investors. This is computed by applying a standard GMM procedure to the unconditional euler equation (9). Each panel shows a different investment horizon: monthly, quarterly and annual. I consider four portfolio allocation problems: two where the investor chooses between the portfolio of short-term or long-term nominal Treasury bonds and a risk-free asset and another two where he chooses between a portfolio of short-term or long-term TIPS and a risk-free asset. Weights in the table correspond to the risky asset. In brackets are the Newey-West (12 lags) standard errors. I used U.S. data from January 1, 2004 to December 30 2011.

the portfolio weight by maximizing the representative agent's marginal utility in that state using a GMM inference technique. The portfolio weights that follow from the optimization of the expected utility under MV and CRRA preferences are presented in this section.

Table 3 shows estimates of the optimal conditional portfolio choice of investors (Weight) and their corresponding standard errors (Std) obtained by applying the Politis and Romano (1994) bootstrap procedure which is described in Appendix A. I use this stationary bootstrap procedure to preserve autocorrelation properties of the data in the bootstrap samples.[14] The standard errors are presented only in order to assess the precision of the non-parametric method used. Each panel shows a different investment horizon (monthly, quarterly and annual), and they present the portfolio allocation problems considered before: two, where the investor chooses between the portfolio of short-term or long-term nominal Treasury bonds and a risk-free asset, and another two where the investor chooses between a portfolio of short-term or long-term TIPS and a risk-free asset, with each of them considering a MV and a CRRA investor.

Figures 3 to 6 are the companion graphs to Table 3. Each figure shows the optimal portfolio weight as a function of liquidity $\alpha(Z_t)$ represented by the bold line. Additionally, in each figure the dotted horizontal line represents the optimal unconditional allocation. The bars in the background represent the histogram of liquidity premium (scaled to add up to 30). The left column contains the optimal fraction of wealth allocated to the respective equally-weighted U.S. nominal bond portfolio, and the right column to the equally-weighted TIPS bond portfolio. Finally, in the first row, both the investment horizon and the rebalancing frequency are one-month, in the second row, one-quarter, and in the third row, one-year.

Results presented in Table 3 and in Figures 3 to 6 correspond to the case when the coefficient of relative risk aversion is equal to $\gamma = 20$. The results for the other degrees of risk aversion considered in the unconditional case are not presented here in order to save space. They are available upon request.

the interior 95% of the empirical distribution of the liquidity premium. Alternatively, I also define fifteen not evenly spaced realizations of the liquidity ranging between its minimum and maximum value, however results are broadly the same with both grids.

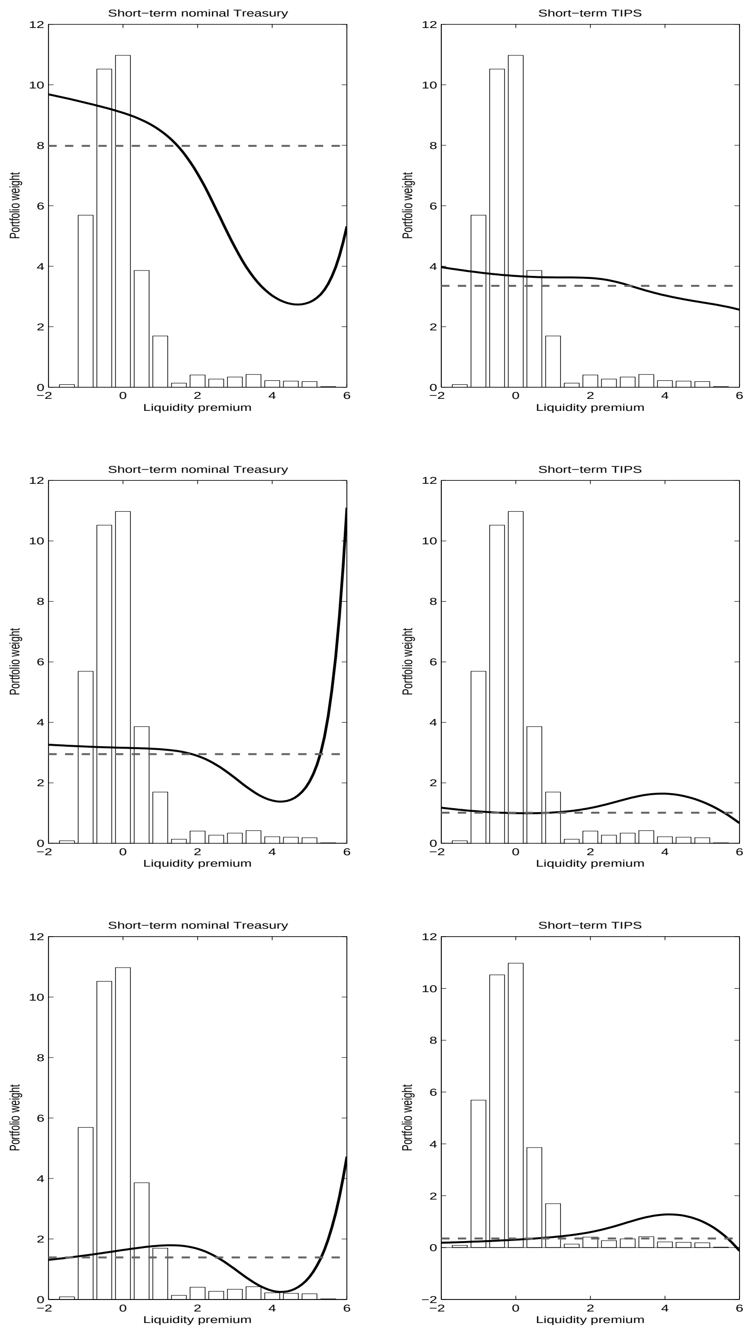
[14] This method is a variation of the standard block bootstrap that manages to create bootstrap series that are strictly stationary which accounts for the autocorrelation in the data.

Table 3: Conditional Portfolio Weights ($\gamma = 20$)

	Mean-Variance investor								Power Utility investor							
	Treasury				TIPS				Treasury				TIPS			
	Short-term		Long-term		Short-term		Long-term		Short-term		Long-term		Short-term		Long-term	
Z	Weight	Stdev	Weight	Stdev	Weight	Stdev	Weight	Stdev	Weight	Stdev	Weight	Stdev	Weight	Stdev	Weight	Stdev
								Monthly frequency								
-1	9.41	0.72	2.05	0.17	3.80	0.50	1.31	0.19	7.00	0.00	2.21	0.26	2.13	0.52	0.99	0.18
0	9.07	0.72	2.02	0.17	3.68	0.55	1.33	0.19	7.00	0.00	2.10	0.24	2.01	0.52	0.98	0.18
2	7.06	0.93	1.42	0.20	3.61	0.66	1.38	0.21	7.00	0.00	1.51	0.23	1.90	0.57	1.03	0.18
4	2.99	0.95	0.41	0.17	3.15	0.50	0.82	0.20	7.00	0.00	0.55	0.32	2.56	0.77	1.19	0.26
5	2.31	NaN	0.27	NaN	3.16	NaN	0.65	NaN	7.00	0.00	0.33	0.42	3.33	1.06	1.12	0.43
								Quarterly frequency								
-1	1.46	0.16	0.75	0.07	0.24	0.07	0.51	0.08	3.48	0.36	0.88	0.10	0.74	0.15	0.46	0.08
0	1.63	0.16	0.73	0.07	0.30	0.08	0.49	0.09	3.48	0.36	0.85	0.09	0.71	0.16	0.43	0.08
2	1.68	0.21	0.61	0.08	0.60	0.10	0.58	0.11	3.51	0.39	0.68	0.08	0.79	0.16	0.49	0.09
4	0.24	0.21	0.07	0.11	1.41	0.19	0.76	0.21	2.43	1.05	0.08	0.14	1.38	0.28	0.84	0.17
5	-0.21	NaN	-0.03	NaN	1.65	NaN	0.77	NaN	2.15	1.44	-0.03	0.19	1.88	0.47	1.04	0.27
								Annual frequency								
-1	0.24	0.08	0.59	0.06	0.24	0.08	0.29	0.04	1.63	0.17	0.71	0.07	0.23	0.08	0.31	0.05
0	0.30	0.08	0.62	0.06	0.30	0.09	0.30	0.05	1.78	0.17	0.72	0.07	0.30	0.08	0.32	0.05
2	0.60	0.09	0.50	0.08	0.60	0.11	0.39	0.06	1.72	0.17	0.42	0.08	0.52	0.09	0.38	0.06
4	1.41	0.21	-0.18	0.08	1.41	0.22	0.68	0.12	0.26	0.31	-0.18	0.07	1.20	0.16	0.76	0.12
5	1.65	NaN	-0.32	NaN	1.65	NaN	0.67	NaN	-0.23	0.42	-0.35	0.10	1.71	0.25	1.08	0.20

