Dynamic Moral Hazard, Risk-Shifting, and Optimal Capital Structure*

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Abstract

We develop an analytically tractable model integrating the risk-shifting problem between bondholders and shareholders with the moral hazard problem between shareholders and the manager. The presence of managerial moral hazard exacerbates the risk-shifting problem. The flexibility of the optimal contract allows shareholders to relax financial constraints precisely when it is most valuable to do so, thus increasing shareholder appetite for risk-shifting. In fact, the model predicts a non-monotonic relation between risk-shifting and financial distress, thereby reconciling seemingly contradictory empirical evidence. Moreover, firms with greater concern for moral hazard issue less debt and choose lower leverage. The model is qualitatively consistent with stylized facts on the survival of firms. Implications for business cycles are also considered.

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Keywords: risk-shifting, moral hazard, principal-agent problem, capital structure, leverage.

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1 Introduction

Shareholders may have incentives to undertake risky projects with negative net present value (NPV), since they benefit from the upside if things go well, whereas the bondholders face the losses on the downside, if things go poorly. Since Jensen and Meckling (1976) introduced the risk-shifting problem, most theoretical models have ignored that corporate decisions are taken by managers whose interest are not perfectly aligned with those of shareholders. In particular, we consider the case in which managerial effort is not observable, and thus shareholders need to provide incentives for the manager to work. Hence, we construct a model in which we jointly model the risk-shifting problem between shareholders and bondholders, and the moral hazard problem between shareholders and managers. Using this unified framework we ask two interrelated set of questions:

1. How does the presence of managerial moral hazard affect the risk-shifting problem? In particular, does managerial moral hazard compound or mitigate the desire of shareholders to engage in risky activities?

2. How do the optimal policies of the firm in terms of leverage, managerial compensation, and investment decisions change when the two problems are present?

In order to address these questions we consider three types of players in our model: shareholders, bondholders, and a manager (agent), all of which are risk-neutral. Bondholders and shareholders discount the future at rate r, whereas the manager is impatient and has discount factor $\gamma > r$. The cash-flows of the project follow a diffusion process in which the drift of the process is the product of the manager's effort and the quality of the project. The quality of the project is time varying and its variance depends on the amount of risk-shifting that is chosen.

Once debt is in place shareholders have an incentive to increase the riskiness of the firm's cash-flows because of limited liability. If things don't go well shareholders exercise their option to default and walk away from their debt obligations. Bondholders have to face the costs of bankruptcy. Moreover, they have rational expectations and correctly anticipate the instances in which the shareholders will default. Therefore, the optimal capital structure of the firm will trade-off the tax advantage of debt with the costs of bankruptcy that results from risk-shifting.

Furthermore, shareholders need to compensate the manager for her work. In the case in which effort is observable, shareholders pay the manager her outside option immediately (since the manager is impatient), and she exerts effort until termination. We solve this model in closed-form and obtain the optimal amount of risk-shifting, total firm value, and leverage. This will be our benchmark model without moral hazard which will serve as a reference point of comparison once we introduce the agency problem.

However, when effort is not observable, shareholders need to provide incentives for the manager to work. Thus, shareholders design a contract that specifies required effort, deferred compensation, amount of risk-shifting, and termination as a function of the observed history of output. The firm's output history determines the manager's current expected utility, which we refer to as the manager's continuation value W. The continuation value of the manager and the mean quality of the project μ will be our two state variables which will encode the contract-relevant history of the firm.

The contract exhibits deferred compensation, in which the manager is only paid after a sufficiently good history of output. Deferred compensation optimally trades off the cost of delaying payments to an impatient manager, with the benefit of postponing her compensation, thereby relaxing the incentive constraint. When the continuation value of the manager runs out, the contract is terminated. Since termination of the contract is costly, it is natural to interpret the continuation value of the manager as a proxy for financial distress. Moreover, because equity is more sensitive to financial distress than debt, financial distress and leverage move in the same direction.

The model yields the following results. First, the optimal amount of risk-shifting in the presence of managerial moral hazard is larger than in the benchmark case without moral hazard. We decompose this result into two components: *i) leverage channel* and *ii) internal hedging channel*. The leverage channel states that highly levered firms (which are closer to default) have a greater incentive to engage in risk-increasing activities. Since equity can be viewed as a call option on the firm's assets with strike price 0, firms that are closer to default benefit more from the convexity of the call option if the risk increases. Thus, holding the amount of debt constant, moral hazard creates a deadweight loss that reduces the total value of the firm. As a consequence leverage and risk-shifting increase. The internal hedging channel emerges as a result of the optimal contract's adjustments to the manager's continuation value in response to the realized quality of the project. Intuitively, the optimal contract attempts to minimize the probability of liquidation when the firm has a good project. Therefore, the continuation value of the manager is increased when the firm draws a good project, thereby relaxing the incentive constraint. Conversely, the contract stipulates a reduction in the continuation value of the manager when a bad project is drawn. In this case the firm need not ensure to profit from these cash-flows for a prolonged period of time, and finds it optimal to increase the probability of default. Hence, the internal hedging channel is a result of the dynamic nature of the optimal contract that allows the shareholders to relax the incentive constraint precisely when it needs it the most. Consequently the benefits from the upside are amplified and the shareholders find it desirable to engage is more risky activities. The internal hedging channel is reminiscent of what is known as the Oi-Hartman-Abel effect (after Oi 1961; Hartman 1972; Abel 1983). This effect emphasizes the possibility that if firms can expand in response to good outcomes and hedge against bad outcomes, they may become risk-loving. In our case, the possibility of increasing the continuation value of the manager when a good project is drawn, is akin to expanding the size of the firm, and explains why risk-shifting is higher in the presence of managerial moral hazard above and beyond what can be explained purely by leverage.

