BUSINESS BANKRUPTCY: AN AGENT-BASED MODEL, A MONETARY ANALYSIS

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Abstract. This work studies the dynamics of business bankruptcy within a model made of mutual-interacting agents. Following the monetary analysis *à la* Schumpeter, these interactions are thought of as payments between different types of agents, in relation with the credits that banks grant for financing part of these payments. The model deals with a simple economy made of three agents acting like a business, which may go bankrupt, and a single bank. For each period, the model specifies: i) how each agent pays the others; ii) how payments are financed, including by debt; and iii) how the incapacity of firms to fully repay debts in due time is dealt with, including by bankruptcy itself. The way firms execute their payments and run into debt cannot be arbitrary in order to avoid business bankruptcies as much as possible. Thus, besides explaining factors of business bankruptcies like macroeconomic aggregates or individual characteristics, the overall configuration of payments and credits shall matter too. This invites to inquire more into this kind of configuration, within more complex models in order to be closer to economic reality.

1. Introduction

Business bankruptcy (or 'business failure' interchangeably) cannot be separated from the issue of macroeconomic stability. On the one hand, a fundamental aspect of the bankruptcy procedure is to find a way out for the economic agent who faces solvency problems and who is unable to fully repay its debts. The debtor's assets are taken over in order to sell them and, eventually, to orderly distribute the resulting proceeds to the creditors[1]. Once achieved this process, the debtor is no longer required to meet its current financial obligations, even if asset liquidation does not make possible the full repayment of debts[2]. On the other hand, if bankruptcy affects a company, the latter has to stop all operations and can no longer make economic decisions. Bankruptcy is the 'death penalty' for the company, who then ceases to exist from an economic point of view[3]. Thus, the number of business

Date: October 29, 2014.

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[1]In the United States, the main bankruptcy procedure is described in Chapter 7 (Title 11 of the US Code) entitled *Liquidation*. The procedure begins with a petition filled by the debtor with the bankruptcy court (or by the creditors under some specific conditions; see Chapter 3, §301-303); then, the court appoints a trustee who carries out bankruptcy.

[2]This is even more the case since the debtor i) may be *discharged* from liability for certain types of debts (see Chapter 5, §523; and Chapter 7, §727); and ii) can *retain* some assets, which are therefore exempted from forfeiture by the trustee (see Chapter 5, §522(b)).

[3]The bankruptcy procedure described in Chapter 11 is an alternative to bankruptcy *per se* as put by Chapter 7, as it aims at *reorganizing* the business in order to repay debts over time according to a 'plan' (which may include exempt assets and a debt discharge, as put by §1123). Chapter

bankruptcies should not be 'too' high with respect to the total number of firms within an economy and to the size of those that go bankrupt, in order to ensure as much as possible a smooth functioning of the production activities and eventually of the economy as a whole. From this macroeconomic point of view, to understand why a business goes bankrupt does matter.

This paper points out new insights about business bankruptcy, through an agent-based model within which three firms interact with one another and with a bank. The interactions at issue are thought of in terms of *payments* and of *bank credits* in the framework of *monetary analysis*, which has as its core principle to think of the economic process on the basis of *money.* The main argument, which accounts for the different results extracted from the model, is that *the way firms execute their payments and run into debt cannot be arbitrary in order to avoid business bankruptcies as much as possible.* Thus, *besides explaining factors of business bankruptcies like macroeconomic aggregates or individual characteristics, the overall configuration of payments and credits shall be of interest too.* This invites to further research with more complex models in order to be closer to economic reality.

Section 2 explains why an agent-based model derived from monetary analysis should be of interest for business bankruptcy to be studied. Section 3 elaborates the model and section 4 presents and discusses the results.

2. The theoretical framework

We first review how business bankruptcy has been understood so far, thus reaching the need for a model elaborated in terms of *mutual-interacting agents* (2.1). Then we explain why the interactions at issue shall be accounted for by payment and by credit within the framework of monetary analysis, rather than by *supply* and by *demand* within the framework of *real analysis* (2.2 and 2.3).

2.1. **Studying business bankruptcy through agent-based modeling.** In order to understand why some firms go bankrupt, the focus is usually on two different types of factors. First, these factors identify with *macroeconomic aggregates* like GDP, money stocks, interest rates, the price/wage level, stock prices or the exchange rate (see, among others: Liu, 2004, 2009; Hunter and Isachenkova, 2006; Bhattacharjee et al., 2009a,b; Santoro and Gaffeo, 2009; Salman et al., 2011; Jin et al., 2013). Then, they consist of *individual firm characteristics* like age, size, management skills, technical/cost efficiency or profitability (see, among others: Thornhill and Amit, 2003; Pusnik and Tajnikar, 2008; Bhattacharjee et al., 2009a,b; Coleman et al., 2013).

A third type can shed light on business bankruptcy too: the *mutual interactions* between agents. Besides macroeconomic aggregates and individual characteristics, business bankruptcy can also be explained through how agents interact with one another through their decisions. In this framework, economic outcomes are endogenously emerging from individual behaviors and from local interactions between agents (Colander et al., 2008). Eventually, from this bottom-up point of view, business bankruptcy might show some patterns which are worth highlighting.

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11 gives to the firm the opportunity to stay in business (see Aivazian and Zhou, 2012), whereas Chapter 7 does not. Here, the focus is on business bankruptcy *per se* outside reorganization. Note that, under Chapter 7, the court may authorize a business to continue for a limited period of time, if such operation allows enhancing the proceeds to be distributed to the creditors; see §721.

With respect to the latter, a way to inquire is to incorporate a set of interacting agents into a model, with the aim to extract logical statements about these patterns.

Some works can be associated to the aforesaid third type. Following the seminal contributions of Shubik (1973) and Shubik and Wilson (1977), these works elaborate models within which agents interact one with each other by *supplying* and by *demanding goods* on some *markets*, while the overall set of these interactions is supposed to result in agents operating under *general equilibrium*, that is to say, the equality between the total supply of every good and the total demand for it. In this class of models, the aim is to link the fact that some agents do not honor their promises to pay with the *incompleteness*[4] of markets. These models first stem from Dubey et al. (1989), Geanakoplos and Dubey (1989) and Zame (1993), before some further contributions from Dubey et al. (2000, 2005), Araujo and Psacoa (2002), Geanakoplos (2003), Sabarwal (2003), Maldonado and Orrillo (2007) and Geanakoplos and Zame (2013), among others[5]. More recently, the same type of models has been used in order to inquire about the bankruptcy of households on their loans, in particular mortgages (Goodhart et al., 2005, 2006; Li and Sarte, 2006; Mateos-Planas and Seccia, 2006; Mateos-Planas, 2013; Eichberger et al., 2014).

It is worth reminding the following. In the previous works, the fact that debt service is not fully paid – to wit, *default* – does not *always* imply bankruptcy. Notably, in the case of 'strategic' default, debtors deliberately choose not to meet the legal obligation of debt repayment although they could do it thanks to the availability of wealth/income (Dubey et al., 2005); and they do *not* go bankrupt for that reason. Still, bankruptcy occurs if the debtor has no means to pay back creditors, as wealth and income do not cover the outstanding debt.

The main point here is that, to the best of our knowledge, the aforementioned works do not study *business* bankruptcy itself. Either they deal with pure-exchange economies, wherein there are not producers who then might face solvency problems; or, despite the introduction of some producers, no room is left for their bankruptcy. Actually, in Bisin et al. (2009, 2011) and Carvalho et al. (2013), firms might default on some debts. However, these models are made of two periods only, with default occurring in the second one. As a consequence, they cannot capture the effects of default, as the earliest of these effects logically take place in the third period. In particular, they cannot capture the possibility for default to lead to business bankruptcy itself, the former being necessary – yet no sufficient – for the latter to occur.

An exception is De Walque et al. (2010), as they suggest a multi-period model. Nonetheless, as put by the authors themselves (p. 1237), 'we assume that defaulters are not excluded from the market but bear costs'. Still, such an exclusion is necessary for business bankruptcy to be thought of, due to the very disappearance

[4]As put by Wilson (1987, p. 759), 'markets are complete when every agent is able to exchange every good either directly or indirectly with every other agent'. See Magill and Quinzii (2002).

[5]For instance, Modica et al. (1998) deal with complete markets and argue that, in this case, failures to pay debts stem from the *bounded rationality* of agents as they cannot foresee all the possible future contingencies; see the comment by Sabarwal (2003, pp. 5-6).

of the firm that goes bankrupt. Again, the model leaves room for default, but does not for business bankruptcy[6].

To sum up, business bankruptcy remains to be explained into models made of agents who interact zith one another over several periods, and which should leave room for the exit of the firms that face solvency problems over time.

2.2. **The need for an alternative conception of the economic interactions.** In this paper, the choice is to elaborate an agent-based model that does *not* to account for the interactions between agents through supply and demand, contrary to the previous works. In this respect, one can stress the *stability* problem, to wit, the fact that (individual) supplies and demands seldom result in a (unique) general equilibrium under economically plausible conditions (see e.g. Saari, 1985; Fisher, 1989, 2005; Kirman, 1989, 2010; Ingrao and Israel, 1990; Hildenbrand and Kirman, 1991; Kehoe, 1998). Consequently, either one has to assume right from the start that the general equilibrium of a model calibrated with statistical data is stable; or, this model is *intrinsically* incapable of generating logical statements which then can be empirically tested. Here, another argument is brought to the fore, according to the following three interrelated points.

First, at the time of incorporating supplies and demands into a model, the starting point is always a *predetermined list* of goods, which are distinguished from one another by four characteristics: their physical attributes, the date of their delivery, the location of their delivery and the state of nature; for instance, 'umbrellas to be delivered in Cambridge on Christmas Day 1980 if it rains' (Hahn, 1981, p. 124). In line with Arrow and Debreu (1954) and Arrow and Hahn (1971), in every model based upon general equilibrium, each agent is supposed to supply and demand *only* the goods that incorporate the list at issue.

Such a 'commodity space' (Debreu, 1959, p. 32) would not seem questionable. According to Hildenbrand and Kirman (1991, p. 53), 'we assume that there is only a finite number l of commodities. Note that this does not impose any real restriction, since all that we are assuming is that the agents in an economy are only capable of distinguishing between a finite number of commodities'. Nevertheless (second point), the authors do not underlie that *all* the agents distinguish between the *same* commodities *before* they enter into interactions. The commodity space, which is introduced for the model to be elaborated, thus becomes a *common prior knowledge* for every agent that enters the model itself. It is as if all the agents agreed *ex ante* upon the goods that will then enter their decision-making process. Or, it could also be that these goods were imposed by a *deus ex machina*, be it the Walrasian auctioneer or any other fictitious agent for the economy as a whole.

Hence (third point), a major difficulty arises. *Decentralization* is a core feature of the economic process. This means that 'a large number of individuals make economic decisions which, in the light of market and other information, they consider most advantageous. They are not guided by the social good, nor is there an overall plan in the unfolding of which they have preassigned roles' (Hahn, 1981, p. 123). In particular, decentralization implies that each agent determines which goods it should supply and demand *outside* any overall prior agreement with the others, and outside some exogenous directives that specify what can be produced,

[6]Actually, this would be difficult in the model at issue: production is accounted for through a representative firm, whose bankruptcy would annihilate the production of the economy as a whole.

exchanged and consumed (Benetti and Cartelier, 1980). So, contrary to what induced by decentralization, the commodity space implies that agents have 'little room to breathe'. If a model thinks of the economic interactions through supply and demand, then the nature of goods should incorporate the decision-making process of agents instead of incorporating the framework that surrounds such process. Otherwise, the model hardly fits with decentralization.

Two solutions are possible. This first one consists in leaving aside the commodity space (and finding a way out with respect to the stability problem) while keeping the conception of the economic interactions in terms of supply and demand, if possible[7]. The second one consists in elaborating a model with another conception, in order to avoid the difficulties induced by the commodity space (and by stability) at the time of modeling. The second solution is chosen in this paper.

2.3. **From supply and demand to payment and credit.** Let us first highlight the following: describing the economic interactions through supply and demand is part of a specific way of thinking of the economic process, named *real analysis*, which consists in making *goods* the core concept of economic theory. 'Real Analysis proceeds from the principle that all the essential phenomena of economic life are capable of being described in terms of goods and services, of decisions about them, and of relations between them' (Schumpeter, 1954, p. 277). This applies to modern economics. Indeed, in addition to focus on the supplies of goods and on the demands for them, goods themselves are the basis for supplies/demands to be explained, through preferences (with respect to goods), endowments (in goods) and production techniques (of goods). Moreover, the *numéraire* is the good whose quantities measure prices (x tons of iron costs, say, y meters of cloth). Even labor is seen as a mere good among others, whose quantity is the time worked and whose price is the wage paid.

Consequently, little room is left for *money* in real analysis. Money is seen as a mere good among others (be it 'immaterial' in the case of the so-called 'fiat' money) which plays 'the modest role of a technical device that has been adopted in order to facilitate transactions. This device can no doubt get out of order, and if it does it will indeed produce phenomena that are specifically attributable to its *modus operandi*. But so long as it functions normally, it does not affect the economic process, which behaves in the same way as it would in a barter economy: this is essentially what the concept of Neutral Money implies' (Schumpeter, 1954, p. 277). To think of money as a mere appendix, which might disturb the 'true' goods-founded outcomes of the economic process, is indeed something usual in modern economics (see Parkin, 2000; Friedman, 2010; Werner, 2011; Argitis, 2013).