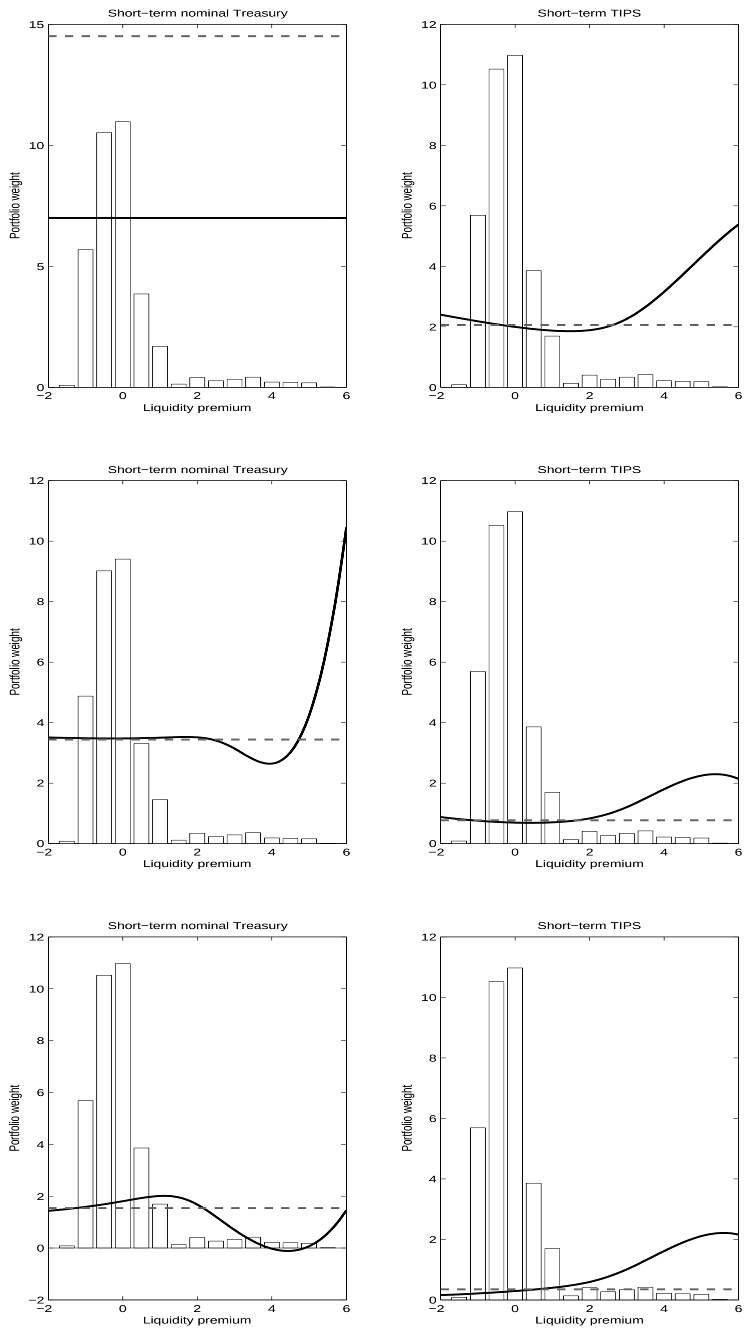
This table shows estimates of the optimal conditional portfolio choice of investors. This is computed by applying a standard GMM procedure to the conditional euler equation (8). Each panel shows a different investment horizon: monthly, quarterly and annual. I consider four portfolio allocation problems: two where the investor chooses between the portfolio of short-term or long-term nominal Treasury bonds and a risk-free asset and another two where he chooses between a portfolio of short-term or long-term TIPS and a risk-free asset. Weights correspond to the risky asset. Standard errors (Std) are obtained applying the Politis and Romano (1994) bootstrap procedure. I used U.S. data from January 1, 2004 to December 30 2011.

Figure 3: Optimal portfolio weights as a function of 10-year liquidity premium (Mean-Variance investor)



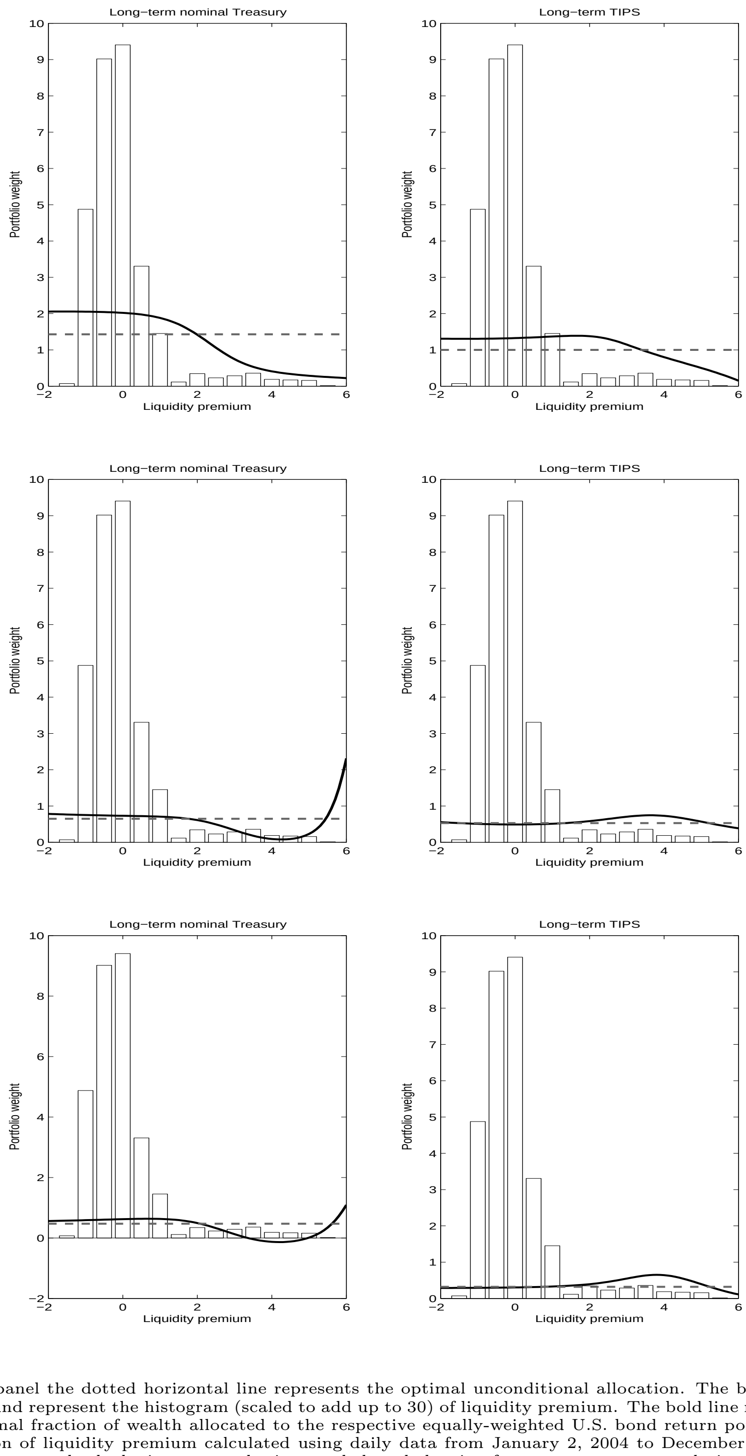
In each panel the dotted horizontal line represents the optimal unconditional allocation. The bars in the background represent the histogram (scaled to add up to 30) of liquidity premium. The bold line represents the optimal fraction of wealth allocated to the respective equally-weighted U.S. bond return portfolios as a function of liquidity premium calculated using daily data from January 2, 2004 to December 30, 2012. In the first row, both the investment horizon and the rebalancing frequency are one-month; in the second row, one-quarter; and in the third, one-year.