Second, the model predicts a non-monotonic relation between risk-shifting and leverage. This result has the potential to reconcile seemingly contradictory empirical evidence relating risk-shifting and financial distress. On the one hand Eisdorfer (2008) shows that financially distressed firms increase their investment in response to a raise in uncertainty. Moreover, the investment undertaken by financially distressed firms has negative NPV. Together, he interprets this as evidence of a positive relation between risk-shifting and financial distress. On the other hand, Rauh (2009) compares the asset allocation of pension funds across firms. He finds that firms with poorly funded pension plans and low credit ratings invest a greater share of their portfolios in safer securities such as government bonds and cash, while firms with well-funded plans and high credit ratings allocate a larger proportion to riskier assets such as stocks. Therefore, risk-shifting seems to be negatively related to financial distress. In our model risk-shifting is initially increasing in financial distress as documented by Eisdorfer (2008), but becomes decreasing for high levels of financial distress as in Rauh (2009). Moreover, these results imply that in the presence of managerial moral hazard standard linear models relating risk-shifting to measures of financial distress are misspecified, and a non-linear relation should be estimated instead.

Third, firms in which there is greater concern for moral hazard issue less debt, and choose lower levels of leverage. As discussed above, moral hazard increases risk-shifting thereby increasing the expected costs of bankruptcy. Therefore, the equilibrium price of debt will be lower. As a result, the firm finds it optimal to reduce the risk-shifting incentives of the shareholders by lowering its initial level of leverage.

Fourth, the model illustrates a potential amplification mechanism of output shocks via counter-cyclical risk-shifting. Since the optimal contract exhibits deferred compensation for the manager, her continuation value on average tends to increase. This brings the firm away from financial distress. When firms are not financially distressed leverage and risk-shifting are low. However, a sufficiently bad sequence of negative output shocks erodes the continuation value of the manager and brings the firm into financial distress. As a consequence, the firm increases its amount of risk shifting, making the probability of filing for bankruptcy all the more likely. Thus, the initial negative shock is amplified by the aggregate deadweight cost of bankruptcy.

Finally, our model captures the stylized fact that younger firms are more fragile, and have lower survival rates. The initial promised value to the manager represents a tradeoff between the benefits associated with having a highly incentivized manager, and the cost of delivering her enough consumption as to fulfill the promises made. In general, the latter effect tends to dominate and young firms start out with relatively low levels of continuation value. Thus, young firms are financially distressed, engage in more risky activities, and as a consequence have lower survival probabilities. As the continuation value of the manager grows over time, risk-shifting decreases, and firms become more stable.

This paper belongs to the growing literature on dynamic moral hazard that uses recursive techniques to characterize optimal dynamic contracts. We rely on the martingale techniques developed in Sannikov (2008) to deal with the principal-agent problem in a continuous time environment in which output follows a diffusion process. Our paper is most closely related to the seminar work of DeMarzo and Sannikov (2006), and Bias, Mariotti, Plantin and Rochet (2007) in which the agent is risk-neutral. Moreover, the extension to point processes developed by Piskorski and Tchistyi (2010) in the context of optimal mortgage design, allows us to capture the risk-shifting problem. The main contribution of this paper is to integrate the risk-shifting problem into this framework, explore the interaction between moral hazard and risk-shifting, the consequences to the optimal capital structure of the firm, and the life-cycle of the firm.

DeMarzo, Fishman, He and Wang (2010) and Bias, Mariotti, Rochet, and Villanueve (2010) embed investment with adjustment costs into the principal-agent problem. They find that financially constrained firms have lower investment rates, and that investment is below the first best benchmark when moral hazard is absent. They key difference with our paper is that they consider risk-less investment with positive NPV, while we focus on risky investments with negative NPV, but which can be desirable for shareholders protected by limited liability in the presence of debt commitments.

In the context of dynamic models of risk-shifting Leland (1998) finds that the costs of the risk-shifting problem are small when compared to the tax advantage of debt, and should not affect the leverage choice significantly. Ericsson (2000) reaches the opposite conclusion and shows that risk-shifting can lower the firm's optimal leverage up to 20%. Both of these papers suppose that managers behave in shareholder's interest hence assumimg away the moral hazard problem.