So, in order to think of the economic interactions in another way, the starting point can be the opposite principle: to make money the core of a conceptual framework while giving secondary importance to goods; hence *monetary analysis*[8]. As

[7]To our knowledge, no work has attempted to put an end to the commodity space, which remains widely unquestioned. This even applies to the latest works that suggest some solutions to the stability problem, for example Bodenstein (2013), Hatfield et al. (2013) or Hsu and Shih (2013).

[8]Actually, goods and the related phenomena can be introduced into monetary analysis (obviously outside a commodity space), in order to design more specific theories and models. Still, such an introduction is not necessary *a priori*. In monetary analysis, 'real' phenomena are supposed not to play a major role for the understanding of the economic life. This does not mean that they are not important in real-world economies. Still, the key-feature of monetary analysis is to

put by Schumpeter (1954, p. 278), 'Monetary Analysis introduces the element of money on the very ground floor of our analytic structure (...)'. Indeed, monetary analysis leads to another conception of the economic interactions, as put below.

First, these interactions are accounted for by the *payments* between agents, leaving aside the 'real' phenomena – goods, production techniques, preferences and endowments – that underlie the payments themselves. Two agents interact every time one of them is the *payer* and the other the *payee*, instead of describing the former as someone seeking some goods and the latter as someone providing them. This shift stems from money as the core concept. In monetary analysis (and as it can be observed), money is first of all what allows agents to execute payments; so, the focus on the former logically leads to the latter (Cartelier, 1996). Money is first the *unit* (say, dollar) for prices and other magnitudes (like wages, profit or capital) to be expressed[9] (say, x tons of iron costs $\$X$). Then, money identifies with the *objects* denominated in the monetary unit and whose transfer between agents eventually permit the payments associated to economic magnitudes. These *means of payment* are first *coins*, to wit, melted pieces of metal which then are minted in order to be associated to, say, $\$x$. So, if the (monetary) price of some good is $\$X$, then the buyer transfers X/x coins to the seller for the former to pay the latter. The payer can also transfer *notes*, to wit, pieces of paper which then are printed in order to be denominated in the monetary unit.

Above all, the payer can transfer *acknowledgments of debts by banks*, provided that these acknowledgments amount to $\$X$[10]. This last type of means of payment is predominant today[11]. It underlies the double-entry bookkeeping in bank accounts: a payment of $\$X$ is executed by debiting the payer's bank account and by crediting the payee's one for the same amount (Rossi, 2007). As a result, for the economic interactions to be thought of, banks must be distinguished from the other agents, and the former interact with the latter through *credits* and through the payments that reimburse them. In order to explain that, we start by the fact that agents must *finance* their payments, to wit, they must dispose of means of payments in order to execute the payments themselves. Accordingly, a way to finance payments is to be paid beforehand. If an agent is paid, means of payment are received, which then can be used for new payments. Now, from a logical point of view, there must be a process which creates means of payment independently of any prior payment. This process consists in melting/minting metal in the case of coins, and in printing pieces of paper in the case of notes; in both cases, this process is undertaken by the central bank. Besides, the process is *bank credit* for the acknowledgments of debts by (private) banks. Contrary to a widely shared belief, 'loans make deposits'. Banks can issue acknowledgments of debts, which then are lent as means of payment. Thereafter, these acknowledgments return to the banking system when used to pay

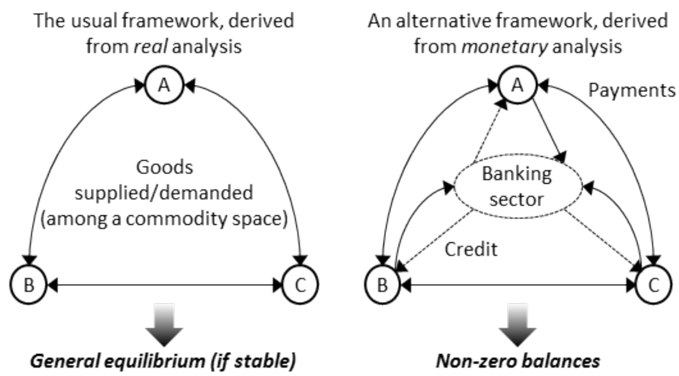
carry out such an understanding only by money and by the related phenomena every time this is possible. The aim is to dispose of the most easy-to-use and manageable theories and models, in accordance with the *modus operandi* of social sciences (Stellian, 2012).

[9] There are no longer 'real' prices as quantities of the *numéraire*, which would be the prices that would truly matter under the so-called 'veil' of monetary prices (Tricou, 2013).

[10] In this respect, agents do not ask banks to settle their debts, which are no longer considered as such, but as a mean to pay. See Parguez and Seccareccia (2005).

[11] The term 'object' aims at emphasizing that coins/notes/debts are not (dematerialized) goods, but the representation of a (purely abstract) monetary unit for payments to be executed. Also, the monetary objects can be used as a *store of value*; see Graziani (1996).

FIGURE 1. From one theoretical framework to another for agent-based economic modeling: real analysis *versus* monetary analysis



banks for the reimbursement of the credits granted before. Banks then decide either to lend again their acknowledgments (thus used as means of payment again), or to destroy them[12] (Keen, 2009).

As today means of payment mainly consist in these acknowledgments, bank credit is the central issuing principle of means of payment; hence the focus on the aforesaid interactions between banks and non-bank agents[13].

Besides, as the interactions are not thought of through supplies/demands, then their result cannot be thought of as a general equilibrium (if stable). With payments and credits, general equilibrium is replaced with *balances*, which are defined as the difference, for each agent, between the payments received from the others – *receipts* – and the payments executed by the former – *expenditures* (Benetti and Cartelier, 1987). A balance can be negative, to wit, a *deficit*. Nothing implies that every indebted agent will be paid by the others up to what is needed to fully repay debt service. So, some agents may record a deficit: they paid more than they were paid by the other agents. The *settlement* of deficits is compulsory, according to different ways: a new credit, the sale of some assets, to issue shares (in the case of firms), or even bankruptcy itself. To the contrary, a balance can be positive, to wit, a *surplus*: some agents paid less than they were paid, thus giving rise to a 'surplus' of means of payment. This surplus can then be used for financing new payments.

To sum up, in monetary analysis, agents interact by executing payments and those that are banks also interact with the other agents through credit relationships; and, the overall result of these interactions is the balances recorded by the agents (see figure 1). This way to account for the economic interactions will be used for the model to be elaborated in this paper.

[12]This also implies some payments and credits between two private banks, as well as between each of them and the central bank, who also issues its own acknowledgments of debts (see Rossi, 2007). Here, these interbank relationships will not be explicitly dealt with, as explained later.

[13]In relation with the previous footnote, the interactions between banks and non-bank agents imply some underlying interactions within which private banks and the central bank are involved, again in terms of payments and of credits (see Rossi, 2007).

3. The model

The modeling will be undertaken as in *agent-based computational economics* (ACE). As put by Tesfatsion (2002, p. 56), 'ACE researchers rely on computational laboratories to study the evolution of decentralized market economies under controlled experimental conditions. (...) As in a culture-dish laboratory experiment, the ACE modeler starts by constructing an economy with an initial population of agents. The ACE modeler specifies the initial state of the economy by specifying the initial attributes of the agents. (...) The economy then evolves over time without further intervention from the modeler. All events that subsequently occur must arise from the historical time-line of agent-agent interactions' (see also: Tesfatsion, 2003; LeBaron and Tesfatsion, 2008; Chen, 2012; Lengnick, 2013). The model has to determine: i) the amount of every payment from a prior typology; ii) their financing; iii) how the balances that result from the payments at issue are dealt with; and iv) how firms go bankrupt. The section begins with a general description of the model (3.1), before introducing the preliminary notations (3.2). Then, the following subsections unfold the model (3.3 to 3.6).

3.1. **Structure of the model.** The model deals with a simple economy made of two types of agents: firms and banks. Payments and the related magnitudes – expenditures, receipts, debts, balances, and so on – are measured in a given monetary unit, say, dollar ($). Means of payments consist of the acknowledgments of debts issued by a single bank through credits granted to the firms (there is no 'cash'). So, each firm holds an account within that bank, who then debits/credits these accounts for payments to be executed on behalf of the former. Note that only these elements – the unit of account, the type of means of payment and the way to issue them – along with the obligation to pay debts frame economic interactions; this is far more compatible with decentralization (contrary to the commodity space) and confirms the usefulness of monetary analysis. Last, banking activities – debiting/crediting accounts and credit granting/monitoring – are supposed to be without cost.

Figures 2 and 3 show the structure of the model. For each period t, the model first begins by checking if a given firm has been affected by bankruptcy in some past period (that is to say, in $t-1$, $t-2$, $\cdots$, 0). At the beginning of the first period $t=0$, no firm is supposed to go bankrupt as the economic process has not yet begun. In case of bankruptcy, the firm no longer exists and cannot execute payments or run into debt in t. If the firm remains in business, then it first *plans* what its payments toward the other firms should in t, before taking into account the past bankruptcies (if any) in order to pass from these *planned* payments to the *effective* ones. A firm cannot pay those that went bankrupt. Thus, in the model, an effective payment equals zero and differs from the related planned one if bankruptcy has affected the payee; and, even if the payee did not go bankrupt, the model assumes that the effective payment might be inferior to (and thus different from) the planned one due to the bankruptcy of a third firm, as all of them are interdependent with one another. If no firm has been affected by bankruptcy, then every effective payment equals the planned one.

In order to deduce the planned payments themselves in t, the model determines their *total* and applies some coefficients that achieve the *distribution* of this total from the firm that makes such a planning to the others. Logically, the value of each coefficient is between 0 and 1 and their sum for a given firm equals 1. For example, 10% of the total payments planned by A in t is to be directed toward B, 25% toward C... and so on, until 100% of the total is distributed. In this respect, two variables enter the model. The first one is the receipts expected by the firm in t. If higher, these *expected receipts* lead the firm to plan more payments in t. Receipts primarily derive from the sale of some production. So, if more receipts are expected, then one can reasonably assume that the firm has to produce (at least) more and eventually has to pay more in order to acquire some intermediary/capital goods necessary for production to be increased (everything else being equal). Expected receipts thus act as an 'effective-demand' scheme.

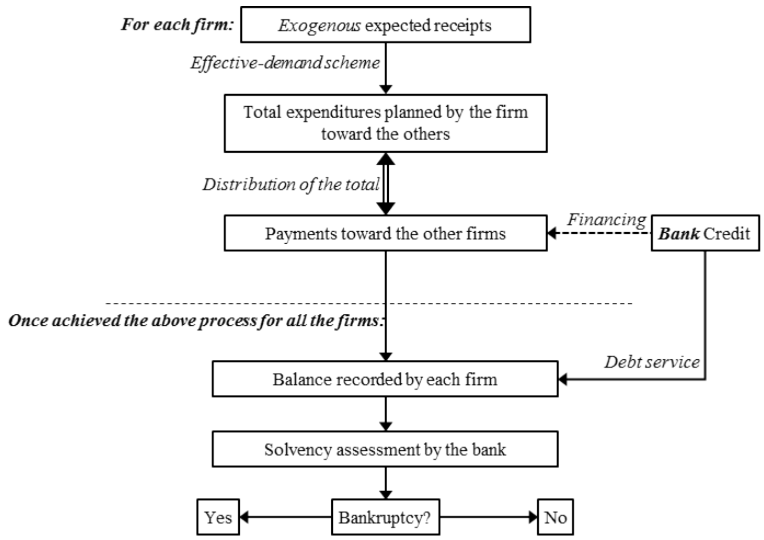
The second variable is the *average balance* recorded by the firm over some past periods $t-1$, $t-2$, $\cdots$, $t-p$. The firm plans more expenditures in t if its average balance turns positive (or, if already positive, higher). If positive, such an average means that the firm tended to record a surplus in the past. Linking the former to expenditures acts as a 'surplus-seeking scheme' and corresponds to a stylized fact: economic agents – first of all firms – are motivated by accumulating a surplus of money (Tricou, 2013). Thus, with a (higher) surplus, the firm plans to spend (at least) more. The objective is to record even more sales and eventually to maintain/intensify surplus accumulation, as it proved to occur in the past. In the first period $t = 0$, the aforementioned past trend has not yet occurred and thus does not enter the determination of the planned payments.

The expected receipts in t are themselves determined as follows. For the initial period ($t = 0$), they are exogenous. They constitute the initial conditions of the model and, as such, the beginning of the decision-making process for the firms. For the following periods ($t \geq 1$), they are determined according to an *adaptive mechanism*: in the previous period, if the expected receipts match the *effective* ones, then the former remain the same; otherwise, they are more or less adjusted taking into account the effective receipts from some past periods.

Thereafter, the model determines how the firm finances its (effective) payments in t. If the firm has recorded a surplus in the previous one ($t-1$), meaning that means of payment are available, then it uses this surplus for financing its payments in t; and if that previous surplus is not enough, then the bank grants a credit to the firm in t in order to cover the financing needs. This amounts to assume that the bank does not apply quantitative restrictions on credits, as long as the firm is considered as solvent by the bank itself. Note that, in the first period $t = 0$, there is no previous surplus by definition, so that only credit finances payments in $t = 0$. Last, if the firm has recorded a deficit in $t-1$, then no surplus is available, so that bank credit covers all the financing needs in t. Credit also settles the deficit at issue: the bank accepts to lend money for such deficit to be settled.