Figure 4: Optimal portfolio weights as a function of 10-year liquidity premium (CRRA investor)



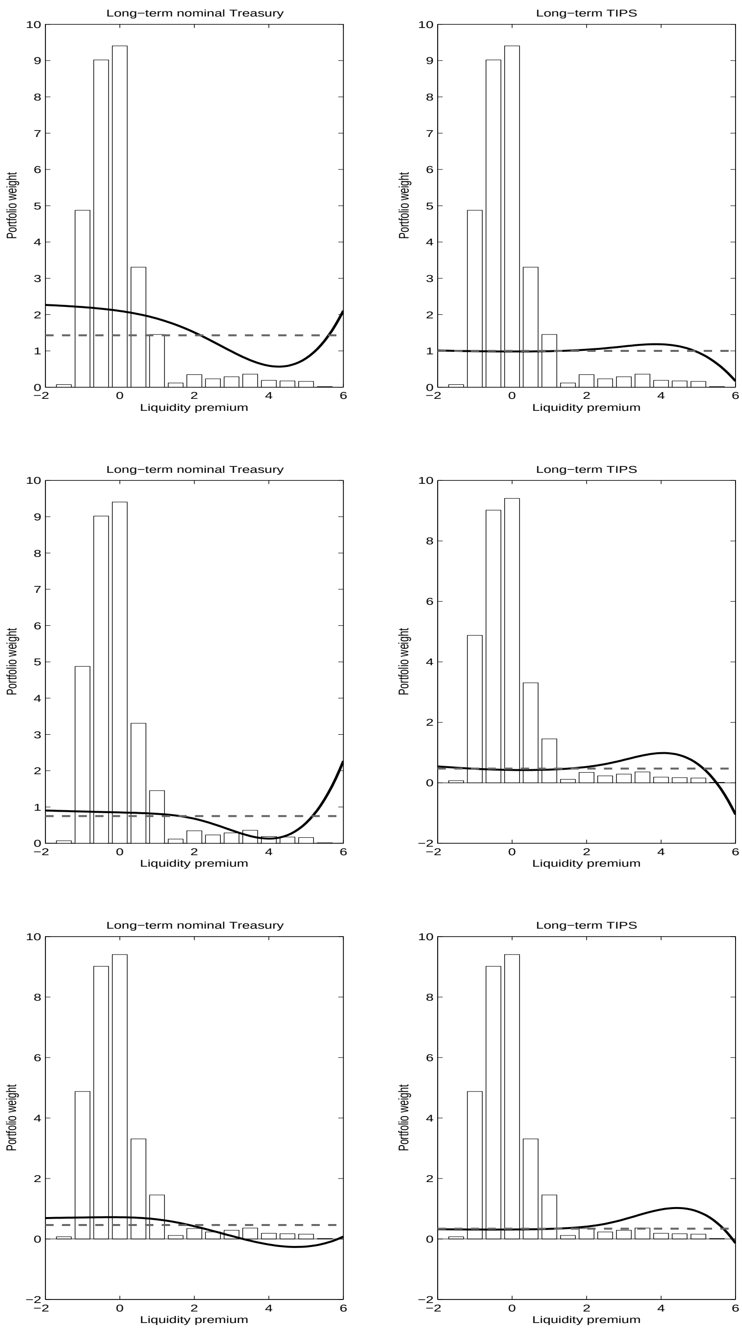
In each panel the dotted horizontal line represents the optimal unconditional allocation. The bars in the background represent the histogram (scaled to add up to 30) of liquidity premium. The bold line represents the optimal fraction of wealth allocated to the respective equally-weighted U.S. bond return portfolios as a function of liquidity premium calculated using daily data from January 2, 2004 to December 30, 2012. In the first row, both the investment horizon and the rebalancing frequency are one-month; in the second row, one-quarter; and in the third, one-year.

Figure 5: Optimal portfolio weights as a function of 10-year liquidity premium (Mean-variance investor)



In each panel the dotted horizontal line represents the optimal unconditional allocation. The bars in the background represent the histogram (scaled to add up to 30) of liquidity premium. The bold line represents the optimal fraction of wealth allocated to the respective equally-weighted U.S. bond return portfolios as a function of liquidity premium calculated using daily data from January 2, 2004 to December 30, 2012. In the first row, both the investment horizon and the rebalancing frequency are one-month; in the second row, one-quarter; and in the third, one-year.

Figure 6: Optimal portfolio weights as a function of 10-year liquidity premium (CRRA investor)



In each panel the dotted horizontal line represents the optimal unconditional allocation. The bars in the background represent the histogram (scaled to add up to 30) of liquidity premium. The bold line represents the optimal fraction of wealth allocated to the respective equally-weigthed U.S. bond return portfolios as a function of liquidity premium calculated using daily data from January 2, 2004 to December 30, 2012. In the first row, both the investment horizon and the rebalancing frequency are one-month; in the second row, one-quarter; and in the third, one-year.

A number of results emerge from this analysis. First, the liquidity premium seems to be a significant determinant of the portfolio allocation to U.S. government bonds. For instance, for a MV investor and at the monthly horizon, liquidity is a strong determinant of the allocation to short-term and long-term nominal bonds, with the optimal weight ranging from 9.41 at Liquidity= (-1) to 2.31 at Liquidity=5, as Table 3 shows. This indicates that an increase in the liquidity premium (i.e., liquidity conditions worsen) is accompanied by a strong decrease in the optimal allocation in short-term nominal bonds.

I have a similar result for the long-term nominal bonds with weights ranging from 2.05 to 0.27. Furthermore, liquidity also seems to be an important determinant of the allocation to TIPS. In this case, an increase in liquidity premium produces a decrease in the optimal allocation to both short-term, and long-term TIPS bonds. However, the effect is less strong with weights ranging from 3.80 to 3.16 for short-term, and from 1.31 to 0.65 for long-term for liquidity ranging between -1 and 5, respectively.

At quarterly and annual frequencies, optimal allocation still responds to changes in liquidity but mainly at high levels of liquidity premium. What we see is that the conditional weight is very close to the unconditional weight for low levels of liquidity (i.e. liquidity= -1 to 2), however optimal allocation starts to respond to changes in the liquidity when market liquidity conditions worsen (i.e. liquidity > 2). Interestingly, the investor tends to substitute cash for nominal bonds, and TIPS bonds for cash when the liquidity rises above its mean plus about 4 standard deviations, as Figures 3 and 5 show.

Second, conditional allocations in risky assets decrease as liquidity conditions worsen. In particular, an increase in the liquidity differential between nominal and TIPS bonds lead to: lower optimal portfolio allocations on nominal Treasury bonds, and also lower optimal portfolio allocations in TIPS, but at different levels of liquidity. When the liquidity premium is low (i.e. the liquidity differential between nominal and TIPS bonds is small), we see that the optimal allocation to either nominal or TIPS bonds is mostly unresponsive to liquidity premium, and it is very close to unconditional allocation. This occurs in the negative range of liquidity and also in the center of the distribution.

When the liquidity premium is high (i.e. in presence of big liquidity differentials between nominal and TIPS bonds), portfolio allocation on both nominal bonds and TIPS bonds decreases. However, this occurs at different levels of liquidity. In particular, the investor starts to decrease their position in nominal bonds at liquidity=2, but when there is insufficient liquidity, the investor holds a larger position in nominal bonds. On the other hand, portfolio allocation on TIPS bonds behaves in the reverse direction. That is, the investor only decreases asset allocation to TIPS in the upper positive part of liquidity (i.e. when the liquidity premium is very high), while between liquidity=2 and liquidity=4 TIPS bonds allocations increases, being above the unconditional value. Thus, in general, portfolio allocation for each type of bonds (nominal and TIPS) moves in cycles and each of them has its own cycle. Typically, when one type of bond is performing well, the other may not be performing as well in terms of liquidity, and the allocation rule reflects this situation.