Our paper is also related to the literature that studies how managerial compensation can mitigate the risk-shifting problem. John and John (1993) in a three period model show that reducing the pay-to-shareholder wealth sensitivity of the manager in response to higher debt can help her internalize the cost of bankruptcy, thus reducing the incentive to take risks. Subramanian (2003) in the context of Leland (1998) shows that the managers optimal compensation is proportional to the firm's cash-flows, but subject to a ceiling and a floor.

Finally, our work is consistent with the empirical findings of Eisdorfer (2008), and Panousi and Papanikolaou (2012) who show that firms in which the interests of shareholders and managers are more closely aligned engage in more risk-shifting. In particular, Eisdorfer (2008) finds that firms in which managers hold a greater share of the firm's total equity engage in more risky investment. Panousi and Papanikolaou (2012) show that during the great recession investment declined significantly as a result of the rise in uncertainty. However, they showed that firms in which managers are compensated with options the reduction in investment was substantially smaller.

The paper is organized as follows. Section 2 presents the model. Section 3 formulates our benchmark case in the absence of moral hazard. Section 4 explores the moral hazard case, and characterizes the optimal contract. Section 5 presents the implementation of the optimal contract. Section 6 presents the empirical implications of the model, and compares our results to the benchmark case without moral hazard. Section 7 concludes. Proofs are relegated to the appendices.

2 The Model

In this section we lay out the model. We first present the players preferences, the timing of events, and the firm's technology. Second, we introduce the risk-shifting problem between the bondholders and the shareholders. Finally, we delineate the moral hazard problem between the shareholders and the manager, and formulate the optimal contract.

2.1 Preferences, Timing, and Technology

Time is continuous and infinite. There are three type of players: bondholders, shareholders and a manager (agent). Everyone is risk neutral and has rational expectations about the future. The bondholder and the shareholders discount the future at rate r, while the manager is more impatient and discounts the future at rate $\gamma > r$.

The initial shareholders of the firm have access to a project with a stream of cumulative cash-flows Y_t that evolves according to:

$$dY_t = a_t \mu_t dt + \sigma dB_t \tag{2.1}$$

where $a_t \in \{0, 1\}$ denotes the amount of effort that the manager exerts, μ_t is the mean cash-flow of the firm, B_t is a standard brownian motion process with respect to the filtration $F_{t.}$, and σ is the volatility.

We interpret the mean cash-flow μ_t as the underlying quality of the project, which has initial value μ_0. Importantly, the mean cash-flow μ_t is time varying. As to focus on the risk-shifting problem, we assume that the manager can choose from a continuum of risky investments $i \in [0, I]$. By selecting investment i the mean cash-flow is subject to a Poisson shock with arrival rate α_i. Upon arrival of the shock, the mean cash-flow will jump to a new value that is drawn independently from a normal distribution centered at $\mu_0$with variance σ_μ. Moreover, by choosing investment i the firm has to pay a flow cost $c(\alpha_i)dt$ that is increasing in α. The idea is that managers who want to engage in risk-shifting will have to choose projects with more negative NPV. In other words, by assuming a negative relationship between the project riskiness and the its NPV we get rid of risk-return tradeoff. Thus, we focus exclusively in the risk-shifting motive as the sole driver of investment choices. Formally, μ_t satisfies:

$$d\mu_t = (\Phi - \mu_0) dJ_t \tag{2.2}$$

where $\Phi \sim N[\mu_0, \sigma_\mu]$, $J = \{J_t, F_t; 0 \leq t < \tau_S\}$ is a standard compound Poisson process with intensity α_t, and τ_S denotes the arrival time of the first (and only) Poisson shock.

At time $t = 0$ the initial shareholders need to decide on the amount of debt issuance. Debt is issued once and for all at time 0. We assume that debt takes the form of a perpetuity that makes coupon payments C per period and pays $(1-\phi)\mu/r$ upon the firm's default. We interpret ϕ as the fraction of firm value that is lost as a result of bankruptcy. We assume that debt is subject to a tax-shield ψ. Thus, the optimal amount of debt will have to trade-off the costly bankruptcy with the tax advantage of debt. Once debt is in place, the firm is entirely controlled by the remaining shareholders.

The bondholders purchase this debt at fair value. The shareholders have limited liability and default once the value of the firm is equal to zero. Once debt is in place shareholders do not internalize the cost of bankruptcy imposed on the debtholders. Hence, bondholders anticipate the instances in which the shareholders will endogenously default, and will price in these expectations in their demand for debt. Thus, the value of debt D_0 will be given by

$$D_0 = E\left[\int_0^\tau e^{-rt} C dt + e^{-r\tau}(1-\phi)\frac{\mu_\tau}{r}\right] \tag{2.3}$$

where τ is the endogenous time of default chosen by the shareholders.