The firm reimburses a credit granted in t by payments of the same value during t as well as during some next periods (that is say, during t, $t+1$, $t+2$, $\cdots$). These payments include some interest charges, which first are proportional to the credit. Moreover, for every period after the first one ($t = 0$), the firm may record a deficit in the past, as put by a negative average balance over $t-1$, $t-2$, $\cdots$, $t-p$. If so, the firm does not show the ability to fully reimburse its debts. So, the bank increases the interest charges in response to a more risky borrower. Reciprocally, if the firm tended to record a surplus in the past, as put by a positive average balance, then it shows the ability to fully reimburse its debts. So, the bank decreases the interest

FIGURE 2. The structure of the model in the initial period $t = 0$



charges in response to a less risky borrower. Notwithstanding, the interest charges cannot be less than a minimal proportion of the related credit.

At this stage, all the firms have determined their payments in t – including those for credits to be reimbursed – and have been granted with new credit too. So, for every of those that has not been affected by bankruptcy, it is possible to establish its *effective* receipts in t and, along with its expenditures, its balance for the period. Then, the balances have consequences over how to finance the payments in the next period $t + 1$, as put before. Balances also determine if firms go bankrupt in t. The bank calculates which percentage of debt service in t is met by each firm, thanks to its receipts in t. Doing so for each period, the bank then calculates an average percentage over some past periods. If the average is inferior to a given level, the bank refuses to lend to the firm anymore, which in turn has shown to be unable to fully repay debts; bankruptcy is thus the sole issue[14].

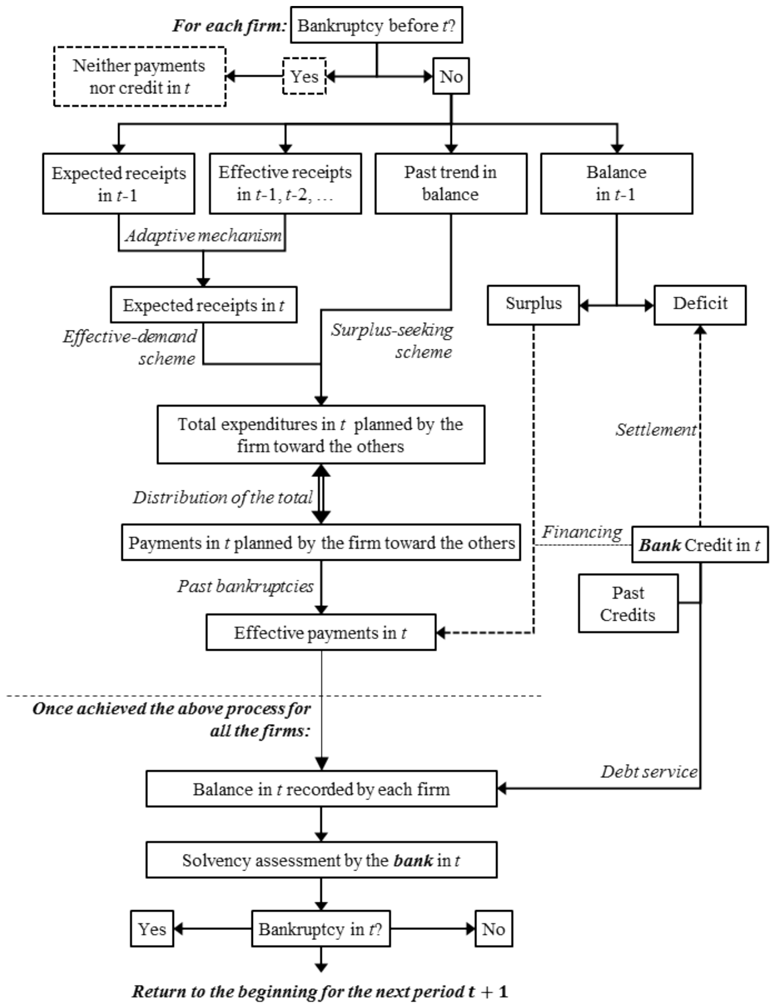
In the end, passing from one period to another, some firms may go bankrupt. Therefore, it is possible to extract the number of bankruptcies within the model, after firms and banks have interacted throughout a given number of periods.

3.2. **Preliminary notations.** $J \in \mathbb{N} \setminus \{0; 1\}$ is the initial number of agents acting like a business. The economic process unfolds over $T + 1 \in \mathbb{N}^*$ periods from the first one $t = 0$ to a last one $t = T$.

$d_{ij}^{(t)} \in \mathbb{R}_+$ is the payment from a firm-like agent i to another $j \neq i$ in t, with $i, j \in \{1; 2; \cdots ; J\}$. $l_i^{(t)} \in \mathbb{R}_+$ is the payment from i to the bank in t in order to reimburse (part of) some credits granted in t and/or before t. $m_i^{(t)} \in \mathbb{R}_+$ is the payment from i to the bank in t as (part of) the interest charges applied to some credits granted in t and/or before t.

[14]We leave aside the fact that an agent may acquire a business that otherwise would go bankrupt.

FIGURE 3. The structure of the model from $t = 1$ onwards



$f \in \{0; 1; 2; \cdots ; J\}$ is the number of firm-like agents that did go bankrupt after the $T + 1$ periods. To this purpose, let $solv_i^{(t)} \in \{0; 1\}$ be a solvency indicator of i in t. If $solv_i^{(t)} = 1$ then i is solvent in t and does not go bankrupt; and if $solv_i^{(t)} = 0$ then i is *not* solvent in t and does go bankrupt. Whether $solv_i^{(t)}$ equals 0 or 1 is an endogenous result of the model and will be dealt with later (in subsection 3.6). For the time being, let $SOLV_i^{(t)} := \left(solv_i^{(0)}; solv_i^{(1)}; \cdots ; solv_i^{(t)}\right)$ be the vector made of every $solv_i^{(p)}$ from $p = 0$ to $p = t$. Thus, $0 \in SOLV_i^{(T)}$ amounts to the bankruptcy of i during the economic process. Eventually:

$$f := \# \left\{ i : \ 0 \in SOLV_i^{(T)} \right\} \in \{0; 1; 2; \cdots ; J\} \tag{1}$$

3.3. **The determination of $d_{ij}^{(t)}$.** For every $(i;j;t)$, let $\tilde{d}_{ij}^{(t)} \in \mathbb{R}_+$ be the payment that i *plans* to execute toward $j \neq i$ in t; $\theta_{ij}^{(t)} \in \{0;1\}$; $0 \leq \theta_i^{(t)} \leq 1$. We write:

$$d_{ij}^{(t)} = \tilde{d}_{ij}^{(t)} \left(\theta_{ij}^{(t)} \cdot \theta_{ji}^{(t)} \cdot \prod_{k=1}^{J} \theta_k^{(t)} \right) \tag{2}$$

The *effective* payment $d_{ij}^{(t)}$ equals the *planned* payment $\tilde{d}_{ij}^{(t)}$ adjusted by some θ-type coefficients. We first begin with these coefficients.

$$\begin{cases} \text{If } t = 0 \text{ then } \theta_{ij}^{(t)} = \theta_{ji}^{(t)} = \theta_i^{(t)} = 1 \\ \text{else} \begin{cases} \text{If } 0 \in SOLV_i^{(t-1)} \text{ then } \begin{cases} \theta_{ij}^{(t)} = \theta_{ji}^{(t)} = 0 \\ 0 \leq \theta_i^{(t)} \leq 1 \end{cases} \\ \text{else } \theta_{ij}^{(t)} = \theta_{ji}^{(t)} = \theta_i^{(t)} = 1 \end{cases} \end{cases} \tag{3}$$

$0 \in SOLV_i^{(t-1)}$ amounts to the bankruptcy of i before t. As a result, i no longer exists from an economic point of view, so that i cannot execute payments towards any other firm $j \neq i$ in t (as well as during the next periods). To put $\theta_{ij}^{(t)} = 0$ is the way to deal with such a situation. Indeed, with that value the product of $\theta_{ij}^{(t)}$ by the planned payment $\tilde{d}_{ij}^{(t)}$ reduces the effective payment $d_{ij}^{(t)}$ to zero, which is consistent with the bankruptcy of i.

If i has not faced bankruptcy before t – to wit, $0 \notin SOLV_i^{(t-1)}$ – but if this is the case of another firm $j \neq i$ – to wit, $0 \in SOLV_j^{(t-1)}$ – then i cannot pay j in t. Although $\theta_{ij}^{(t)} = 1$ we have $\theta_{ji}^{(t)} = 0$. With that value of $\theta_{ji}^{(t)}$ the product of this coefficient by the planned payment $\tilde{d}_{ij}^{(t)}$ reduces the effective payment $d_{ij}^{(t)}$ to zero, which is consistent with the bankruptcy of j.

If neither i nor $j \neq i$ have been affected by bankruptcy in t or before t – to wit, $0 \notin SOLV_i^{(t-1)}$ and $0 \notin SOLV_j^{(t-1)}$ – then $\theta_{ij}^{(t)} = \theta_{ji}^{(t)} = 1$. $\tilde{d}_{ij}^{(t)}$ remains unchanged after its product with that value of these coefficients. However, if bankruptcy affects a third firm $k \neq i, j$ before t, we assume that this might in turn affect the other payments outside those from and toward k in t (as well as during the next periods). At least these payments remain the same. Hence $0 \leq \theta_k^{(t)} \leq 1$ which is the 'reduction coefficient' of every planned payment in t by the bankruptcy of k, as put by the product of $\tilde{d}_{ij}^{(t)}$ by $\prod_{k=1}^{J} \theta_k^{(t)}$.

If no firm has been affected by bankruptcy, then $\theta_{ij}^{(t)} = \theta_{ji}^{(t)} = \theta_k^{(t)} = 1 \; \forall i, j, k$; so every effective payment is equal to the planned one. If not, then the two payments are different ($\theta_{ij}^{(t)} = 0$ and/or $\theta_{ji}^{(t)} = 0$) or might be different ($0 \leq \theta_k^{(t)} \leq 1$).

Let us continue with the determination of $\tilde{d}_{ij}^{(t)}$ in order to achieve the very determination of $d_{ij}^{(t)}$. Let $X_i^{(t)} \in \mathbb{R}_+$ be the overall sum of payments that i plans to execute in t toward the other firms, to wit:

$$X_i^{(t)} := \sum_{\substack{j=1 \\ j \neq i}}^{J} \tilde{d}_{ij}^{(t)} \tag{4}$$

Besides, let $\alpha_{ij} \in [0;1]$ $\forall i$ and $\forall j \neq i$ with $\sum_{\substack{j=1 \\ j \neq i}}^{J} \alpha_{ij} = 1$. We write:

$$\tilde{d}_{ij}^{(t)} = \alpha_{ij} \cdot X_i^{(t)} \tag{5}$$

$\tilde{d}_{ij}^{(t)}$ is proportional to $X_i^{(t)}$ scaled by a 'distribution coefficient' α_{ij}. Logically, the value of every α_{ij} is between 0 and 1 (a negative value would be meaningless, while bounded by 100% of the total) and the sum of them for a given i equals 1 (the total is entirely distributed, otherwise it would not be a total as such). As such, every α_{ij} is not supposed to change over time.

Thereafter, one must deal with the determination of $X_i^{(t)}$. To this purpose, let $R_i^{(t)} \in \mathbb{R}_+$ be the overall sum of payments that i *expects* to benefit from the other firms in t; $\Pi_i^{(t)} \in \mathbb{R}$ the balance recorded by i in t; and $\bar{\Pi}_i^{(t)} \in \mathbb{R}$ the discounted average of $\Pi_i^{(t)}, \Pi_i^{(t-1)}, \cdots, \Pi_i^{(t-p)}$, with $p \in \{t; t_i\}$, $t_i + 1$ being the maximum number of past periods for the calculation of that average by i, and $\eta_i \in [0;1]$ being the (time-constant) discount factor used by i, to wit:

$$\begin{cases} \text{If } t < t_i \text{ then } \bar{\Pi}_i^{(t)} := \frac{1}{\eta} \cdot \sum_{p=0}^{t} (1-\eta_i)^u \, \Pi_i^{(t-p)} \text{ with } \eta := \sum_{p=0}^{t} (1-\eta_i)^p \\ \text{else } \bar{\Pi}_i^{(t)} := \frac{1}{\eta} \cdot \sum_{p=0}^{t_i} (1-\eta_i)^p \, \Pi_i^{(t-p)} \text{ with } \eta := \sum_{p=0}^{t_i} (1-\eta_i)^p \end{cases} \tag{6}$$

Besides, let $f_i : \mathbb{R}_+ \to \mathbb{R}_+$ and $g_i : \mathbb{R}_+ \to \mathbb{R}_+$. We write:

$$\begin{cases} \text{If } t = 0; \text{ or if } t \geq 1 \text{ and } \bar{\Pi}_i^{(t)} \leq 0 : X_i^{(t)} = f_i\left(R_i^{(t)}\right) \text{ with } \frac{\partial f_i}{\partial R_i^{(t)}} \geq 0 \\ \text{else } X_i^{(t)} = f_i\left(R_i^{(t)}\right) + g_i\left(\bar{\Pi}_i^{(t-1)}\right) \text{ with } \frac{\partial g_i}{\partial \bar{\Pi}_i^{(t-1)}} \geq 0 \end{cases} \tag{7}$$

$X_i^{(t)}$ is a function of the 'proceeds' expected by i in t – $R_i^{(t)}$ – and possibly of the surplus that i tended to record – $\bar{\Pi}_i^{(t-1)} \geq 0$. $R_i^{(t)}$ acts as the aforesaid effective-demand scheme (f_i and g_i) and $\bar{\Pi}_i^{(t-1)} \geq 0$ as the surplus-seeking scheme (g_i).