Third, I find in general that the shape of the optimal portfolio policy functions of mean-variance and CRRA investors, with the same degree of risk aversion, are similar even though they have different levels (see Figure B.1 in the Appendix B). This suggest that investors seems to be primarily affected in their decisions by the first two return moments. Thus, the effect of higher order moments of CRRA investors exist but it seems not to be strong enough. However, this is not true at the monthly frequency. In this case, portfolio policies differ substantially which can be attributed to time variation in the higher order moments of the return distribution. This result is not induced by the choice of the kernel bandwidth, given that I explicitly control for it by constraining the kernel to be the same for the mean-variance and the CRRA preferences.[15]

Fourth, the effect of liquidity is a decreasing function of the investment horizon. For a given degree of risk aversion, the size of the optimal portfolio weight differs considerably across investment horizons. I find that as investment horizons became longer, the smaller the optimal portfolio weight, and the less that is invested in the risky asset. In particular, for the same degree of risk aversion investors react less abruptly to an increase in the liquidity premium when the investment horizon is one-year, than when the investment horizon is one-month.

For instance, we can see from Table 3 that when liquidity is equal to its mean ($Z_t = 0$) a MV investor with $\gamma = 20$ reduces the cash holdings from 2.02 to 0.62 when the investment horizon increases from one-month to one-year. This means that the investor borrows 102% of wealth at the risk-free rate to invest a total of 202% in short-term nominal bonds when the investment horizon is one-month. However, when the investment horizon becomes larger, the investor takes a long position in both assets holding 62% of their wealth in

[15] Non-parametric methods are typically indexed by a bandwidth or tuning parameter which controls the degree of complexity. The choice of bandwidth is often critical to implementation. In this application, the bandwidth is given by: $h = \lambda \sigma_z T^{-1/K+4}$, where $K = 1$ which is the dimension of Z (I am considering only one predictor variable which is liquidity), $\sigma(Z)$ is the standard deviation of the predictor variable, $T = 2086$ is the sample size and λ is a constant. For a big enough value of λ, I obtain a flat portfolio weight and small λ produce a very noise portfolio weight function. I consider values ranging from 9 to 3 for λ. These values guarantee bigger weight to an observation located at the mean of liquidity variable (which is zero), smaller weights to observations located one standard deviation away from the mean ($Z_t = \pm 1$), and even smaller weights to observations located two standard deviation away from the mean, etc. The results presented in this section correspond to $\lambda = 6$.

short-term nominal bonds and 38% in cash. The same occurs when I consider a CRRA investor. For example, considering the same case, but for long-term TIPS bonds, a CRRA investor reduces their bonds positions from 98% to 32%, as Table 3 shows.

Fifth, different degrees of risk aversion mainly change the level of the portfolio function but have little impact on the shape of this function, as is shown in Figure 7. In this figure, I only plot the portfolio policies for the long-term nominal (left column) and TIPS bonds (right column) for a one-year investment horizon. The first row in the figure corresponds to a mean-variance investor, and the second row to a CRRA investor. Finally, in each panel bold black lines represent an investor with $\gamma = 5$, the bold grey line with $\gamma = 10$ and the dotted line with $\gamma = 20$. Looking at Figure 7, we see that the more risk-averse the investor becomes, the smaller the optimal portfolio weight, so the less that is invested in the risky asset. Furthermore, more risk-averse investors react less abruptly to an increase in the liquidity premium.

To summarize, and in general, results consistently show that the optimal allocation to short-term or long-term bonds is mostly unresponsive to changes in liquidity conditions at low levels (i.e. at liquidity= -1 to 4). However once liquidity reaches certain levels (liquidity $>$ 4), which indicates that market liquidity conditions have worsened, then the investor starts to respond by decreasing the positions in TIPS and increasing the position in nominal bonds.

Additionally, the above conclusion is not determined by the level of risk aversion, the investment horizon or the investor preferences. The relation between optimal portfolio weights and liquidity premium remains the same for different values of risk-aversion, different investment horizons and also across investors' preferences. The characteristics mainly change the level of the portfolio function that have a small impact on the function shape, except for the monthly frequency.

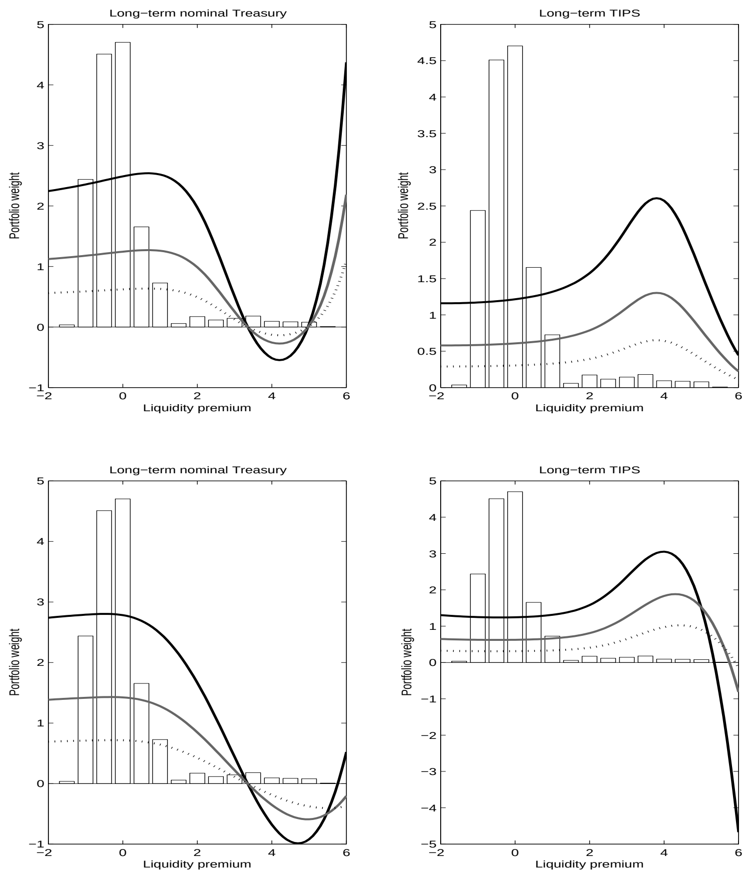
4.2.2 Do weights really respond to changes in liquidity?

The main question of this paper is whether or not the weights respond to changes in liquidity. To test whether or not a portfolio weight is statistically different from zero is pointless in this context, simply because it does not provide an answer for the question asked above. What I do next, following Ghysels and Pereira (2008), is to statistically test this question by using the following approximation:

$$H_0 : \frac{\partial \alpha(Z)}{\partial Z} \mid_{Z=\bar{Z}} \quad \cong \frac{\alpha(\bar{Z}+0.1) - \alpha(\bar{Z}-0.1)}{0.2} = 0 \qquad (11)$$

where the first derivative of $\alpha(Z)$ is approximated by a finite difference which allows me to compute the slope of the optimal portfolio weight function at

Figure 7: Optimal portfolio weights as a function of 10-year liquidity premium (Mean-variance and CRRA investor with different values for γ)



The bars in the background represent the histogram (scaled to add up to 30) of liquidity premium. The lines represent the optimal fraction of wealth allocated to the respective equally-weighted U.S. bond return portfolios as a function of liquidity premium calculated using daily data from January 2, 2004 to December 30, 2012. Bold black line represent an investor with $\gamma = 5$, the bold grey line for $\gamma = 10$ and dotted line for $\gamma = 20$. In the first row correspond to the case of mean-variance investor and the second row to the CRRA investor. The investment horizon and the rebalancing frequency in this figure correspond to one-year.

each value of the predictor variable.

Table 4 shows the point estimate slopes and t-stat computed using the standard errors obtained also from the Politis and Romano (1994) stationary bootstrap procedure. I draw one main conclusion from this table which is consistent with the results presented above. It is clear that optimal portfolio policy is not linear or constant in liquidity. For the two investor preferences the short-term nominal and the TIPS bonds portfolio policy responds to changes in liquidity. This conclusion is derived from the fact that the null hypothesis is rejected indicating that all slopes are statistically significant at the 10% level or less. The only case where slopes are not statistically significant is for short-term TIPS bonds with MV preferences. The other case where we can not reject the null hypothesis is for short-term nominal bonds with CRRA preferences. In this case, the optimal portfolio function is constant but smaller than the unconditional weight.