2.2 The Risk-Shifting Problem

Shareholders value the stochastic cumulative cash-flows from the firm net of coupon payments and payments to the manager:

$$E\left[\int_0^\tau e^{-rt}(dY_t - c(\alpha_t)dt - (1-\psi)C_t dt - dP_t)\right] \tag{2.4}$$

where P_t are the cumulative payments made to the manager, and $c(\alpha)$ is an increasing function that reflects the decreasing returns of engaging in more risky investments. Because of limited liability shareholders get zero upon default at time τ. Once debt is in place shareholders cannot commit to internalize the cost of bankruptcy incurred by the bondholders. Thus, because shareholders have limited liability, they have an incentive to choose risky investments. Intuitively, the larger α is the more risky the investment is as the shock will occurs earlier. Since the shock is drawn from a mean-preserving distribution, higherα implies a higher variance of future cash-flows. We formalize this intuition in the following lemma:

Lemma 1. *Let Y_t be cumulative cash-flows at some arbitrary time $t > 0$, and $\tau_S \in (0, t)$ the time of the shock. Then:*

1. $Y_t/\tau_S \sim N(\mu_0 t, \sigma t + \sigma_\mu (t - \tau_S))$
2. $Var(Y_t)$ *is an increasing function of* α.

Lemma 1 shows that the expected value of cash-flows is not affected by the time of the shock τ_S. However, the variance of the cash-flows is a decreasing function of $\tau_{S.}$. That is, the earlier the shock occurs, the higher is the variance of the cumulative cash-flows at a future time t. Since α is the arrival rate it is intuitive that increasing α increases the variance of future cash-flows. The above argument justifies our interpretation of α as a measure of the amount of risk-shifting. The managers will be instructed by the shareholders to optimally choose the amount of risk-shifting as to benefit from the option to default when the realized cash-flows are low. [1] Because of this conflict of interests between the bondholders and shareholders, at time 0 the optimal capital structure will have to trade-off the tax advantage of debt with the expected costs of bankruptcy resulting from the risk-shifting behavior of the shareholders.

2.3 The Moral Hazard Problem

In this section we introduce an agency conflict resulting from the unobservability of managerial effort. We recall from (2.1) how manager's effort a_t influences cash-flows Y_t. However, the amount of effort the manager exerts is her private information, and shareholders need to infer effort from the realized path of cash-flows. Moreover, when the manager exerts effort $a_t \in \{0, 1\}$ she enjoys private benefits at the rate $\lambda(1 - a_t)\mu_t$ where $0 \leq \lambda \leq 1$. We say that the manager *works* if $a_t = 1$ and *shirks* if $a_t = 0$. Alternatively, we could interpret $1 - a_t$ as the fraction of cash that is diverted by the manager for her private benefit, with $1 - \lambda$ being the the fraction loss by the diversion. In either case, λ captures the magnitude of the agency problem, and as we will see later it will pin down the incentives required to motivate the manager to work. Moreover,

[1] Through out the paper we assume that managers are only responsible to shareholders (Allen, Brealey, and Myers (2006))

we also assume that the manager controls the amount of risk-shifting α. However, we assume that the amount of risk-shifting is observable, and that it is costless for the manager to choose an arbitrary α. While the effort the manager exerts is not observable, it is realistic to assume that the type of investment chosen is public information. For example, it is public information whether a pharmaceutical company has decided to open a new R&D laboratory , or if a retail firm has decided to open stores in a foreign market.

We assume that the firm's cash-flows Y_t, the mean cash-flows μ_t, and the amount of risk shifting α_t are observable and contractible. The shareholders design a contract (α, P, τ_T) that specifies the firm's investment choice α, the cumulative compensation to the manager P, and the termination of the contract τ_T,[2] all of which depend on the realized history of output $Y_{t,}$, and on the mean cash-flows μ_t. Limited liability by the manager requires that $dP_t \geq 0$. Moreover, if the manager's saving interest rate is lower than the principal's discount rate DeMarzo and Sannikov (2006) show that there is an optimal zero savings contract. Under this condition, it is without loss of generality that we equate the manager's cumulative consumption with $P_{t.}$. Henceforth denote an arbitrary contract by $\Gamma = \Gamma(\alpha, P, \tau_T)$ and relegate further regularity conditions to the Appendix. We assume the shareholders and the manager can commit to such a contract. Moreover, we assume that the manager can be replaced and that the cost of replacing the manager M is a linear function of the mean cash-flow i.e. $M = \kappa\mu$.