As a starting point, f_i and g_i can be linear functions. Let $(\beta_{i1}; \beta_{i2}) \in \mathbb{R}_+^2$:

$$\begin{cases} f_i\left(R_i^{(t)}\right) = \beta_{i1} \cdot R_i^{(t)} \\ g_i\left(\bar{\Pi}_i^{(t-1)}\right) = \beta_{i2} \cdot \bar{\Pi}_i^{(t-1)} \end{cases} \tag{8}$$

The next step is to determine $R_i^{(t)}$ and $\Pi_i^{(t-1)}$. The latter will be dealt with in subsection 3.6 along with the other pending variable $solv_i^{(t)}$. With respect to $R_i^{(t)}$, let $Q_i^{(t)} \in \mathbb{R}_+$ be the total receipts *effectively* recorded by i in t, to wit:

$$Q_i^{(t)} := \sum_{\substack{j=1 \\ j \neq i}}^{J} d_{ji}^{(t)} \tag{9}$$

Besides, let $\bar{Q}_i^{(t)} \in \mathbb{R}_+$ be the η_i-discounted average of $Q_i^{(t)}, Q_i^{(t-1)}, \cdots, Q_i^{(t-p)}$, with $p \in \{t; t_i\}$, to wit:

$$\begin{cases} \text{If } t < t_i \text{ then } \bar{Q}_i^{(t)} := \frac{1}{\eta} \sum_{p=0}^{t} (1-\eta_i)^p \, Q_i^{(t-p)} \text{ with } \eta := \sum_{p=0}^{t} (1-\eta_i)^p \\ \text{else } \bar{Q}_i^{(t)} := \frac{1}{\eta} \sum_{p=0}^{t_i} (1-\eta_i)^p \, Q_i^{(t-p)} \text{ with } \eta := \sum_{p=0}^{t_i} (1-\eta_i)^p \end{cases} \tag{10}$$

The adaptive mechanism that derives $R_i^{(t)}$ from $\bar{Q}_i^{(t-1)}$ and from $R_i^{(t-1)}$ is:

$$\begin{cases} \text{If } t = 0 \text{ then } R_i^{(t)} \text{ is exogenous} \\ \text{else} \begin{cases} \text{If } R_i^{(t-1)} = Q_i^{(t-1)} \text{ then } R_i^{(t)} = R_i^{(t-1)} \\ \text{else } R_i^{(t)} = \delta_i \cdot \bar{Q}_i^{(t-1)} + (1-\delta_i)R_i^{(t-1)} \text{ with } 0 \leq \delta_i \leq 1 \end{cases} \end{cases} \tag{11}$$

δ_i is the coefficient that measures the respective importance of $\bar{Q}_i^{(t-1)}$ and of $R_i^{(t-1)}$ for $R_i^{(t)}$ to be determined if i did not correctly anticipate its prior receipts, to wit, if $R_i^{(t-1)} \neq Q_i^{(t-1)}$. For instance, if $\delta_i = 1$ then i bases its expected receipts in t only upon the past trend in effective receipts $\bar{Q}_i^{(t-1)}$. If $\delta_i = 0$ then i bases its expected receipts in t only upon the past expected receipts $R_i^{(t-1)}$. Every value of δ_i between 0 and 1 accounts for an intermediate situation, wherein both $R_i^{(t-1)}$ and $\bar{Q}_i^{(t-1)}$ matters to a more or less extent for the determination of $R_i^{(t)}$.

3.4. **The financing of $d_{ij}^{(t)}$ and the settlement of deficits.** Once given every $d_{ij}^{(t)}$ through the above equations, one has to inquire about their financing. If $d_{ij}^{(t)} = x$ then i must possess means of payments up to \$$x$ in t in order to finance $d_{ij}^{(t)}$. Bank credit allows firms to finance some of their payments, and to settle their previous deficits – i.e. $\Pi_i^{(t-1)} \leq 0$. Firms can also use (part of) their previous surplus – i.e. $\Pi_i^{(t-1)} \geq 0$ – in order to finance some payments.

Let $Y_i^{(t)} \in \mathbb{R}_+$ be the total payments *effectively* executed by i in t, to wit:

$$Y_i^{(t)} := \sum_{\substack{j=1 \\ j \neq i}}^{J} d_{ij}^{(t)} \tag{12}$$

Besides, let $L_i^{(t)} \in \mathbb{R}_+$ be the credit granted by the bank to i in t and $\Phi_i^{(t)} \in \mathbb{R}_+$ the part of $\Pi_i^{(t-1)} > 0$ that is not used by i in t in order to finance payments within $Y_i^{(t)}$. We first write:

$$\text{If } t = 0 \text{ then: } L_i^{(t)} = Y_i^{(t)}; \text{ and } \Phi_i^{(t)} = 0 \tag{13}$$

In the initial period $t = 0$, i logically cannot record a previous surplus which could help for financing purposes. As a result, $\Phi_i^{(0)} = 0$ and only bank credit can finance the payments decided by i in $t = 0$, to wit $L_i^{(0)} = Y_i^{(0)}$.

Then, we write:

$$\text{If } t \geq 1 \text{ and if } 0 \in SOLV_i^{(t-1)} \text{ then } L_i^{(t)} = \Phi_i^{(t)} = 0 \tag{14}$$

If i has been affected by bankruptcy before $t \geq 1$ then i cannot benefit from a credit in t; hence $L_i^{(t)} = 0$. On the other hand, the bankruptcy of i is the result of the accumulation of deficits over time. So, there cannot be any surplus, even less part of this surplus which is unused after deciding of a financing plan; hence $\Phi_i^{(t)} = 0$.

Last:

$$\text{If } t \geq 1 \text{ and if } 0 \notin SOLV_i^{(t-1)} \text{ then:} \\ \begin{cases} \text{If } \Pi_i^{(t-1)} < 0 \text{ then } L_i^{(t)} = Y_i^{(t)} + \left|\Pi_i^{(t-1)}\right| \text{ and } \Phi_i^{(t)} = 0 \\ \text{If } 0 \leq \Pi_i^{(t-1)} \leq Y_i^{(t)} \text{ then } L_i^{(t)} = Y_i^{(t)} - \Pi_i^{(t-1)} \text{ and } \Phi_i^{(t)} = 0 \\ \text{If } \Pi_i^{(t-1)} \geq Y_i^{(t)} \text{ then } L_i^{(t)} = 0 \text{ and } \Phi_i^{(t)} = \Pi_i^{(t-1)} - Y_i^{(t)} \end{cases} \tag{15}$$

If $\Pi_i^{(t-1)} < 0$ (first case) then a bank credit in t finances all the payments within $Y_i^{(t)}$. This credit also finances the deficit previously recorded by i. Logically, as i does not record a surplus in $t-1$, there is not part of this surplus after financing (part of) the above-mentioned payments. If $\Pi_i^{(t-1)} \geq 0$ then this gives rise to the two other cases. Either the surplus previously recorded by i in $t-1$ is not enough in order to finance all the payments within $Y_i^{(t)}$ (second case). A bank credit is thus needed in order to finance what the previous surplus cannot. In this respect, the surplus is totally used for financing purposes. Or, the surplus previously recorded by i in $t-1$ is enough in order to finance all the payments within $Y_i^{(t)}$ (third case). No bank credit is needed and the previous surplus even remains partly unused.

3.5. **The determination of $l_i^{(t)}$ and $m_i^{(t)}$.** The next step is to set how firms reimburse their credits; hence the determination of the two other types of payments from the typology, $l_i^{(t)}$ and $m_i^{(t)}$.

We begin with $l_i^{(t)}$. Let $p_i \in \mathbb{N} \setminus \{0\}$. We first write:

$$\text{If } t = 0 \text{ then } l_i^{(t)} = \frac{1}{p_i} L_i^{(t)} \tag{16}$$

In the initial period, i has to reimburse $100/p_i\%$ of $L_i^{(0)}$.

Then:

$$\text{If } t \geq 1 \text{ and if } 0 \in SOLV_i^{(t-1)} \text{ then } l_i^{(t)} = 0 \tag{17}$$

If i has been affected by bankruptcy before t then i no longer exists from an economic point of view, so that i cannot reimburse any credit in t; hence $l_i^{(t)} = 0$.

Last:

$$\text{If } t \geq 1 \text{ and if } 0 \notin SOLV_i^{(t-1)} \text{ then:}$$
$$\begin{cases} l_i^{(1)} = \frac{1}{p_i} L_i^{(0)} + \frac{1}{p_i} L_i^{(1)} = \frac{1}{p_i}\left(L_i^{(0)} + L_i^{(1)}\right) \\ l_i^{(2)} = \frac{1}{p_i} L_i^{(0)} + \frac{1}{p_i} L_i^{(1)} + \frac{1}{p_i} L_i^{(2)} = \frac{1}{p_i}\left(L_i^{(0)} + L_i^{(1)} + L_i^{(2)}\right) \\ \vdots \\ \text{If } t < p_i \text{ then } l_i^{(t)} = \frac{1}{p_i}\sum_{p=0}^{t} L_i^{(t-p)} \text{ else } l_i^{(t)} = \frac{1}{p_i}\sum_{p=t-p_i+1}^{t} L_i^{(t-p)} \end{cases} \tag{18}$$

The same part $\frac{1}{p_i}$ of $L_i^{(t)}$ has to be paid back during each period from t to $t+p_i-1$. Accumulating several credits over time, we then have the knowledge of every $l_i^{(t)}$.

Now, let us focus on $m_i^{(t)}$. Let $M_i^{(t)} \in \mathbb{R}_+$ be the total interest charges applied to $L_i^{(t)}$; $\bar{\Pi}_{ib}^{(t)} \in \mathbb{R}$ the η_b-discounted average of $\Pi_i^{(t)}, \Pi_i^{(t-1)}, \cdots, \Pi_i^{(t-p)}$, t_b+1 being the maximum number of past periods for the calculation of that average by the bank, and $\eta_b \in [0;1]$ being the (time-constant) discount factor used by the bank, to wit:

$$\begin{cases} \text{If } t < t_b \text{ then } \bar{\Pi}_{ib}^{(t)} := \frac{1}{\acute{\eta}}\sum_{p=0}^{t}(1-\eta_b)^p\,\Pi_i^{(t-p)} \text{ with } \acute{\eta} := \sum_{p=0}^{t}(1-\eta_b)^p \\ \text{else } \bar{\Pi}_{ib}^{(t)} := \frac{1}{\acute{\eta}}\sum_{p=0}^{t_b}(1-\eta_b)^p\,\Pi_i^{(t-p)} \text{ with } \acute{\eta} := \sum_{p=0}^{t_b}(1-\eta_b)^p \end{cases} \tag{19}$$

$\bar{\Pi}_{ib}^{(t)}$ is different from $\bar{\Pi}_i^{(t)}$ if the bank does not have the same time-horizon as i – i.e. if $t_b \neq t_i$ – and/or if the bank does not apply the same discount factor as i – i.e. if $\eta_b \neq \eta_i$. Besides, let $(\tau_{i1}; \tau_{i2}; \tau_{i3}; \tau_{i4}) \in \mathbb{R}_+^4$. We first write:

$$\text{If } t = 0 \text{ then } M_i^{(t)} = \tau_{i1} \cdot L_i^{(t)} \tag{20}$$

Table 1. The bank account of i in t

(a) $\Pi_i^{(t-1)} < 0$:

Debit	Credit
$\left\lvert \Pi_i^{(t-1)} \right\rvert$	$L_i^{(t)}$
$Y_i^{(t)}$	$Q_i^{(t)}$
$l_i^{(t)}$	
$m_i^{(t)}$	

(b) $0 \leq \Pi_i^{(t-1)} < Y_i^{(t)}$:

Debit	Credit
$Y_i^{(t)}$	$\Pi_i^{(t-1)}$
$l_i^{(t)}$	$L_i^{(t)}$
$m_i^{(t)}$	$Q_i^{(t)}$

(c) $\Pi_i^{(t-1)} \geq Y_i^{(t)}$:

Debit	Credit
$Y_i^{(t)}$	$\Pi_i^{(t-1)}$
$l_i^{(t)}$	$Q_i^{(t)}$
$m_i^{(t)}$	

In the initial period, interest charges are proportional to the granted credit.

Then, we write:

$$\text{If } t \geq 1 \text{ and if } 0 \in SOLV_i^{(t-1)} \text{ then } M_i^{(t)} = 0 \tag{21}$$

If i has been affected by bankruptcy before t and therefore cannot benefit from a credit in t, then i does not pay any interest charge.

Last:

$$\text{If } t \geq 1 \text{ and if } 0 \notin SOLV_i^{(t-1)} \text{ then:}$$
$$\begin{cases} \text{If } \bar{\Pi}_{ib}^{(t-1)} < 0 \text{ then } M_i^{(t)} = \tau_{i1} \cdot L_i^{(t)} + \tau_{i2} \cdot \left\lvert \bar{\Pi}_{ib}^{(t-1)} \right\rvert \\ \text{else: } \begin{cases} \text{If } \tau_{i1} \cdot L_i^{(t)} - \tau_{i3} \cdot \bar{\Pi}_{ib}^{(t-1)} \geq \tau_{i4} \cdot L_i^{(t)} \\ \text{then } M_i^{(t)} = \tau_{i1} \cdot L_i^{(t)} - \tau_{i3} \cdot \bar{\Pi}_{ib}^{(t-1)} \\ \text{else } M_i^{(t)} = \tau_{i4} \cdot L_i^{(t)} \end{cases} \end{cases} \tag{22}$$

if i tended to record a deficit according to the bank – i.e. $\bar{\Pi}_{ib}^{(t-1)} < 0$ – (first case) then the bank increases $M_i^{(t)}$ as the borrower is more risky, as put by $\tau_{i2} \cdot \left\lvert \bar{\Pi}_{ib}^{(t-1)} \right\rvert$ in addition to $\tau_{i1} \cdot L_i^{(t)}$. Now, if i tends to record a surplus according to the bank – i.e. $\bar{\Pi}_{ib}^{(t-1)} < 0$ – then i then the bank decreases $M_i^{(t)}$ as the borrower is less risky, as put by $-\tau_{i3} \cdot \bar{\Pi}_{ib}^{(t-1)}$. Still, interest charges cannot be less than a minimal proportion τ_{i4} of $L_i^{(t)}$. The parameter τ_{i4} sets the 'liquidity trap' under which the interest charges cannot decrease (second case).