For long-term TIPS, $\alpha(Z_t)$ is almost constant and statistically not different from zero over the negative range of liquidity until $Z_t = 2$. After that the slopes are positive and over the last range of liquidity they are negative and statistically significant. I find the same results for both investor preferences. The optimal portfolio function for long-term nominal bonds goes in the opposite way. It starts by being flat and statistically not different from zero, then slopes become negative, and over the the end range of liquidity, slopes are positive and statistically significant.

Overall, I can conclude that optimal portfolio choice is unresponsive over the negative and first positive range of liquidity, however portfolio allocations start to react as liquidity conditions worsen. This conclusion regarding the general shape of the portfolio weight functions is reliable in the sense that non-parametric techniques used here produce a consistent estimator of the portfolio functions.

4.2.3 Robustness analysis: Parametric portfolio functions

Parametric portfolio functions

To analyze whether or not the shape of the optimal portfolio functions presented in this section are robust to a particular choice for the constant λ, I also estimate a parametric portfolio function. This is to confirm the results obtained above. In accordance with the shapes of the portfolio functions obtained before, I use a third degree polynomial in liquidity (Z_t) to approximate the estimated non-parametric portfolio policy function

$$\alpha^p(Z_t) = a_0 + a_1 Z_t + a_2 Z_t^2 + a_3 Z_t^3. \tag{12}$$

The parametric optimal portfolio weight is computed by applying a standard GMM procedure to the conditional Euler equation. In this case the moment condition is

$$\hat{\mathbb{E}}\left[\frac{\partial u(W_{t+1})}{\partial W_{t+1}}\frac{\partial W_{t+1}}{\partial \alpha} \mid Z_t = \bar{z}\right] = \frac{1}{T}\Sigma_{t=1}^{T}\left(\frac{\partial u(W_{t+1})}{\partial W_{t+1}}\frac{\partial W_{t+1}}{\partial \alpha}\right) \otimes g(Z_t) = 0, \tag{13}$$

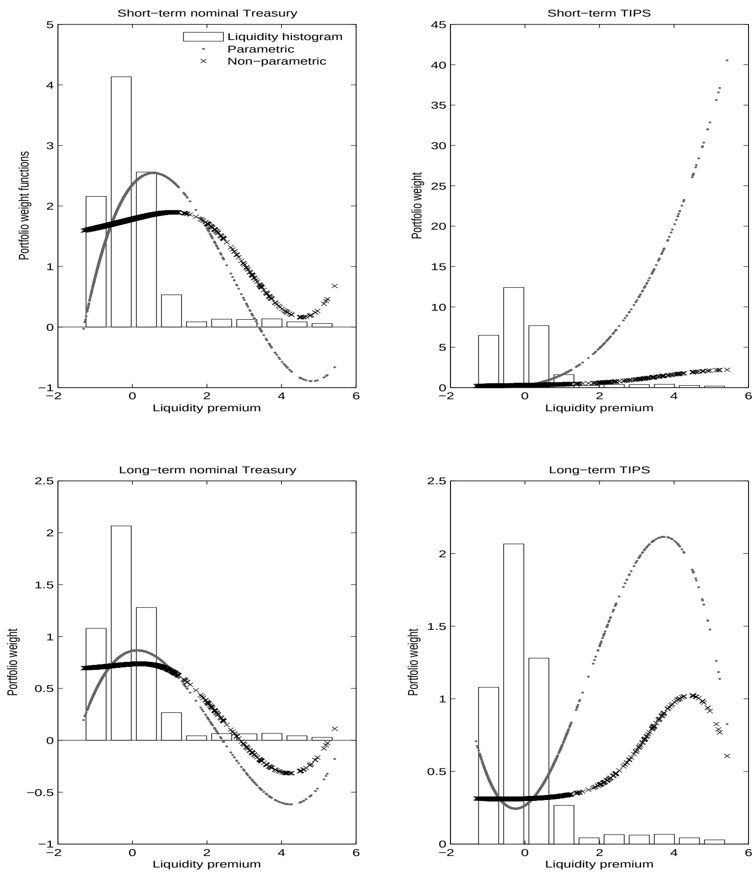
where $g(Z_t) = [1, Z, Z^2, Z^3]$. The constants a_0, a_1, a_2, a_3 are reported in Table 5, and a comparison between the parametric and non-parametric optimal portfolio functions is plotted in Figure 8.

Table 4: Point estimates for the slope of the conditional portfolio weight function

	Mean-Variance investor								Power Utility investor							
	Treasury				TIPS				Treasury				TIPS			
	Short-term		Long-term		Short-term		Long-term		Short-term		Long-term		Short-term		Long-term	
Z	Slope	t-stat	Slope	t-stat	Slope	t-stat	Slope	t-stat	Slope	t-stat	Slope	t-stat	Slope	t-stat	Slope	t-stat
								Monthly frequency								
-1	-0.30	-1.62	-0.01	-0.31	-0.15	-1.14	0.01	0.18	0.00	-0.03	-0.08	-1.49	-0.14	-2.02	-0.01	-0.53
0	-0.40	-2.19	-0.06	-1.10	-0.08	-0.58	0.03	0.76	0.00	-0.26	-0.15	-2.39	-0.11	-1.63	0.00	-0.05
2	-2.08	-3.33	-0.67	-4.76	-0.07	-0.35	-0.07	-0.62	0.00	0.07	-0.47	-3.96	0.05	0.58	0.06	1.42
4	-1.01	-2.01	-0.19	-1.64	-0.14	-0.52	-0.24	-2.75	0.00	0.61	-0.31	-2.06	0.66	1.68	-0.01	-0.03
4.5	-0.71	-1.67	-0.14	-1.27	-0.03	-0.05	-0.18	-2.51	0.00	-1.76	-0.23	-1.62	0.76	1.77	-0.07	-0.31
6.0	2.43	4.31	-0.10	-1.30	-0.01	-0.05	-0.13	-2.35	0.00	0.65	0.42	0.93	0.91	2.24	-0.06	-0.28
								Quarterly frequency								
-1	0.17	4.26	-0.03	NaN	0.06	3.08	-0.03	-1.60	-0.02	-0.24	-0.03	-1.08	-0.05	-2.43	-0.03	-1.06
0	0.18	2.81	-0.02	-1.00	0.08	3.95	0.00	-0.05	0.01	0.10	-0.03	-1.04	-0.02	-1.10	-0.01	-0.72
2	-0.35	-2.85	-0.18	-4.56	0.28	6.51	0.10	2.42	-0.08	-0.33	-0.23	-3.31	0.13	3.78	0.08	3.26
4	-0.62	-3.18	-0.17	-1.98	0.31	2.52	0.01	0.04	-0.55	-1.05	-0.18	-2.29	0.46	2.21	0.24	1.84
4.5	-0.53	-2.82	-0.13	-1.61	0.27	2.18	0.00	0.03	-0.44	-0.84	-0.14	-1.79	0.48	2.18	0.23	1.71
6.0	0.93	2.76	0.65	1.65	-0.31	1.96	-0.05	-0.91	0.57	0.54	0.45	1.68	0.53	2.07	-0.43	-1.08
								Annual frequency								
-1	0.06	3.23	0.03	3.00	0.06	2.78	0.01	0.66	0.14	5.12	0.02	1.12	0.05	3.06	0.00	-0.15
0	0.08	4.26	0.03	1.48	0.08	3.39	0.02	1.46	0.15	3.41	-0.02	-0.61	0.07	3.74	0.01	0.64
2	0.28	6.16	-0.26	-4.02	0.28	5.94	0.10	4.41	-0.45	-3.13	-0.29	-5.80	0.19	5.90	0.08	3.34
4	0.31	1.83	-0.21	-3.64	0.31	2.04	0.04	0.54	-0.65	-3.75	-0.24	-4.48	0.48	3.96	0.30	2.78
4.5	0.24	1.26	-0.15	-2.61	0.24	1.54	-0.01	-0.21	-0.52	-3.04	-0.18	-3.45	0.50	3.66	0.31	2.55
6.0	0.87	1.45	0.54	2.34	-0.33	1.98	-0.11	-0.76	0.37	1.92	0.06	1.57	0.12	3.25	-0.34	-2.48

This table shows the point estimates slopes and their standard errors obtained from Politis and Romano (1994) stationary bootstrap procedure. This is computed by approximating the first derivative of $\alpha(Z)$ by equation (10). Each panel shows a different investment horizon: monthly, quarterly and annual. I consider four portfolio allocation problems: two where the investor chooses between the portfolio of short-term or long-term nominal Treasury bonds and a risk-free asset and another two where he chooses between a portfolio of short-term or long-term TIPS and a risk-free asset. I used U.S. data from January 1, 2004 to December 30 2011.