Now, consider an arbitrary contract Γ, the manager chooses an effort process a as to maximize her expected utility at time $t = 0$:

$$
\begin{aligned}
W(\Gamma) \quad = \quad & \max_{a \in A} E^a \Bigg\{ \int_0^{\min\{\tau_S, \tau_T\}} e^{-\gamma t}(\mu_t \lambda(1 - a_t)dt + dP_t) + 1_{\{\tau_S > \tau_T\}} e^{-\gamma \tau_T} R \\
& + 1_{\{\tau_S \leq \tau_T\}} e^{-\gamma \tau_S} \int_{\tau_S}^{\tau_T} e^{-\gamma(t-\tau_S)}(\mu_t \lambda(1 - a_t)dt + dP_t) + 1_{\{\tau_S \leq \tau_T\}} e^{-\gamma \tau_T} R \Bigg\}
\end{aligned}
$$

where $A = \{a_t \in \{0, 1\} : 0 \leq t < \tau\}$ is the set of effort process that are measurable with respect to F_t, and the manager receives utility R from her outside option if the contract is terminated, irrespective of whether the shock takes place or not. The first two terms correspond to the utility the manager derives when the contract is terminated before the shock. The third and fourth terms correspond to the utility the manager de-

[2]When the manager is not replaced termination is equivalent to liquidation $\tau_T = \tau$.

rives from the moment the shock occurs until the contract is terminated. For simplicity we assume that the outside option of the manager $R = 0$ for the rest of the paper. [3] Henceforth, we focus on the case in which it is optimal for the shareholders to make the manager work $a_t = 1$ at all times. Intuitively this is true when the private benefit the manager derives from shirking is small compared to the gain shareholders derive from a manager that works. In propositions (4) and (8) below we provide a sufficient condition for the optimality of implementing work. For the remaining of the paper we use the expectations operator $E(.)$ to denote the expectation induced under $a_t = 1$ at all times. We say that a contract $\Gamma = \Gamma(\alpha, P, \tau_T)$ is *incentive compatible* if the agent's expected utility is maximized by choosing work.

The shareholders problem (upon debt issuance) is to solve the following maximization problem:

$$\max_{\Gamma} E\left\{\int_0^{\min\{\tau_S,\tau_T\}} e^{-rt}(dY_t - c(\alpha)dt - (1-\psi)C_t dt - dP_t)\right. \tag{2.5}$$

$$\left. +1_{\{\tau_S \leq \tau_T\}} e^{-r\tau_S}\int_{\tau_S}^{\tau_T} e^{-r(t-\tau_S)}(dY_t - (1-\psi)C_t dt - dP_t) + e^{-r\tau_T}\bar{F}(W_{\tau_T})\right\}$$

$$s.t\ \Gamma\ \textit{is incentive compatible and}\ W(\Gamma) = W_0 \geq 0$$

where W_0 is the initial expected utility to the manager,[4] and $\bar{F}(W_{\tau_T})$ represents the payoff the shareholders will receive upon termination of the contact.[5] Shareholders maximize the expected present value of firm's cash-flows net of the flow cost associated with the risky investment, the coupon payments made to the bondholders, and the payments made to the manager. As to simplify our analysis we will assume that shareholders have the full bargaining power when choosing the initial expected utility of the manager W_0. Thus, shareholders will choose $W_0 \geq 0$ as to maximize their expected profits.

[3]Relaxing this assumption is straight forward but does not contribute much to our analysis.

[4]For simplicity we have assumed that the outside option of the manager is 0. It is straight forward to generalize our framework to the case in which the outside option of the manager is strictly positive.

[5]In particular, if the manager is not replaced the shareholders will default and get zero. However, if the manager is replaced $\bar{F}(W_{\tau_S})$ will represent the expected profits from the new contract net of the cost of replacing the manager.

3 Solution without Moral Hazard

In this section with solve the case in which the manager's choice of effort is observable by the shareholders. This case will serve as a benchmark of the risk-shifting problem in the absence of moral hazard. First, we solve for the equilibrium outcomes after the shock. Then, we solve the problem prior to the shock, and characterize the optimal amount of risk-shifting in the absence of moral hazard. Finally, we solve the initial shareholders problem and solve for the optimal capital structure at time $t = 0$.

3.1 Solution After the Shock

Assume for the moment that debt with coupon payment C is already in place. Later we will find the optimal coupon payment chosen at the initial time. By assumption the outside option of the manager is 0. Since there is no moral hazard it is optimal for the shareholders to pay nothing to the manager, and implement effort a_t =1 at all times. We denote with a hat the quantities after the shock. Recall that for simplicity we assume that once the shock occurs, the mean cash-flow μ stays permanently at that value. Thus, after that there is no longer a risk-shifting problem. Let $\hat{F}(\mu)$ be the shareholder's value function when the mean cash-flow is μ. $\hat{F}(\mu)$ satisfies:

$$\hat{F}(\mu) = \max\left\{\frac{\mu - (1-\psi)C}{r}, 0\right\}$$

The shareholders can choose to either receive the stream of cash-flows from the project net of of the coupon payments, or default and get 0. Therefore, shareholders will continue to service their debt if their mean cash-flow is large enough, i. e. $\mu \geq (1-\psi)C$.