Once known $M_i^{(t)}$, then $m_i^{(t)}$ is determined in the same way as $l_i^{(t)}$ with respect to $L_i^{(t)}$:

$$\begin{cases} \text{If } t = 0 \text{ then } m_i^{(t)} = \frac{1}{p_i} M_i^{(t)} \\ \text{If } t \geq 1 \text{ and if } 0 \in SOLV_i^{(t-1)} \text{ then } m_i^{(t)} = 0 \\ \text{else: } \begin{cases} \text{If } t < p_i \text{ then } m_i^{(t)} = \frac{1}{p_i} \sum_{p=0}^{t} M_i^{(t-p)} \\ \text{else } m_i^{(t)} = \frac{1}{p_i} \sum_{p=t-p_i+1}^{t} M_i^{(t-p)} \end{cases} \end{cases} \tag{23}$$

3.6. **Balances and solvency.** We are now able to determine the balance of i in t, to wit, $\Pi_i^{(t)}$. The bank account of i in t can be summarized by the following entries, as in table 1, depending on the way i finances its payments in t and on the necessity or not to settle a previous deficit. Whatever the case, we can show:

Proposition 1. $\Pi_i^{(t)} = Q_i^{(t)} + \Phi_i^{(t)} - \left(l_i^{(t)} + m_i^{(t)}\right) \; \forall i, t$

Proof. In case (a), $\Pi_i^{(t)} = L_i^{(t)} + Q_i^{(t)} - \left(\left|\Pi_i^{(t-1)}\right| + Y_i^{(t)} + l_i^{(t)} + m_i^{(t)}\right)$. As $L_i^{(t)} = Y_i^{(t)} + \left|\Pi_i^{(t-1)}\right|$ then $\Pi_i^{(t)} = Y_i^{(t)} + \left|\Pi_i^{(t-1)}\right| + Q_i^{(t)} - \left(\left|\Pi_i^{(t-1)}\right| + Y_i^{(t)} + l_i^{(t)} + m_i^{(t)}\right)$ which can be arranged as $\Pi_i^{(t)} = Q_i^{(t)} - \left(l_i^{(t)} + m_i^{(t)}\right)$. Now, as $\Phi_i^{(t)} = 0$ then we can write $\Pi_i^{(t)} = Q_i^{(t)} + \Phi_i^{(t)} - \left(l_i^{(t)} + m_i^{(t)}\right)$. In case (b), $\Pi_i^{(t)} = \Pi_i^{(t-1)} + L_i^{(t)} + Q_i^{(t)} - \left(Y_i^{(t)} + l_i^{(t)} + m_i^{(t)}\right)$. As $L_i^{(t)} = Y_i^{(t)} - \Pi_i^{(t-1)}$ then $\Pi_i^{(t)} = \Pi_i^{(t-1)} + Y_i^{(t)} - \Pi_i^{(t-1)} + Q_i^{(t)} - \left(Y_i^{(t)} + l_i^{(t)} + m_i^{(t)}\right)$ which can be arranged as $\Pi_i^{(t)} = Q_i^{(t)} - \left(l_i^{(t)} + m_i^{(t)}\right)$. Now, as $\Phi_i^{(t)} = 0$ then we can write $\Pi_i^{(t)} = Q_i^{(t)} + \Phi_i^{(t)} - \left(l_i^{(t)} + m_i^{(t)}\right)$. Last, in case (c), $\Pi_i^{(t)} = \Pi_i^{(t-1)} + Q_i^{(t)} - \left(Y_i^{(t)} + l_i^{(t)} + m_i^{(t)}\right)$. As $\Phi_i^{(t)} = \Pi_i^{(t-1)} - Y_i^{(t)}$ then we can directly write $\Pi_i^{(t)} = Q_i^{(t)} + \Phi_i^{(t)} - \left(l_i^{(t)} + m_i^{(t)}\right)$. □

As for $solv_i^{(t)}$, we first derive $\tilde{\Pi}_i^{(t)} \in \mathbb{R}$ from $\Pi_i^{(t)}$:

$$\tilde{\Pi}_i^{(t)} := Q_i^{(t)} + \Phi_i^{(t)} - \rho_i \left(l_i^{(t)} + m_i^{(t)}\right) \quad \text{with } \rho_i \in [0;1] \tag{24}$$

For example, if $\rho_i = 0.5$, then $\tilde{\Pi}_i^{(t)} < 0$ means that i is not able to repay even 50% of what is due to the bank in t thanks to its receipts in t and from its past surplus (if any); and *vice-versa* if $\tilde{\Pi}_i^{(t)} \geq 0$. The bank calculates a trend in $\tilde{\Pi}_i^{(t)}$ in order to asses the solvency of i in t. This trend is written $\bar{\tilde{\Pi}}_i^{(t)} \in \mathbb{R}$ and is as follows:

$$\text{If } t \geq t_b \text{ then:} \quad \begin{cases} \text{If } 0 \in SOLV_i^{(t-1)} \text{ then } \bar{\tilde{\Pi}}_i^{(t)} = \bar{\tilde{\Pi}}_i^{(t-1)} \\ \text{else: } \bar{\tilde{\Pi}}_i^{(t)} = \frac{1}{\acute{\eta}} \cdot \sum_{p=0}^{t} (1-\eta_b)^p \, \tilde{\Pi}_i^{(t-p)} \text{ with } \acute{\eta} := \sum_{p=0}^{t} (1-\eta_b)^p \end{cases} \tag{25}$$

$\bar{\tilde{\Pi}}_i^{(t)}$ is calculated for each period starting from t_b and is the η_b-discounted average of $\tilde{\Pi}_i^{(t)}$ calculated over $t_b + 1$ past periods. If i has already faced bankruptcy – $0 \in SOLV_i^{(t-1)}$ – then $\bar{\tilde{\Pi}}_i^{(t)}$ does not need to be recalculated, so that it remains the same as in the previous period. If $\bar{\tilde{\Pi}}_i^{(t)} < 0$ then i has tended not to repay even $\rho_i/100$% of what is due to the bank. In this case, the solvency problems accumulated by the firm leads to its bankruptcy[15]. As a result:

$$\text{If } t \geq t_b \text{ then: if } \bar{\tilde{\Pi}}_i^{(t)} \geq 0 \text{ then } solv_i^{(t)} = 1\text{; else } solv_i^{(t)} = 0 \tag{26}$$

As $\bar{\tilde{\Pi}}_i^{(t)}$ can be calculated starting from t_b, the bank is not supposed to assess the solvency of i before t_b. It is as if the bank was waiting for a trend in order to assess the solvency of i. So, we write:

$$\text{If } t < t_b \text{ then } solv_i^{(t)} = 1 \tag{27}$$

The knowledge of every $solv_i^{(t)}$ is thus possible, which in turn allows to determine every $SOLV_i^{(T)}$ and eventually the number f of business bankruptcies after the T+1 periods.

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[15] In this paper, we do not explicitly deal with the losses recorded by the bank due to business bankruptcies. Due to this losses, some firms might doubt about the solvency of the bank itself; this could lead to a bank run, which might affect the viability of the monetary system itself and, eventually, the very possibility to execute payments.

Here ends the construction of the model, which is a dynamic system. Indeed, as put by various equations, a given variable in $t \geq 1$ may depend on some others associated to the past periods: $t-1$, $t-2$, and so on, until a 'first' period $t-p$. Logically, this iterative process starts from some exogenous variables in the initial period $t=0$; these variables are the initial conditions of the system. In this respect:

Proposition 2. *The initial conditions of the model are made of the vector* $R := \left(R_1^{(0)}; R_2^{(0)}; \cdots; R_J^{(0)}\right) \in \mathbb{R}_+^J$ *that describes the receipts expected by each firm for the initial period.*

Proof. Let $\Gamma(t)$ be the vector made of all the variables of the system in t; S a set of substitutions applied between the variables into $\Gamma(0)$. Looking for this set, it is easy to show that $\exists S[\Gamma(0)] = \left(\Pi_1^{(0)}; \Pi_2^{(0)}; \cdots; \Pi_J^{(0)}\right) \in \mathbb{R}_J$ and that $\Pi_i^{(0)} = \sum_{\substack{j=1 \\ j \neq i}}^{J} \alpha_{ji} \cdot \beta_{j1} \cdot R_j^{(0)} - \frac{\beta_{i1}}{u_i}(1+\tau_1) R_i^{(0)} =: s_i(R)$ with $s_i : \mathbb{R}_+^J \to \mathbb{R}\ \forall i$. So, $\Gamma(0) = S^{-1}(s_1(R); s_2(R); \cdots; s_J(R))$ which means that $\Gamma(0)$ is deduced from R through S and $\{s_1; s_2; \cdots; s_J\}$. Then, the iterative process embedded into the model unfolds and allows to pass from $\Gamma(0)$ to $\Gamma(1)$, then to $\Gamma(2)$... and so on until $\Gamma(T)$. □

Note that nowhere goods, preferences, production techniques or initial endowments enter the picture, as real phenomena are not considered as necessary *a priori* within monetary analysis; yet, this does not prevent the model, built upon the sole monetary/financial phenomena, from being fully-consistent and without subdetermination.

4. Studying business bankruptcy in the model

The aim is to deduce the number of firms that go bankrupt after T periods within the model. Due to the numerous 'if-then-else' statements and the related threshold effects, an analytical solution cannot be made explicit as for most of the nonlinear dynamic systems. A solution is to perform numerical simulations, in line with ACE. The simulations and their results are first outlined (4.1). The values attributed to the parameters and to the initial conditions are then detailed (4.2), followed by the results extracted from the related simulations (4.3).

4.1. **Numerical simulations and their results: a general account.** All the simulations are set for three firms (alongside the single bank). Though real-world economies are made of numerous firms, this does not precludes us from extracting insightful results from this baseline model. The latter can also be used as a benchmark for further investigations by complexifying it. In this respect, this work should be considered in a dynamic context: its results are preliminary (not definitive) and pave the way for others in further research[16].

The following settings are also common to every simulation: i) the number of periods ($T = 29$); ii) the determination of debt service; iii) the way to adjust expected receipts from one period to another; iv) the way to assess solvency and, thus, to go bankrupt; and v) the consequence of a given bankruptcy on the payments outside those executed/received by the firm that goes bankrupt.

[16]This comment also applies to the treatment of the banking system, here reduced to a single bank for temporary simplification purposes.

TABLE 2. Summary of the settings that characterize the numerical simulations

Setting	**Meaning**
'Structure'	The configuration according to which each firm considers the distribution of its planned payments among the others
Sensitivity	The extent of the change in the vector made of the total expenditures planned by each firm, depending on changes in their expected receipts and in their average balance
The receipts initially expected by each firm	*The initial conditions of the model*

Each simulation differs from the others according to three settings, summarized in table 2. The first one is the structure according to which the total expenditures planned by each firm in t is supposed to be distributed between the two others (that is to say, if none of these two has been affected by bankruptcy before t). As a starting point, three different distributions are possible (independently of t). Either a firm considers an *equal* distribution, meaning that each one of the two others should benefit from 50% of the total planned by the former (if both remain in business); or the whole total should be paid to a single firm, whereas the other should not benefit from any payment; or *vice versa*. With three firms, $3^3 = 27$ different 'planned payment structures' – hereafter 'structures' more briefly – are thus studied.

The second setting is the *sensitivity* to expected receipts and to average balances of the vector made of the total expenditures planned by the firms. As already put, each firm increases its total expenditures if it expects more receipts and/or if its average balance turns positive (or, if already positive, higher). A simulation sets the magnitude of that increase for each firm. From one simulation to another, if such a magnitude is higher for at least one firm and/or if a smaller magnitude for a given firm is more than counterbalanced by a higher one for another, then the second simulation is said to show a higher sensitivity. Each firm is endowed with five possible magnitudes (independently of t). So, three firms lead to $5^3 = 125$ different sensitivities.

Last, the third setting is the receipts initially expected by each firm. As already put, this identifies with the initial conditions of the model and, from an economic point of view, constitutes the beginning of the decision-making process. Each firm is endowed with 13 possible values, beginning with \$1 and ending with \$5000 in order to dispose of a broad spectrum of initial conditions. So, three firms lead to $13^3 = 2197$ different ways for them to initiate their decision-making process.

Eventually, each simulation is associated to a 'structure' (as defined before) among 27 of them, to a sensitivity among 125 of them, and to a vector of initially expected receipts among 2197 of them; hence, $27 \times 125 \times 2197 = 7\,414\,875$ different simulations are performed. With such a number, the aim is to bring to the fore real tendencies about the number of business bankruptcies within the model.