Figure 8: Optimal portfolio weights as a function of 10-year liquidity premium (Parametric vs Non-parametric functions)



The bars in the background represent the histogram (scaled to add up to 10) of liquidity premium. The lines represent the optimal fraction of wealth allocated to the respective equally-weighted U.S. bond return portfolios as a function of liquidity premium calculated using daily data from January 2, 2004 to December 30, 2012. The investment horizon and the rebalancing frequency in this figure correspond to one-year and I assume CRRA preferences.

Figure 8 compares the parametric optimal portfolio function, obtained from the polynomial model, with the non-parametric function in the case of CRRA preferences. What we see is that both functions have approximately the same shape (except for short-term TIPS), which is an indication that results are broadly consistent. I confirm that portfolio policies are nonlinear, and the comparison of policies at different points of the liquidity distribution shows a large variation in the optimal allocation, being the effect of liquidity strong when it is greater than three standard deviations above its mean. However, results must be interpreted with caution since the data density at the margins of the empirical distribution of the predictor variable is small. It is well known that an empirical distribution is a noisy model of the true distribution in the tail area.

Table 5 gives estimates of the parameter a_i for the optimal portfolio function defined in equation (11) for a CRRA investor. I will focus on the annual frequency, which contains the companion results for Figure 8, and we see that most of the coefficients are statistically significant at 5% o more. a_1, which is the slope of the portfolio function in the center of the distribution ($Z_t = 0$), is positive and statistically significant for short-term nominal and TIPS bonds, but it is not statistically significant for long-term bonds. This means that portfolio weights do not respond to changes in liquidity at the central range of liquidity. The result is consistent with the non-parametric results.

Alternative market-based measure of liquidity

As an additional robustness check, I also consider an alternative market-based measure of liquidity. This examines whether or not the results presented here depend on a particular way to proxy the liquidity differential between inflation-indexed bonds and nominal bonds (liquidity premium), represented by Z_t. Figure C.1, in Appendix C, shows the optimal fraction of wealth allocated to equally-weighted U.S. bond portfolios as a function of liquidity premium measures using Gomez (2013). Looking at Figure 10, first I confirm the conclusion that liquidity constitutes relevant conditioning information in the portfolio choice problem. Second, I conclude that results are robust to the liquidity premium measure used, in the sense that the shape of the optimal allocation policy is approximately the same with both measures of liquidity. Finally, liquidity measures are available for different maturities (10-years and 20-years), however results do not depend on a particular choice of the maturity of the liquidity premium (results are available upon request). This implies that both market-based measures of liquidity are capturing time variations in investment opportunities.

4.3 Does bond return predictability imply improved asset allocation and performance?

From the standpoint of practical advice to portfolio investors, an additional natural question to ask is whether or not the bond return predictability translates into improved out-of-sample asset allocation and performance. The idea is that at the start of each period (one-month, one-quarter or one-year), one investor makes portfolio allocations based on the belief that bond returns are predictable by liquidity. I compare his/her performance to that of another investor who believes that bond returns are independent and identically

Table 5: Parametric conditional portfolio weight function (CRRA investor)

Monthly frequency

	Short-term Nominal				Short-term TIPS			
	a_0	a_1	a_2	a_3	a_0	a_1	a_2	a_3
Estimate	10.81	-6.35	25.75	-4.61	1.65	-1.19	2.17	0.06
t-stat	4.37	-2.17	2.00	-1.92	3.53	-1.76	2.29	0.11
p-value	0.00	0.03	0.05	0.06	0.00	0.08	0.02	0.91
	Long-term Nominal				Long-term TIPS			
	a_0	a_1	a_2	a_3	a_0	a_1	a_2	a_3
Estimate	2.06	-0.51	0.08	-0.01	0.91	0.00	0.30	-0.06
t-stat	9.05	-1.42	0.40	-0.42	4.21	0.00	1.41	-1.63
p-value	0.00	0.16	0.69	0.67	0.00	1.00	0.16	0.10

Quarterly frequency

	Short-term Nominal				Short-term TIPS			
	a_0	a_1	a_2	a_3	a_0	a_1	a_2	a_3
Estimate	3.48	-0.01	0.05	-0.03	0.49	0.02	2.06	-0.38
t-stat	6.64	-0.02	0.09	-0.30	2.62	0.05	4.05	-4.14
p-value	0.00	0.99	0.93	0.77	0.01	0.96	0.00	0.00
	Long-term Nominal				Long-term TIPS			
	a_0	a_1	a_2	a_3	a_0	a_1	a_2	a_3
Estimate	0.83	-0.20	-0.01	0.00	0.35	-0.06	0.52	-0.10
t-stat	6.67	-1.37	-0.07	0.04	3.41	-0.33	3.00	-3.07
p-value	0.00	0.17	0.94	0.97	0.00	0.74	0.00	0.00

Annual frequency

	Short-term Nominal				Short-term TIPS			
	a_0	a_1	a_2	a_3	a_0	a_1	a_2	a_3
Estimate	2.35	0.72	-0.72	0.09	0.25	0.62	0.60	0.12
t-stat	8.19	3.35	-4.16	2.46	1.99	2.05	2.38	0.54
p-value	0.00	0.00	0.00	0.01	0.05	0.04	0.02	0.59
	Long-term Nominal				Long-term TIPS			
	a_0	a_1	a_2	a_3	a_0	a_1	a_2	a_3
Estimate	0.86	0.07	-0.28	0.04	0.27	0.17	0.31	-0.06
t-stat	7.27	0.93	-4.44	3.65	3.49	1.39	2.93	-2.59
p-value	0.00	0.35	0.00	0.00	0.00	0.16	0.00	0.01

Each panel gives estimates of the parameter c_i for the optimal portfolio function defined in equation (11). These estimates are computed through GMM on the moment condition (12), using a Newey and West estimator of the spectral density matrix. I consider four portfolio allocation problems: two where the investor chooses between the portfolio of short-term or long-term nominal Treasury bonds and a risk-free asset and another two where he chooses between a portfolio of short-term or long-term TIPS and a risk-free asset. Each panel shows a different investment horizon: monthly, quarterly and annual. I used U.S. data from January 1, 2004 to December 30 2011.

distributed (i.i.d.), and ignores any evidence of bond return predictability in making his/her portfolios allocation choices.

I used rolling estimation approach, which consists of estimating a series of out-of-sample portfolio returns by using a rolling estimation window over the entire data set. Specifically, I choose an estimation window of length M=260 days (1 year). In each day, starting from $t = M + 1$, I use the data in the previous M days to estimate the optimal portfolio weights. In other words, each investor has an investment horizon of one-year and uses all data available until period $T - M$ to choose his/her first portfolio weights. Next, I use those weights to compute the portfolio returns. Repeating this procedure, involve adding the information for the next period in the data set and dropping the earliest period (keeping the window length fixed), until the end of the data set is reached. In this way, I obtain a time series of portfolio returns for each (unconditional and conditional) strategy.