3.2 Solution Before the Shock

Let us now solve the shareholders problem before the shock. The shareholder's value function $F(\mu_0)$ solves:

$$F(\mu_0) = \max_{\alpha} E\left[\int_0^{\tau_S} e^{-rt}(dY_t - (1-\tau)Cdt - c(\alpha)dt) + e^{-r\tau_S}\int_{\mathbb{R}} \hat{F}(x)dN(x)\right]$$

The shareholders receive the projects cash-flows net of the debt payments and the operating costs of the selected investment until the shock occurs. After the shock, shareholders get the value function averaged out over the possible realizations of the shock, where $dN(.)$ is the density of a normal distribution centered at μ_0 with variance σ_μ. For the remainder of this paper we assume the functional form $c(\alpha) = \frac{\theta\alpha^2}{2}$. The parameter θ describes the rate at which riskier projects become less profitable. Large θ implies that riskier projects have higher operating costs and tend to discourage the shareholders from selecting them.

We solve this problem recursively. The shareholders value function satisfies:

$$rF(\mu_0) = \max_{\alpha} \left\{ \mu_0 - (1-\psi)C - \frac{1}{2}\theta\alpha^2 + \alpha \left[\int_{\mathbb{R}} \hat{F}(\hat{\mu}) dN(\hat{\mu}) - F(\mu_0) \right] \right\}$$

The flow value of equity for the shareholders equals the expected cash-flows from the project net of the coupon payments and the operating cost of investment, plus the expected capital gain upon arrival of the shock. The FOC with respect to the optimal amount of risk-shifting α is:

$$\alpha = \frac{1}{\theta} \left[\int_{\mathbb{R}} (\hat{F}(x) - F(\mu_0)) dN(x) \right] \tag{3.1}$$

The optimal amount of risk-shifting is proportional to the expected capital gain, and is inversely proportional to θ.

Plugging back α we solve for $F(\mu_0)$ in closed form:

$$F(\mu) = \int_{\mathbb{R}} \hat{F}(x)] dN(x) + \theta r - \sqrt[2]{\theta^2 r^2 + 2\theta \left(r \int_{\mathbb{R}} \hat{F}(x) dN(x) - (\mu_0 - (1-\psi)C) \right)} \tag{3.2}$$

Plugging this expression back in (3.1) yields:

$$\alpha = \frac{1}{\theta} \left[-\theta r + \sqrt[2]{\theta^2 r^2 + 2\theta \left(r \int_{\mathbb{R}} \hat{F}(x)] dN(x) - (\mu_0 - (1-\psi)C) \right)} \right] \tag{3.3}$$

We call the risk-shifting in the case without moral hazard the second best risk-shifting benchmark denoted α^{SB}. This is because in first best risk-shifting would be identically zero since we have assumed that riskier projects have negative NPV.

Panel B in Figure 3.1 traces risk-shifting and leverage for different values of the coupon payment C. A larger coupon payment implies higher leverage and thus shareholders are more exposed to the upside of their risks, but can default on the downside. That is, because of limited liability their loses are bounded, while their gains are un-

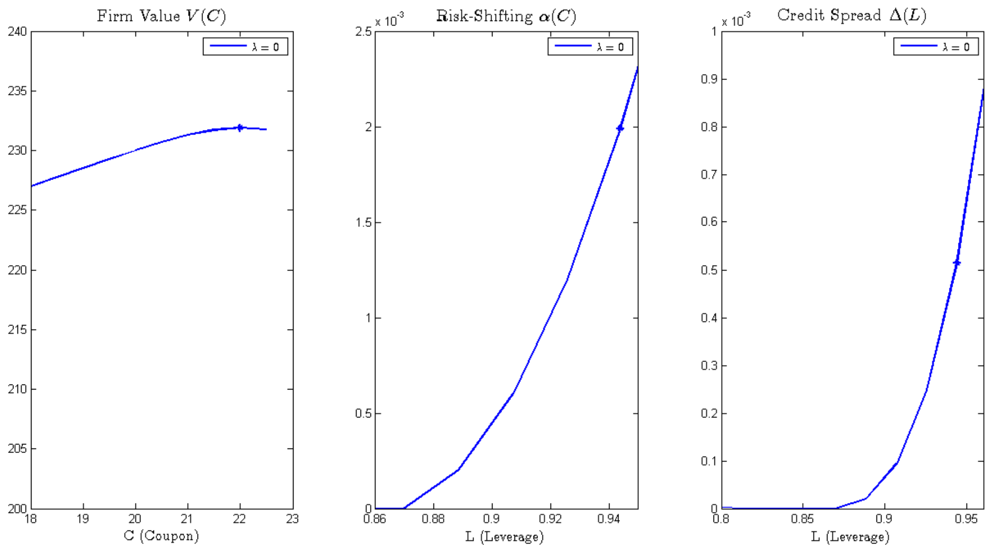

Figure 3.1: **Initial firm value, and risk-shifting.** The parameter values are $\mu_0 = 20, r = 0.1, \gamma = 0.15, \theta = 50, \psi = 0.2, \phi = 0.5, \sigma = 5, \sigma_\mu = 9.5$

bounded. Alternatively, one can think that highly leveraged firms want to "gamble for resurrection" as they are closer to default. We refer to the mechanism by which leverage induces higher risk shifting as the *leverage channel.* As it can be seen from panel B of Figure 3.1 the most important observation from the benchmark case without moral hazard is that risk-shifting is governed by the *leverage channel*. In the next section, we will see that the presence of moral hazard creates a dynamic effect that amplifies the leverage channel and induces higher risk-shifting.