The following scenario is possible *a priori*: every simulation (or almost every one of them) end with the same number of business bankruptcies. If so, the way firms execute their payments and run into debt, as induced by the previous three settings, would not matter (or only a little) for understanding why some firms go

Table 3. Summary of the concepts used for the analysis of the numerical simulations

Concept	Meaning
Symmetry	Each firm considers an equal distribution of its planned payments among the two others
Asymmetry	One or several firms do not consider an equal distribution of their planned payments
Exclusion	The fact that a firm does not receive any of the payments planned by the others
Degree of asymmetry	The number of firms that do not consider an equal distribution of their planned payments
Pole	A firm that should receive all the payments planned by the others

bankrupt in the model, as such a number is (almost) independent of such a way. Though this scenario would have to be confirmed by other simulations in the model itself, as well as in others of the same type, it would already question the usefulness of the type at issue. Nonetheless, as shown below, the aforementioned scenario does not happen.

We can start with any of the studied structures (first setting). Then, we can count how many business bankruptcies – hereafter 'bankrutpcies' more briefly – there are for each combination made of a given sensitivity (second setting) and of a given way for each firm to start its decision-making process (third setting); to wit, how many among $125 \times 2\,197 = 274\,625$. In this framework, three scenarios can be brought to the fore, as shown by figures 4, 5 and 6. Table 3 summarizes the different concepts hereafter introduced in order to understand the overall set of simulations according the scenarios at issue.

Scenario 1 is described in figure 4 and is made of a specific structure: the one according to which the total expenditures planned by *every* firm are to be equally distributed between the two others. With such a *symmetric* structure, the number of the aforesaid combinations that avoid bankruptcies over time is the *highest.* If a combination does not avoid bankruptcy, then it leads to a unique bankruptcy most of the time. A unique bankruptcy would accordingly undermine the stability of the economic process described in the model, but there would still remain two firms. So, one of these two remaining firms can be the payer and the other the payee, and *vice versa.* In the end, some firms can keep on interacting, and the economic process can keep on unfolding over time.

Thereafter, if the sensitivity (second setting) that enters the combination is high enough, then no firm goes bankrupt *irrespective of* the way they initiate their decision-making process (in other words, irrespective of the third setting). Thus, firms must be willing to implement a symmetric structure and to adjust their expenditures enough with respect to their expected receipts and to their past balances in order to avoid bankruptcies as much as possible; and, logically, this feeds back into their indebtedness.

Notwithstanding, the way firms execute their payments and apply for credits is supposed to reflect their own objectives and constraints, as put by decentralization, but obviously does not reflect the overall number of bankruptcies. So, the structure is unlikely to be symmetric, and the sensitivity is unlikely to be high enough at the

FIGURE 4. Scenario 1: symmetric distribution of the payments planned by the firms

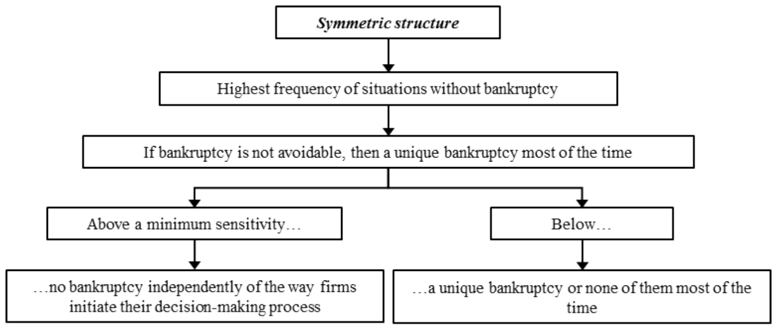
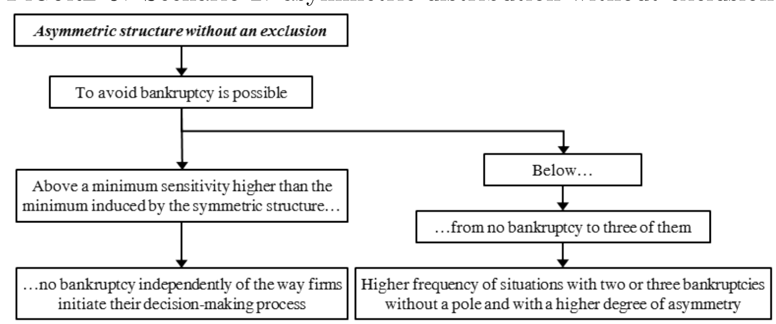


FIGURE 5. Scenario 2: asymmetric distribution without exclusion

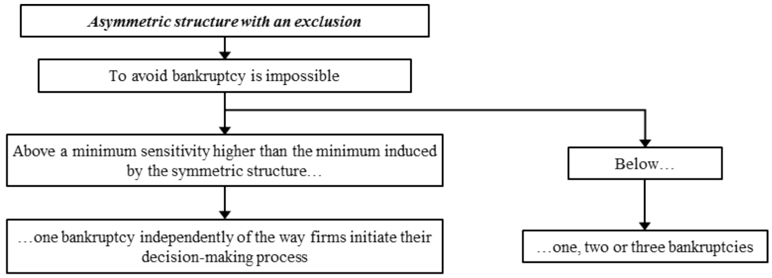


same time. Below the 'minimum' sensitivity, then a unique bankruptcy may occur instead of none of them if (by chance) the structure is symmetric. If the structure is *asymmetric*, meaning that at least one firm does not consider an equal distribution of its planned payments, then follows Scenario 2. More precisely, as put by figure 5, Scenario 2 is made of the asymmetric structures that do not imply *exclusion*. By this word, we mean that one firm among the three of them does not receive any of the payments planned by the two others.

Without exclusion, it remains possible to avoid bankruptcy, albeit to a lesser extent than with the symmetric structure. Like in Scenario 1, a combination must be made of a sensitivity higher than a minimum for no bankruptcy to occur irrespective of the way firms initiate their decision-making process. Contrary to Scenario 1, no bankruptcy or a unique one occurs for most of the combinations that does not fulfill with that minimum *only if* the asymmetry (without exclusion) implies a *pole*. This concept refers to the single firm that should receive all the payments planned by the others (regardless of their number). Also, the minimum sensitivity is higher than with symmetry.

If the asymmetry (without exclusion) does not contain a pole, then the number of combinations that does not fulfill the minimum sensitivity and that lead to two or three bankruptcies may become significant. Actually, two or three bankruptcies occur with almost all the combinations (with a sensitivity below the minimum) if the asymmetry (without exclusion and without pole) is said to be of *third degree*.

FIGURE 6. Scenario 3: asymmetric distribution with exclusion



This concept refers to the number of firms that do not consider an equal distribution of the total expenditures they plan; degree 3 thus points out that no firm is characterized by such an equal distribution. So, this type of asymmetry is very conducive to a non-viable economic process: in the end, no firm can pay another, so that no interaction is possible. With a lesser degree of asymmetry (and still without an exclusion and without a pole), the combinations with a sensitivity below the minimum lead to two or three bankruptcies to a lesser extent; yet this number remains significant and higher than with an asymmetry with a pole (and, logically, higher than with a symmetry).

Last, Scenario 3 is made of the asymmetric structures that contain exclusion. In this scenario, as described in figure 6, it is impossible to avoid bankruptcy. At least one firm goes bankrupt regardless of the sensitivity and of the way each firm starts its decision-making process. The firm subject to the exclusion appears among those that go bankrupt (or is the sole one that goes bankrupt). Again, exclusion remains possible due to decentralization. Nothing prevents two firms from entering into bilateral payments, excluding at the same time a third one from their payments. Like in the two previous scenarios, there exists a minimum sensitivity (higher than with symmetry) which should be met in order to give rise to the same number of bankruptcies irrespective of the way firms initiate their decision-making process. Nonetheless, contrary to the two previous scenarios, this number is no longer 0 but 1.

Up to this point, we thus reach the main result already mentioned in the introduction: *the way firms execute their payments and run into debt cannot be arbitrary in order to avoid business bankruptcies as much as possible.* Accordingly, macroeconomic aggregates and individual characteristics matter, as already suggested by the literature. Nonetheless, the configuration of payments and credits shall matter too.

4.2. **Values of the parameters and of the initial conditions.** We first attribute values to the parameters in table 4. These values will not change from one simulation to another. In particular, $J = 3$ implies to deal with an economy made of three firm-like agents alongside the single bank. As already put, we will obtain some results from this simple economy, before looking in further research for how they change when the model is more complex.

We then set the parameters α_{ij}. As $J = 3$, there are 2 coefficients $(\alpha_{ij}; \alpha_{ik})$ for each i, with $i \neq j \neq k$. We write:

$$(\alpha_{ij}; \alpha_{ik}) \in U := \{(0;1)\,;(0.5;0.5)\,;(1;0)\} \;\; \forall i,j,k \in \{1;2;3\} \text{ with } i \neq j \neq k$$

TABLE 4. Parameters common to all the numerical simulations

	J 3	T 29	η_b 0.025	t_b 5	$\theta_i^{(t)}$ 0.8 $\forall i, t$
$\forall i$	τ_{i1} 0.025 δ_i 0.5	τ_{i2} 0.0375 η_i 0.025	τ_{i3} 0.0125 ρ_i 0.5	τ_{i4} 0.025 t_i 5	u_i 5

In the first case and the third one, a given firm plans to allocate its overall expenditures $X_i^{(t)}$ to another, while the third one does not benefit from payments from the first. In the second case, a given firm plans to equally pay the two others. A point $((\alpha_{12}; \alpha_{13}); (\alpha_{21}; \alpha_{23}); (\alpha_{31}; \alpha_{32}))$ among the Cartesian product U^3 characterizes the first setting for a simulation to be performed. We will deal with all of these $(\#U)^J = 3^3 = 27$ ordered arrangements of 3 elements of U among 3 *with* repetition (a firm endowed with $(0; 1)$ does not prevent another from being endowed with the same parameters).

With respect to the parameters β_{i1} and β_{i2}:

$$(\beta_{i1}; \beta_{i2}) \in V := \{(0.05; 0.2); (1/3; 0.3); (2/3; 0.4); (1; 0.5); (4/3; 0.6)\} \ \ \forall i$$

As $J = 3$, a point $((\beta_{11}; \beta_{12}); (\beta_{21}; \beta_{22}); (\beta_{31}; \beta_{32}))$ among the Cartesian product V^3 characterizes the second setting for a simulation to be performed. We will deal with all of these $(\#V)^J = 5^3 = 125$ ordered arrangements of 3 elements of V among 5 *with repetition*.

In the end, a simulation is associated to the values of the parameters as in table 4 and to a point of $(U \times V)^3$ for a given R, thus giving rise to $(\#U \times \#V)^3 = (3 \times 5)^3 = 3\,375$ different configurations of parameters for a given R. In this respect:

$$R_i^{(0)} \in W := \{1; 2; 5; 10; 15; 20; 50; 100; 200; 500; 1\,000; 2\,000; 5\,000\} \ \ \forall i$$

A point $\left(R_1^{(0)}; R_2^{(0)}; R_3^{(0)}\right)$ among the Cartesian product W^3 characterizes the third setting for a simulation to be performed. We will deal with all of these $(\#W)^J = 13^3 = 2\,197$ ordered arrangements of 3 elements of W among 13 with repetition.

In the end, $3\,375 \times 2\,197 = 7\,414\,875$ simulations are performed (for each set of the initial conditions among the 2 197 of them, 3 375 configurations of parameters are studied) in order to extract results about the number f of bankruptcies within the model after T periods[17]. f depends on: a point in U^3, another in V^3 and a last one in W^3. So, f can be written $f(u; v; w)$ with $(u; v; w) \in U^3 \times V^3 \times W^3$.

4.3. **The results.** For each $u \in U^3$, table 5 describes how many points $(v; w)$ of $V^3 \times W^3$ are associated to each possible number of bankruptcies, to wit, $F(u; \kappa) := \#\left\{(v; w) \in V^3 \times W^3 : f(u; v; w) = \kappa\right\} \in \{1; 2; \cdots; 274\,625\} \ \forall u \in U^3$ and $\forall \kappa \in \{0; 1; 2; 3\}$. We group together the values 2 and 3 of $f(u; v; w)$ as two or three bankruptcies both imply a *non-viable* economic process: with these numbers, no firm can execute payments. With two bankruptcies, there is a unique payer but no payee; with three there are neither payer nor payee. So, firms can no longer interact between them and the related decentralized economy no longer exists. Within the model, the maximum number of bankruptcies for a viable economic process is 1.

[17]The Maple©worksheet elaborated for running the simulations is available on request.