To compute out-of-sample performance of this two different strategies, I compute the out-of-sample Sharpe ratio of strategy j, defined as the sample mean of out-of-sample excess returns (over the risk-free asset), μ_j, divided by their sample standard deviation, σ_j, for strategy $j = U, C$

$$SR_j = \frac{\mu_j}{\sigma_j}. \tag{14}$$

In addition, I calculate the certainty equivalent rates of return (CER) for each strategy to judge its relative performance. The CER represents the risk-free rate of return that investor is willing to accept instead of undertaking the risky portfolio strategy. Formally, I compute the CER of strategy j

$$CER_j = \mu_j - \frac{\gamma}{2}\sigma_j^2, \tag{15}$$

where μ_j and σ_j^2 are the mean and variance of out-of-sample excess returns for strategy $j = U, C$. To test whether or not the Sharpe ratios, and the certainty equivalent returns of two strategies are statistically distinguishable, I test the following null hypothesis $Ho : SR_U - SR_C$ and $Ho : CER_U - CER_C$. This difference represents the gain (or loss) in returns from investing in unconditional strategy versus conditional strategy. I compute the p-value of the differences by using the Politis and Romano (1994) stationary bootstrap procedure ($pv-boot$).[16] Finally, an useful benchmark are the in-sample Sharpe ratios and the certainty equivalent returns (to assess the effect of estimation error), calculated for the different portfolio strategies by using the entire time series of excess returns.

Table 6 shows results assuming both investors are mean-variance optimizer with a one-year investment horizon, and $\gamma = 10$. Panel A shows the CER and the SR calculated with the entire data set (in-sample analysis). The in-sample Sharpe ratios are all positive (except for short-term nominal bonds), being the performing of the conditional strategy better than the unconditional strategy for all portfolios. For instance, for a nominal long-term portfolio the Sharpe ratio of unconditional strategy is equal to 0.12 versus 0.36 of the conditional

[16] I replicate the process described in Appendix A 1000 times. For each such replication, I compute the optimal allocations for each investor through one year (260 days). At every point in time, the investors are allowed to utilize just the information available up to that point in time. I calculate the difference in certainty equivalent between the two strategies and the adjusted Sharpe ratio for each replication. Finally, I count the proportion of times in 1000 replications that these differences exceed the certainty equivalent and adjusted Sharpe ratio based on the original data for a given set of results.

Table 6: Sharpe ratios and certainty equivalent returns (Mean-Variance investor with $\gamma = 10$)

Panel A: In- sample results

		Unconditional		Conditional		Differential		Ho:differential=0	
		SR	CER	SR	CER	$SR_U - SR_C$	$CER_U - CER_C$	$pv - boot$	$pv - boot$
Treasury	Short-term	0.2891	1.5005	-0.1912	0.3447	0.4812	1.1558	0.0460	0.0484
	Long-term	0.1286	0.5358	0.3632	0.6010	-0.2346	-0.0652	0.0479	0.0014
TIPS	Short-term	0.2188	0.3602	0.4114	0.9541	-0.1926	-0.5939	0.0480	0.0004
	Long-term	0.1399	0.3502	0.8102	0.8110	-0.6703	-0.4608	0.0404	0.0004

Panel B: Out-of-sample results

		Unconditional		Conditional		Differential		$p - value$	
		SR	CER	SR	CER	$SR_U - SR_C$	$CER_U - CER_C$	$pv - boot$	$pv - boot$
Treasury	Short-term	0.2823	1.5886	-0.2261	0.3204	0.5084	1.2682	0.0553	0.0548
	Long-term	0.1220	0.5540	0.3413	0.6456	-0.2193	-0.0916	0.0504	0.0040
TIPS	Short-term	0.2085	0.3351	0.3520	1.1332	-0.1435	-0.7981	0.0541	0.0005
	Long-term	0.1295	0.2894	0.7521	0.8846	-0.7551	-0.5952	0.0464	0.0006

This table reports the out-of-sample CER returns for two different investor strategies: unconditional (bond returns are i.i.d) and conditional (bond returns are predictable) strategy. The $p - values$ of the difference between SR, and CER from each strategy are obtained applying the Politis and Romano (1994) bootstrap procedure. The complete data set correspond to U.S. data from January 1, 2004 to December 30 2011.

strategy, indicating that with the conditional strategy the investor takes on less risk to achieve the same return. For the same portfolio, the CER_U is equal to 0.53 vs 0.60 of the CER_C. This means that an investor requires a higher risk-free return to give up the opportunity to invest in the portfolio following a conditional strategy.

Similarly, the difference between the in-sample SR for the unconditional and conditional strategy shows the loss (given that I obtain negative values) from investing, based on the belief that bond returns are i.i.d. This means that the bond return predictability translates into improved in-sample asset allocation and performance. The comparison of in-sample certainty equivalent returns and their differences, confirms the conclusions from the analysis of Sharpe ratios. Finally, the difference between the Sharpe ratios and certainty equivalent returns of each strategy are statistically significant in all cases, as $pv - boot$ values indicate.

Next, I assess the magnitude of the potential gains that can actually be realized by an investor, using the out-of-sample performance of the strategies. From panel B of Table 6, we see that in all cases the SR for the portfolios from the conditional strategy is much higher than for the unconditional strategy. I find the same results for CER. This means that a conditional strategy outperforms the unconditional strategy. This suggests also that conditional strategy might improve, not only in-sample but also out-of-sample performance. The significance of the CER differential and the SR differential, which is measure using the stationary bootstrap technique proposed by Politis and Romano (1994), implies that this result is statistically significant.

Finally, the difference between the in-sample and out-of-sample strategies allows me to gauge the severity of the estimation error. From the out-of-sample Sharpe ratio, reported in Panel B of Table 6, the unconditional strategy does not have a substantially lower Sharpe ratio and certainty equivalent returns out-of-sample than in-sample. This means that the effect of estimation error seems not to be so large. Consequently, it does not erodes the gains from optimal diversification given that differences turn out not to be economically important.

5 Conclusions

Although many studies on the liquidity premium have been conducted, the implications for investors are rarely addressed in any detail. In order to draw conclusions from the effect of the liquidity risk premium from an investor's point of view, it is necessary to specifically analyze optimal portfolio compositions in realistic settings. This is the focus of this paper.

I consider the portfolio problem of a mean-variance and a power utility investor whose portfolio choices are between the asset of interest and a risk-free asset. The investor's problem is to choose optimal allocations to the risky asset as a function of predictor value: liquidity premium. In this paper, I use two alternative measures recently proposed in the literature for the liquidity differential between inflation-indexed bonds and nominal bonds. These assess whether or not liquidity changes influence optimal portfolio allocations in the U.S. government bond market. While these issues have been well studied for stock-only portfolios, in general, less has been done to provide empirical evidence for the optimal portfolio choice of a utility-maximizing risk-averse investor, conditional upon observing a particular liquidity signal.

Overall, results show that optimal portfolios vary substantially with regards to predictor value. In particular, the effect of liquidity is a decreasing function of the investment horizon. Additionally, conditional allocations in risky assets decrease as liquidity conditions worsen. However, once the liquidity differential between U.S. nominal Treasury and TIPS bonds is sufficiently large, it leads to: (*i*) lower optimal portfolio allocations in TIPS; and (*ii*) higher optimal portfolio allocations on nominal bonds with respect to the risk-free bond. To summarize, this paper suggests that market liquidity signals could provide valuable guidance to investors, and adds to the evidence found for stock portfolios by Ghysels and Pereira (2008), which suggests the existence of a dependence of the optimal portfolio choices on changes in liquidity.

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Appendix

A Bootstrap procedure

Algorithm 1 Politis and Romano (1994) stationary bootstrap procedure

Require: Considering the equally weighted bond returns portfolio R_t

Ensure: that the data are re-sampled in blocks where the block length has a geometric distribution with a mean of $1/q$.

1: Randomly select an observation, say, R_t^N, from the original time series

2: With a fixed probability q, select the next observation randomly from the original time series, and with probability $(1-q)$ select it as the next observation to R_t (i.e., select R_{t+1}^N) from the original time series.

3: Repeat this process to generate a pseudo time series of desired length.