Table 5. Distribution of the simulations according to the resulting number of bankruptcies and to the planned payment structure

Case	α-coefficients within u						$F(u;0)$	$F(u;1)$	$F(u;2)$ $+F(u;3)$
	α_{12}	α_{13}	α_{21}	α_{23}	α_{31}	α_{32}			
1	0.5	0.5	0.5	0.5	0.5	0.5	133 120	138 510	2 995
2	0	1	0	1	0.5	0.5	132 328	136 271	6 026
	1	0	0.5	0.5	0	1	132 328	136 271	6 026
	0.5	0.5	1	0	1	0	132 328	136 271	6 026
3	0	1	0.5	0.5	0.5	0.5	128 501	127 157	18 967
	1	0	0.5	0.5	0.5	0.5	128 501	127 157	18 967
	0.5	0.5	0	1	0.5	0.5	128 501	127 157	18 967
	0.5	0.5	1	0	0.5	0.5	128 501	127 157	18 967
	0.5	0.5	0.5	0.5	0	1	128 501	127 157	18 967
	0.5	0.5	0.5	0.5	1	0	128 501	127 157	18 967
4	0	1	1	0	0.5	0.5	120 703	100 710	53 212
	0	1	0.5	0.5	0	1	120 703	100 710	53 212
	1	0	0	1	0.5	0.5	120 703	100 710	53 212
	1	0	0.5	0.5	1	0	120 703	100 710	53 212
	0.5	0.5	0	1	1	0	120 703	100 710	53 212
	0.5	0.5	1	0	0	1	120 703	100 710	53 212
5	0	1	1	0	0	1	88 456	3 768	182 401
	1	0	0	1	1	0	88 456	3 768	182 401
6	0	1	0.5	0.5	1	0	0	222 260	52 365
	1	0	1	0	0.5	0.5	0	222 260	52 365
	0.5	0.5	0	1	0	1	0	222 260	52 365
7	0	1	0	1	0	1	0	221 104	53 521
	0	1	0	1	1	0	0	221 104	53 521
	0	1	1	0	1	0	0	221 104	53 521
	1	0	0	1	0	1	0	221 104	53 521
	1	0	1	0	0	1	0	221 104	53 521
	1	0	1	0	1	0	0	221 104	53 521
						Subtotal	**2 202 240**	**3 915 465**	**1 297 170**
						Total	***7 414 875***		

Example: if $u = ((0.5; 0.5) ; (0.5; 0.5) ; (0.5; 0.5))$ then $F(u; 1) = 138\,510$, meaning that with that planned payment structure 138 510 simulations lead to a unique bankruptcy.

As it can be seen, the 27 points of U^3 are distributed among 7 different cases, each one being associated to the same values of $F(u; 0)$, $F(u; 1)$ and $F(u; 2) + F(u; 3)$. The first case is associated to the *highest* number of simulations without bankruptcy. Also, still within that first case, if $(v; w)$ cannot avoid bankruptcy, then a unique one occurs most of the time, as 2 or 3 bankruptcies only correspond to $2\,995/274\,625 \approx 1.09\%$ of the simulations performed, to wit, the *smallest* number as compared to the other cases. So, the question is: what does make the first case specific? The former is made of $u = ((0.5; 0.5) ; (0.5; 0.5) ; (0.5; 0.5))$, which means that every $X_i^{(t)}$, which is the total expenditures planned by i in t, should be equally distributed among the other firms (that is to say, if they remain in business). For that reason, this case is called the *symmetric* one.

Definition 1. $(\alpha_{i1};\alpha_{i2};\cdots;\alpha_{ii-1};\alpha_{ii+1};\cdots;\alpha_{iJ})$ $\forall i \in \{1;2;\cdots;J\}$, *which describes how each firm considers the distribution of an overall amount of planned payments between the others, is symmetric if* $\alpha_{ij}=\frac{1}{J-1}$ $\forall i$ *and* $\forall j \neq i$.

Remark 1. *As* $J=3$*, the symmetric* u *is given by* $\alpha_{ij}=\frac{1}{3-1}=0.5$ $\forall i$ *and* $\forall j\neq i$.

Let us denote the symmetric u by u_s. We write:

Proposition 3. u_s *being the case according to which each firm considers a symmetric distribution of the payments they plan:*

- $F(u_s;0)=\max\left\{F(u;0):u\in U^3\right\}$
- $F(u_s;1)\to\sum_{\kappa=1}^{3}F(u_s;\kappa)-F(u_s;0)$
- $F(u_s;2)+F(u_s;3)=\min\left\{F(u;2)+F(u;3):u\in U^3\right\}$

Proposition 3 corresponds to the Scenario 1 (see the upper part in figure 4). Now, let us give more details with the distribution of each $(v;w)$ among the different number of possible bankruptcies, within the symmetric case. To this purpose, each v is identified by the *sensitivity* it implies and all the points are thus ranked in order of increasing sensitivity. In order to explain sensitivity, we start from the fact that $\beta_{i1}=\partial f_i/\partial R_i^{(t)}=\partial g_i/\partial R_i^{(t)}$. So, for a given variation of $R_i^{(t)}$, a greater β_{i1} implies a greater variation of $X_i^{(t)}$ (everything else being equal). Similarly, $\beta_{i2}=\partial g_i/\partial\bar{\Pi}_i^{(t-1)}$ with $\bar{\Pi}_i^{(t-1)}>0$. So, for a given variation of $\bar{\Pi}_i^{(t-1)}$ within $\mathbb{R}^+$ or from $\mathbb{R}^-$ to $\mathbb{R}^+$, a greater β_{i1} implies a greater variation of $X_i^{(t)}$ (everything else being equal). In this respect:

Definition 2. *Let* $(\beta_{i1};\beta_{i2})$ $\forall i$ *be the coefficients associated to* $v\in V^J$*;* $(\beta'_{i1};\beta'_{i2})$ $\forall i$ *those associated to* $v'\in V^J$*;* $B(v):=\sum_{i=1}^{J}(\beta_{i1}+\beta_{i2})$ *and* $B(v'):=\sum_{i=1}^{J}\left(\beta'_{i1}+\beta'_{i2}\right)$. $\left(X_1^{(t)};X_2^{(t)};\cdots;X_J^{(t)}\right)$ *is equally or more sensitive to* $\left(R_1^{(t)};R_2^{(t)};\cdots;R_J^{(t)}\right)$ *and to* $\left(\bar{\Pi}_1^{(t-1)};\bar{\Pi}_2^{(t-1)};\cdots;\bar{\Pi}_J^{(t-1)}\right)$ *with* v' *than with* v *if* $B(v')\geq B(v)$.

Example 1. $((0.05;0.3);(0.05;0.3);(0.05;0.3))$ *is the point of* V^3 *with the smallest sensitivity as* $B\left[((0.05;0.3);(0.05;0.3);(0.05;0.3))\right]=1.05=\min\left\{B(v):v\in V^3\right\}$. $((0.05;0.3);(0.05;0.3);(1/3;0.4))$ *is more sensitive than the former point as the related sum equals* 1.4333*, to wit, higher than* 1.05. $((0.05;0.3);(1/3;0.4);(0.05;0.3))$ *also leads to* 1.4333*, so both points share the same sensitivity.*

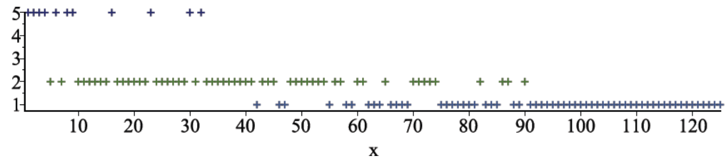
Thus, v_x is some point of V^J and v_{x+1} is another as $B(v_{x+1})\geq B(v_x)$ with $x\in\left\{1;2;\cdots;\#V^J-1\right\}$.

Example 2. *As put by the former example:* $v_1:=((0.05;0.3);(0.05;0.3);(0.05;0.3))$, $v_2:=((0.05;0.3);(0.05;0.3);(1/3;0.4))$ *and* $v_3:=((0.05;0.3);(1/3;0.4);(0.05;0.3))$. *In the same vein, we have:* $v_{123}:=((4/3;0.60);(1.00;0.50);(4/3;0.60))$, $v_{124}:=((4/3;0.60);(4/3;0.60);(1.00;0.50))$ *and* $v_{125}:=((4/3;0.60);(4/3;0.60);(4/3;0.60))$.

In this framework, for every $u\in U^3$, we distribute every $(v_x;w)$ among the following partition made of five sets:

(1) The first one is made of the v_x-points that avoid bankruptcy for all w-points. Put differently, once given a way the firms consider the distribution of the payments they plan (u), the way these payments are sensitive to expected receipts and to past trend in balances always implies a zero-bankruptcy

FIGURE 7. Sensitivity distribution for the symmetric case ($u = u_s = ((0.5; 0.5)\,; (0.5; 0.5)\,; (0.5; 0.5)))$



Note: A coordinate $(x; a)$ means $x \in G_a(u)$ ($x \in \{1; 2; \cdots ; 125\}$ and $a \in \{1; 2; \cdots ; 5\}$).

situation independently of the way firms initiate their decision-making process (by calculating their expected receipts for the initial period). The set is thus written $G_1(u) := \{x : f(u; v_x; w) = 0 \ \forall w \in W^3\}$.

(2) The second set is made of the v_x-points associated to a unique bankruptcy independently of w, to wit, $G_2(u) := \{x : f(u; v_x; w) = 1 \ \forall w \in W^3\}$.

(3) The third set is made of the v_x-points associated to a unique bankruptcy in the worst case and none of them in the best case, to wit $G_3(u) := \{x : f(u; v_x; w) \in \{0; 1\} \ \ \forall w \in W^3\} \setminus (G_1(u) \cup G_2(u))$.

(4) The fourth set is made of the v_x-points associated to two or three bankruptcies independently of w, which amounts to a non-viable economic process. This set is thus written $G_4(u) := \{x : f(u; v_x; w) \in \{2; 3\} \ \ \forall w \in W^3\}$.

(5) Last, the fifth set is made of all the other points. Within that set, the sensitivity levels imply any possible number bankruptcies (from 0 to 3) depending on the initial conditions of the model, that is to say, depending on the way firms initiate their decision-making process (by expecting their receipts for the initial period). This set is thus written $G_5(u) := \overline{\bigcup_{a=1}^{4} G_a(u)}$.

Figure 7 shows the composition of the five sets for u_s. In the horizontal axis, 1 corresponds to v_1 which is the point of V^3 with the lowest sensitivity; 2 corresponds to v_2 as this other point of V^3 implies the second lowest sensitivity; and so on for every $v_x \in V^3$ until v_{125}. In the vertical axis, the numbers 1, 2, 3, 4 and 5 are respectively associated to $G_1(u_s)$, $G_2(u_s)$, $G_3(u_s)$, $G_4(u_s)$ and $G_5(u_s)$. In this vein, if the graph is made of the coordinate (1;5) then it means that 1 in v_1 belongs to $G_5(u_s)$, so that $f(u_s; v_1; w) \in \{0; 1; 2; 3\} \, \forall w \in W^3$. As it can be seen, starting from the lowest values of x then $x \in G(5)(u_s)$, meaning that any possible number of bankruptcy is implied by the lowest sensitivity (in case of symmetry). Thereafter, a higher x tends to be belong to $G(2)(u_s)$ and eventually to $G(1)(u_s)$. Eventually, from $x = 91$ onwards, every x belongs to $G(1)(u_s)$.

Proposition 4. *If the firms consider a symmetric distribution of the payments they plan, then:*

- *The lowest sensitivities of these payments to expected receipts and to past trend in balances lead to any number of bankruptcies, i.e. from 0 to 3.*
- *With a higher sensitivity, the number tends to be reduced to 1 or even to 0.*
- *If sensitivity is high enough, the number is always reduced to 0.*

Like Proposition 3, Proposition 4 corresponds to the Scenario 1 (see the lower part in figure 4). Now, let us continue with the six other cases, which are *asymmetric* and thus correspond to the two other scenarios. These cases form two groups. The first one is made of Cases 2, 3, 4 and 5, within which there is still the possibility to avoid bankruptcy, as $F(u;0) \neq 0$. The second one is made of Cases 6 and 7, within which such a possibility has vanished, as $F(u;0) = 0$. So, we have to find what makes the two groups different. Actually, such a difference identifies with the *first-degree exclusion* implied by the second group but not by the first.

Definition 3. *u is asymmetric if $\exists A(u) := \left\{ i : \alpha_{ij} \neq \frac{1}{J-1} \; \forall j \neq i \right\} \neq \emptyset$. The degree of asymmetry is $m := \#A(u) \in \{1; 2; \cdots; J\}$.*

Remark 2. *Symmetry corresponds to an asymmetry of degree zero.*

Definition 4. *An asymmetric u implies exclusion if $\exists E(u) := \{j : \alpha_{ij} = 0 \; \forall i \neq j\} \neq \emptyset$. The degree of exclusion is $n := \#E(u) \in \{1; 2; \cdots; J-2\}$.*

To sum up, a m^{th}-degree asymmetry refers to m firms that do not consider an equal distribution of their planned payments, which may exclude n of them from benefiting from payments. n is the degree of exclusion.

Example 3. *$u = ((0;1); (0;1); (0.5;0.5))$, which is the first u among Case 2, is a 1^{st}-degree asymmetry: $A(u) = \{1;2\}$. This asymmetry is without exclusion: $\alpha_{21} = 0$ and $\alpha_{31} = 0.5$ mean that $i = 1$ is not paid by $i = 2$ but by $i = 3$; $\alpha_{12} = 0$ and $\alpha_{23} = 0.5$ mean that $i = 2$ is not paid by $i = 1$ but by $i = 3$; and $\alpha_{13} = \alpha_{13} = 1$ mean that $i = 3$ receives all the payments planned by the other firms. To sum up, $B(u) = \emptyset$. On the other hand, $u = ((0;1); (0.5;0.5); (1;0))$, which is the first u among Case 6, is a 2^{nd}-degree asymmetry: $A(u) = \{1;3\}$. This asymmetry implies a 1^{st}-degree exclusion: $\alpha_{12} = \alpha_{32} = 0$ means that $i = 2$ does not receive any payment. In a nutshell, $B(v) = \{2\}$.*

Remark 3. *$J = 3 \Rightarrow n = 1$: with 3 firms, only one of them might be excluded. Besides, $m = 1 \Rightarrow E(u) = \emptyset$: there is no exclusion with a first-degree asymmetry.*

We denote u_e any u with a (first-degree) exclusion; and $u_{\neg e}$ any u without.

Proposition 5. *$f(u_{\neg e}; v; w) = 0 \; \exists (v; w) \in V^3 \times W^3$ whereas $f(u_e; v; w) \neq 0 \; \forall (v; w) \in V^3 \times W^3$.*

Let us focus on the asymmetric cases without exclusion (2, 3, 4 and 5). As shown by table 5, passing from one case to another, *two or three bankruptcies occur more often to the detriment of none of them and of a unique one.* In other words, passing from one case to another, the economic process is less and less viable. Moreover, in Case 5, the number of simulations with two or three bankruptcies is the *highest.* We have to outline what makes each one of these asymmetric cases different from each other. Notably, the degree of asymmetry is not a sufficient criterion, as Cases 2 and 4 share the same degree (2). To this purpose, let us introduce the following definition:

Definition 5. *The firm j is the (unique) pole of a m^{th}-degree asymmetric u if: $m \in \{J-1; J\}; \alpha_{ij} = 1 \; \forall i \in A(u)$.*

Put differently, a firm is a pole if it receives all the payments planned by the others. Let us remark that a 1^{st}-degree asymmetry has no pole. Given definition 5, we can say that Case 2 contains a pole, whereas Cases 3, 4 and 5 do not. So, outside the symmetric case, to avoid bankruptcy the most requires a pole (and therefore a second-degree asymmetry); if not, then it requires the minimum degree of asymmetry. In this respect, we denote u_P any u with a pole (and without

FIGURE 8. Sensitivity distribution for (2nd-degree) asymmetry with pole

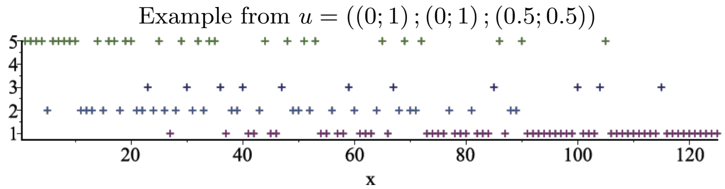
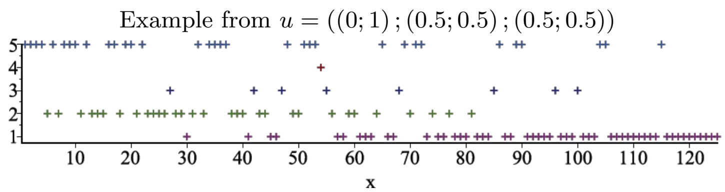


FIGURE 9. Sensitivity distribution for 1st-degree asymmetry (without pole)



exclusion); and u_m any u showing a m^{th}-degree asymmetry *without* a pole (and without exclusion). Thus, u_P refers to Case 2; u_1 refers to Case 3; u_2 refers to Case 4; and u_3 refers to Case 5. We then write:

Proposition 6. *If u is asymmetric and without exclusion, then:*

- $F(u_P;\kappa) \geq F(u_1;\kappa) \geq F(u_2;\kappa) \geq F(u_3;\kappa)\ \forall \kappa \in \{0;1\}$
- $F(u_P;2) + F(u_P;3) < F(u_1;2) + F(u_1;3) < F(u_2;2) + F(u_2;3) < F(u_3;2) + F(u_3;3)$
- $F(u_3;2) + F(u_3;3) = \max\left\{F(u;2) + F(u;3) : u \in U^3\right\}$

Let us continue with, for each aforesaid case, more details about the distribution of each $(v;w)$ among the different number of possible bankruptcies, again using the map $x \mapsto v_x$ and the related distribution of each x among $G_1(u), G_2(u), \cdots, G_5(u)$. Figure 8 shows the distribution pattern with the example of $u = ((0;1)\,;(0;1)\,;(0.5;0.5))$. We can find the same pattern as in Case 1: starting from the lowest values of then $x \in G_5(u)$); thereafter, a higher x tends to be belong to $G_2(u)$ and eventually to $G_1(u)$. Actually, the difference lies in the value of x from which all the following ones x belong to $G_1(u)$. Within the symmetric case, this value is 91. Now, without the symmetry, the value must be higher (here 94).

The aforesaid pattern is found for every other u associated to Case 2[18]. It can also be found for Cases 3 and 4, as illustrated by figures 9 and 10.

Now, with Case 5, the lowest values of x tend to belong to G_4, meaning at least 2 bankruptcies. With a higher value of x emerges the possibility of a smaller number as thereafter x tend to belong to G_1. Eventually, as in the former cases, there is a value of x from which all the following ones belong $G(1)$, still a higher value than with the symmetric case (106) (see figure 11).

[18]The authors made available on request the graphs at issue for all the u-like vectors.

FIGURE 10. Sensitivity distribution for 2nd-degree asymmetry without pole

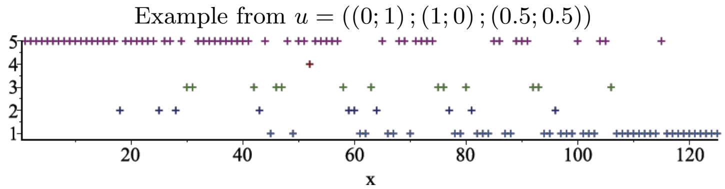


FIGURE 11. Sensitivity distribution for 3rd-degree asymmetry without pole

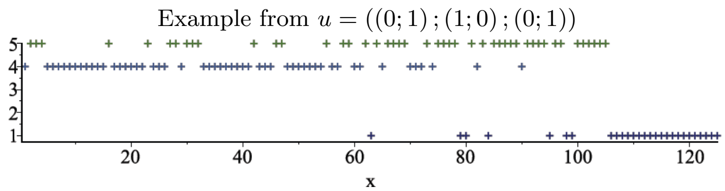
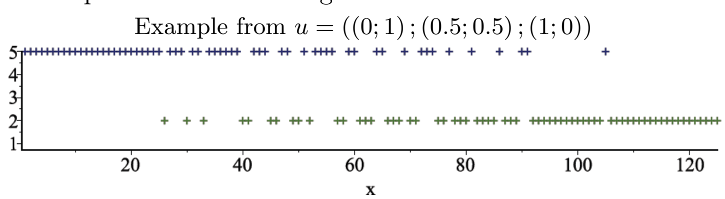


FIGURE 12. Sensitivity distribution for 2nd-degree asymmetry without pole and with 1st-degree exclusion



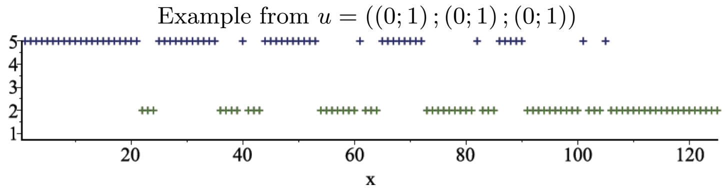
Proposition 7. *If the firms do not consider a symmetric distribution of the payments they plan, and if such an asymmetry does not imply an exclusion, then sensitivity must be higher in order to always reduce the number of business bankruptcies to 0.*

Proposition 8. *If the firms do not consider a symmetric distribution of the payments they plan, and if such an asymmetry is of the highest degree and does not imply an exclusion, then:*

- *The lowest sensitivity tends to lead to 2 or 3 bankruptcies instead of any.*
- *Only with a higher sensitivity, the number tends to be any (before being eventually reduced to 0).*

Last, let us focus on Cases 6 and 7. As already said, both imply a 1st-degree exclusion. Table 5 shows that both are roughly the same. Two or three bankruptcies are a little more frequent in Case 7 than in Case 6 as the degree of asymmetry in Case 7 is higher than in Case 6 (2 versus 3). Besides, starting from the lowest values of x tend to belong to $G(5)$. Thereafter, with a higher value, the number can only be reduced to 1 instead of 0, in accordance with the fact that $F(u;0) = 0$ (see figures 12 and 13).

FIGURE 13. Sensitivity distribution for 3rd-degree asymmetry with asymmetric pole and with 1st-degree exclusion



Proposition 9. *If the firms do not consider a symmetric distribution of the payments they plan, and if such an asymmetry implies an exclusion, then the number of bankruptcies can only be reduced to 1 instead of 0 with the increase in the sensitivity.*

Here end the analysis of the simulations. Table 6 summarizes each case and to which scenario it pertains.

TABLE 6. Cases and scenarios

Case	Description	Scenario
1	Symmetry	1
2	(2nd-degree) asymmetry with pole and without exclusion	2
3	1st-degree asymmetry without pole and without exclusion	2
4	2nd-degree asymmetry without pole and without exclusion	2
5	3rd-degree asymmetry without pole and without exclusion	2
6	2nd-degree asymmetry with exclusion	3
7	3rd-degree asymmetry with exclusion	3

Note: see table 3 for the meaning of the concepts, and figures 4, 5 and 6 for the description of each scenario, in relation with table 2.

5. CONCLUSION

In order to explain why firms go bankrupt into decentralized market economies, the model elaborated in this paper specifies how some firms interact between themselves and with a bank through payments and credits. This way to account for the economic interactions is based on monetary analysis, which allows elaborating models that are able to be consistent with our decentralized economies.

The model shows that each firm must pay each other and thus run into debt into some specific ways in order to avoid their bankruptcy as much as possible. Still, the decision-making process remains decentralized. So, one thing is to design for the economy some configurations related to payments and to indebtedness, in the face of business failure; another thing is the achievement of these configurations. Eventually, *there is not some endogenous mechanism which would avoid business bankruptcies*, even less some disturbed by some 'imperfections'.

Admittedly, as this result applies to the simple economy accounted for by the model, then the former might not be valid for real-world economies. Still, this does not prevent from elaborating upon more complex models with the aim to enhance such a validity. This paper results in some new concepts in order to characterize how payments and therefore credit are connected: planned payment structure, sensitivity, symmetry, asymmetry and its degree, exclusion and its degree, pole. This can be the starting point for further investigations into business bankruptcy, within more complex models in order to get closer to real-world economies.

Appendix. List of the variables that enter the model

Agents and time

Variable	**Domain**	**Meaning**
J	$\mathbb{N} \setminus \{0;1\}$	The initial number of firm-like agents
i, j, k	$\{1;2;\cdots;J\}$	A given firm-like agent
T	$\mathbb{N}^*$	The last (discrete) period of the economic process
t	$\{0;1;2;\cdots;T\}$	A given time period

Balances and solvency

Variable	**Domain**	**Meaning**
f	$\{0;1;2;\cdots;J\}$	The number of firm-like agents that did go bankrupt at the end of the economic process
$solv_i^{(t)}$	$\{0;1\}$	Solvency indicator of i in t
$SOLV_i^{(t)}$	$(\{0;1\})^{t+1}$	$\left(solv_i^{(0)}; solv_i^{(1)}; \cdots; solv_i^{(t)}\right)$
$\Pi_i^{(t)}$	$\mathbb{R}$	The balance of i in t
$\bar{\Pi}_{ib}^{(t)}$	$\mathbb{R}$	η_b-discounted average of $\Pi_i^{(t)}, \Pi_i^{(t-1)}, \cdots, \Pi_i^{(t-p)}$ with $p \in \{t; t_b\}$, $\eta_b \in [0;1]$ and $t_b + 1 \in \{0;1;2;\cdots;T\}$
$\tilde{\Pi}_i^{(t)}$	$\mathbb{R}$	$Q_i^{(t)} + \Phi_i^{(t)} - \rho_i \left(l_i^{(t)} + m_i^{(t)}\right)$ with $\rho_i \in [0;1]$

Payments (with $\mathbb{R}_+$ as their domain)

Variable	**Meaning**
$\tilde{d}_{ij}^{(t)}$	The payment that i *plans* to execute toward $j \neq i$ in t
$X_i^{(t)}$	$\sum_{\substack{j=1 \\ j \neq i}}^{J} \tilde{d}_{ij}^{(t)}$
$d_{ij}^{(t)}$	The payment from i to $j \neq i$ in t
$Y_i^{(t)}$	$\sum_{\substack{j=1 \\ j \neq i}}^{J} d_{ij}^{(t)}$
$Q_i^{(t)}$	$\sum_{\substack{j=1 \\ j \neq i}}^{J} d_{ji}^{(t)}$
$\bar{Q}_i^{(t)}$	The η_i-discounted average of $Q_i^{(t)}, Q_i^{(t-1)}, \cdots, Q_i^{(t-p)}$ with $p \in \{t; t_i\}$; $t_i \in \mathbb{N}$
$l_i^{(t)}$	The payment from i to the bank in t in order to settle (part of) some credits in t and/or before t
$m_i^{(t)}$	The payment from i to the bank in t as (part of) the interest charges of some credits in t and/or before t
$R_i^{(t)}$	The overall sum of payments that i expects to benefit from the other firms in t

The financing of payments and of deficits

Variable	**Domain**	**Meaning**
$L_i^{(t)}$	$\mathbb{R}_+$	The credit granted by the bank to i in t
$\Phi_i^{(t)}$	$\mathbb{R}_+$	The part of $\Pi_i^{(t-1)} > 0$ that is not used by i in t in order to finance payments within $Y_i^{(t)}$
p_i	$\mathbb{N} \setminus \{0\}$	The number of constant payments to be made in order to settle some credit granted by the bank to i
$M_i^{(t)}$	$\mathbb{R}_+$	The total interest charges applied to $L_i^{(t)}$

Other variables

Variable	**Domain**	**Meaning**
$\theta_{ij}^{(t)}$	$\{0;1\}$	Coefficient used in order to pass from $\tilde{d}_{ij}^{(t)}$ to $d_{ij}^{(t)}$
$\theta_i^{(t)}$	$[0;1]$	Coefficient used in order to pass from $\tilde{d}_{ij}^{(t)}$ to $d_{ij}^{(t)}$

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