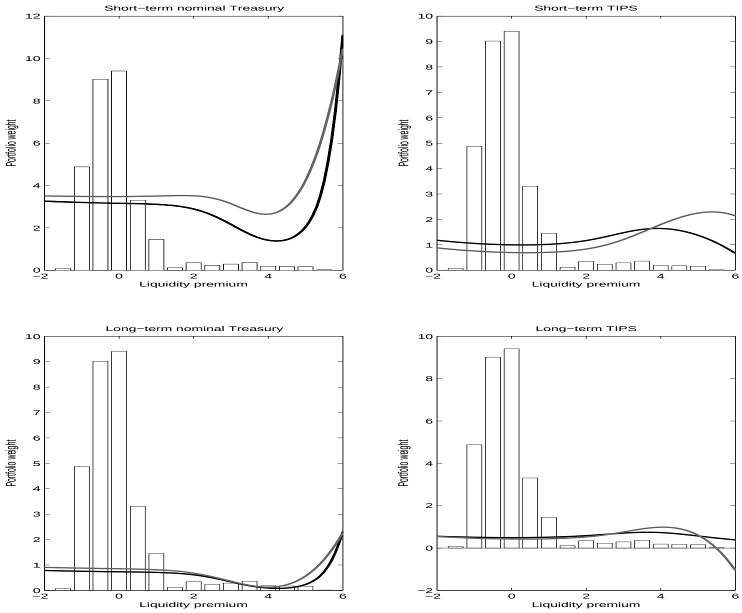
4: Construct bootstrap samples of $rx_{t+1}^{(n)}$ by using the bootstrap samples of $\mathbf{X}_t$ and resampling blocks of w subsequent residuals $\varepsilon_{t+1}^{(n)}$.

5: Repeat the bootstrap procedure 1000 times.

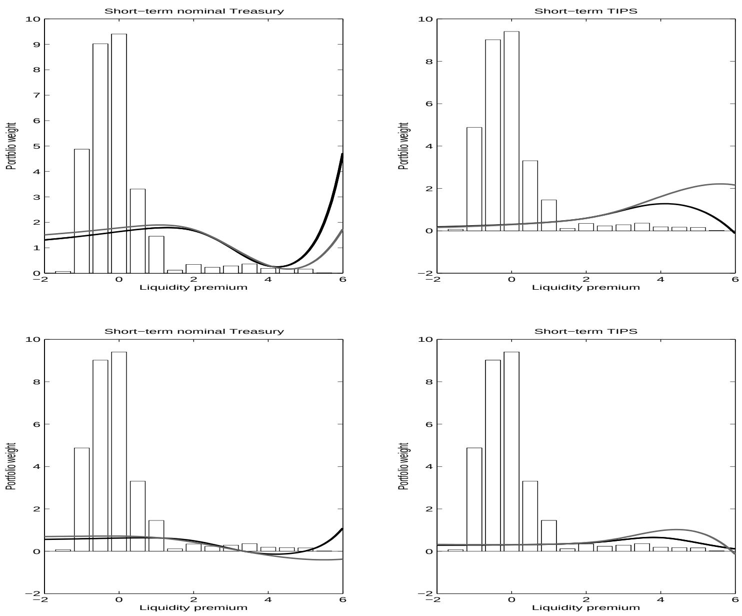
B MV vs CRRA optimal portfolio functions

Figure B.1: MV vs CRRA optimal portfolio weights

(a) Quaterly frequency



(b) Annual frequency

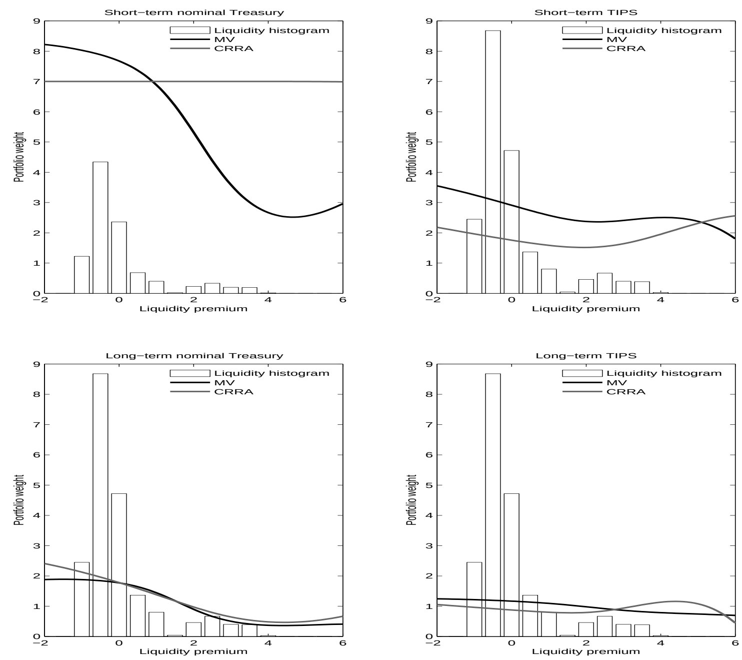


The bars in the background represent the histogram (scaled to add up to 30) of liquidity premium. The black line represent line represent the optimal fraction of wealth allocated to equally-weighted U.S. bond portfolio as a function of liquidity premium for a mean-variance investor. The grey line represents the same for CRRA investor.

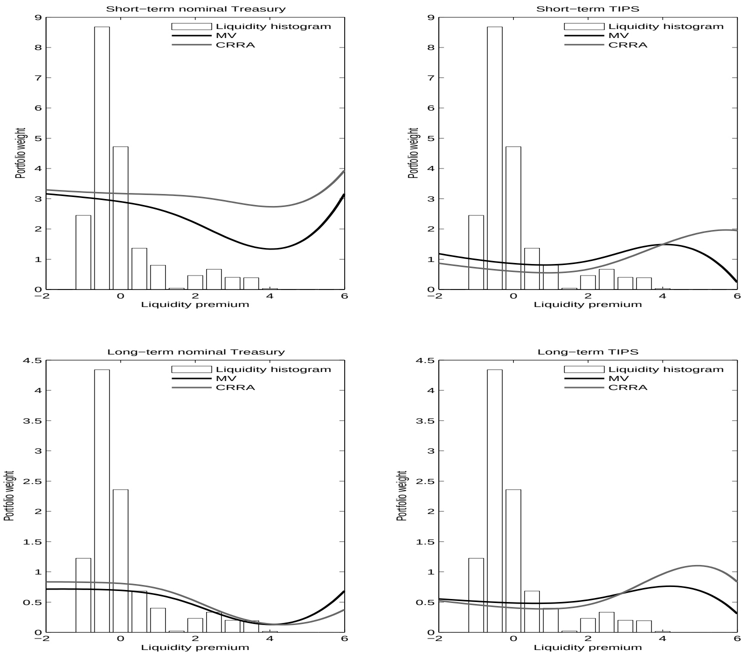
C Optimal portfolio functions considering an alternative liquidity premium measure

Figure C.1: Optimal portfolio weights as a function of 10-year liquidity premium: $L_{10,t}^{z-asw}$

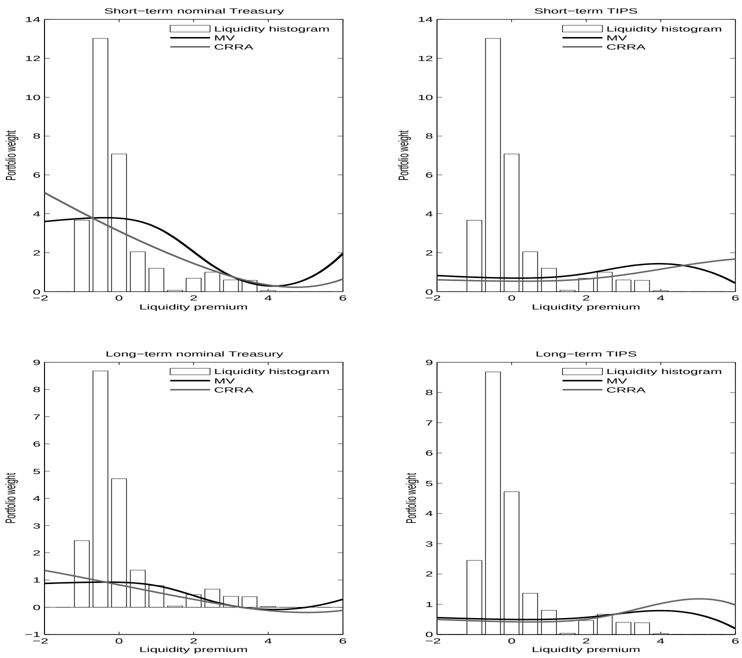
(a) Monthly frequency



(a) Quarterly frequency



(b) Annual frequency



The bars in the background represent the histogram (scaled to add up to 20) of liquidity premium. The lines represent the optimal fraction of wealth allocated to equally-weighted U.S. bond portfolio as a function of liquidity premium.