3.3 Optimal Capital Structure

In this section we solve for the optimal coupon payment C. The initial shareholders need to optimally split the firm into debt and equity as to get the largest possible value from the initial issuance. The value of equity post-issuance is given by (3.2) once the new shareholders take control of the firm. The value of debt satisfies (2.3), which implies that debt is fairly priced. The explicit formulas for the value of debt can be found in the appendix. Formally the initial shareholder's problem is:

$$\max_C D(\mu_0; C) + F(\mu_0; C)$$

s.t (3.2) and (2.3)

The optimal coupon payment will tradeoff the tax benefit of debt and the cost of bankruptcy. Intuitively, for low values of C the firm can take on more debt as to benefit from the tax advantage of debt. However, as the leverage of the firm increases the expected costs of bankruptcy will pile up, and will balance out the tax benefits of debt. Panel A of Figure 3.1 shows a numerical example of the optimal choice of C.

4 Solution with Moral Hazard

In this section we assume that the manager's effort is not observable. Therefore, the optimal contract needs to provide incentives for the manager to work. As in the previous section we start by finding the optimal contract, and the value functions after the shock. Then, we use those expressions to solve the model before the shock. After that, we characterize the optimal amount of risk-shifting in the presence of moral hazard, describing the main features of the optimal contract. Finally, we calculate the optimal capital structure of the firm at time zero.

4.1 Solution After the Shock

Consider the case in which the shock has already taken place. The shareholder's problem consists of finding an optimal contract (P, τ_T) [6] that maximizes shareholder's discounted cash-flows, subject to incentive compatibility and delivering the manager her required payoff W_{τ_S}. W_{τ_S} is the manager's continuation value immediately after the shock. The contract is incentive compatible if the manager's expected utility from τ_S onward given (P, τ) is maximized by choosing $a_t = 1$ at all times.

In order to characterize the optimal contract we write the problem recursively with the continuation value of the manager as the only state variable. For a given contract (P, τ_T) the manager's continuation value W_t given that she will follow effort choice a is given by:

$$W_t = E_t \left[\int_t^{\tau_T} e^{-\gamma(s-t)} (\mu_s \lambda (1 - a_s) ds + dP_s) \right] \tag{4.1}$$

That is, W_t captures the expected utility the manager will derive from this contract

[6] Recall the after the shock there is no more risk-shifting incentive, thus the contract after τ_S will only specify payments to the manager, and a termination clause.

from time t until termination provided that she follows effort choice a. The optimal contract will now be derived using the techniques developed by the seminal work of Sannikov (2008). Proposition 2 represents the dynamics of W_t and provides necessary and sufficient conditions for the contract to be incentive compatible.

Proposition 2. *Given a contract (P, τ) there exists sensitivity β_t that is measurable with respect to F_t such that:*

$$dW_t = \gamma W_t - dP_t - \mu_s \lambda(1 - a_s)ds + \beta_t(dY_t - \mu_t dt) \tag{4.2}$$

for every $t \in (\tau_{S,}\tau)$. The contract is incentive compatible if and only if:

$$\beta_t \geq \lambda \tag{4.3}$$

The first term in the evolution of W_t corresponds to the compensation required by the manager from her time preference. The second and third term correspond to the change in utility induced by the manager's consumption and the disutility of effort. The fourth term captures the sensitivity of the manager's continuation value to the change in output. Exposing the manager to the realizations of output provides her with incentives.

Condition (4.3) states that in order for the manager to work the sensitivity of her continuation value has to be sufficiently large. Intuitively, If the manager deviates and chooses to shirk $a_t = 0$ for an instant dt, output decreases by μdt. Thus, the manager incurs a loss of $\beta_t \mu_t dt$ and gets private benefit $\lambda \mu_t dt$ by (4.2). Therefore, working is optimal for the manager if and only if

$$\beta_t \mu \geq \lambda \mu \ \text{ or } \beta_t \geq \lambda.$$

Let $\hat{F}(W_t, \mu_t)$ denote the shareholders value function after the shock, when they have drawn value μ_t for the mean cash-flow, and the promised utility to the manager is W_t. We suppress the dependence of the value function on μ_t to ease notation. DeMarzo and Sannikov (2006) show that $\hat{F}(W)$ is strictly concave so that it is optimal to set $\beta = \lambda$. Intuitively, it is not optimal to make the manager bear more risk than the minimal required for her to work. Increasing the volatility of the manager's continuation value will increase the probability of inefficiently liquidating the firm. Moreover, since the shareholders can always make a lump-sum payment to the manager it must be the case that $\hat{F}'(W) \geq -1$ for all W. Let $\bar{W}$ be the lowest value such that $\hat{F}'\left(\bar{W}\right) = -1$. $\bar{W}$ will be a reflecting boundary at which $dP_t \geq 0$. Therefore, the manager's continuation value will always be between 0 and $\bar{W}$. Proposition 3 summarizes the optimal contract after the shock:

Proposition 3. *The shareholder's value function $\hat{F}$ satisfies the following differential equation on the interval $[0, \bar{W}]$:*

$$r\hat{F}(W) = \max_{\beta \geq \lambda} \mu - (1-\psi)C + \hat{F}'(W)\gamma W + \frac{\hat{F}''(W)}{2}\sigma^2\beta^2 \tag{4.4}$$

with boundary conditions:

$$\hat{F}(0) = \max\{\max_{W_{reset}} \hat{F}(W_{reset}) - M, 0\}, \quad \hat{F}'(\bar{W}) = -1, \quad \hat{F}''(\bar{W}) = 0.$$

When $W_t \in [0, \bar{W})$, the shareholders make no payments to the manager, and only pay her when W_t hits the boundary $\bar{W}$. The payment dP_t is such that the process W_t reflects on that boundary. If $W_{\tau_S} > \bar{W}$, , the shareholders pay $W_{\tau_S} - \bar{W}$ immediately to the manager and the contract continues with the manager's new initial value $\bar{W}$. Once W_t hits 0 for the first time, the contract is terminated. At this point the shareholders can choose to default and get 0 or hire a new manager and optimally restart the contract. The optimal contract delivers a value of $\hat{F}(W_{\tau_S})$ to the shareholders.

Proof. In the Appendix.

□

Equation (4.4) says that the flow value of the shareholders value function is equal to the sum of the instantaneous expected cash-flow from the project net of debt payments, plus the capital gain induced by the change in the continuation value of the manager.

It is important to mention that in the cases when cash-flows are not large enough to cover debt payments, shareholders find it optimal to default on their debt obligations immediately. In the case with moral hazard that is equivalent to having the liquidation boundary equal to zero. More precisely, one can show that when $\mu_{\tau_S} \leq (1-\psi)C$ the payout boundary $\bar{W} = 0$. Thus, shareholders pay the manager her promised value W_{τ_S} and immediately default.

We end this subsection by providing a necessary and sufficient condition for the manager's high effort to be optimal for any $t \in [\tau_{S,}\tau]$.

Proposition 4. *Implementing high effort is optimal at any time after the shock $t \in [\tau_{S,}\tau]$ if and only if:*

$$\hat{F}(W) \geq \frac{\gamma}{r}\hat{F}'(W)\left(W^S - W\right) + (1-\psi)\frac{C}{r}$$

for all $W \in [0, \bar{W}]$, where $W^S = \frac{\lambda\mu}{\gamma}$ represents the utility of the manager if she shirks forever.

4.2 Solution Before the Shock

As in the previous section assume that debt is already in place. Later we will find the optimal coupon chosen at the initial time. The contracting problem is to find an incentive compatible contract (α, P, τ_T) that maximizes the shareholder's utility subject to delivering the manager her initial required expected utility W_0. We recall, that prior to the shock the contract specifies a required amount of risk-shifting α, in addition to the manager's consumption P, and the termination of the contract τ_T.

Similar to proposition 2 we obtain the following proposition.

Proposition 5. *Given a contract (α, P, τ_T) there exists sensitivities β_t and $\{\Delta W_{\hat{\mu}}\}_{\hat{\mu}\in\mathbb{R}}$ that are measurable with respect to F_t such that:*

$$dW_t = \gamma W_t - dP_t - \mu_t\lambda(1-a_t)dt + \beta_t(dY_t - \mu_t dt) + \int 1_{\{\mu_{t_+}=\hat{\mu}\}}\Delta W_{\hat{\mu}}d\hat{\mu}dJ_t + \rho_t dt \quad (4.5)$$

for every $t \in (0, \tau_{T_0 I})$, where $E_t[\int_{\mathbb{R}} 1_{\{\mu_{t_+}=\hat{\mu}\}}\Delta W_{\hat{\mu}}d\hat{\mu}dJ_t] = \int_{\mathbb{R}} \alpha_t \Delta W_{\hat{\mu}} dN(\hat{\mu})dt = -\rho_t dt$. Moreover, the contract is incentive compatible if and only if:

$$\beta_t \geq \lambda$$

The first fourth terms are the same as in Proposition 2. The fifth term is new and captures the exposure the manager has to the realized value of the shock. That is, immediately after the shock occurs if the realized value of the mean cash-flow is $\mu_{t_+} = \hat{\mu}$, then the continuation value of the manager will be adjusted by an amount $\Delta W_{\hat{\mu}}$.

Because the contract specifies payments that are contingent on the particular draw of the new mean cash-flow, the adjustment to the continuation value is different for each realization of the draw. The final term $\rho_t dt$ is a compensating trend required to deliver the manager her promised value. It is important to note that while in principle the effort choice of the manager affects the magnitudes of the adjustments to her continuation value $\Delta W_{\hat{\mu}}$, from the perspective of the manager these adjustments have a zero effect on her expected utility. This is because, if the manager deviates, and shirks, she will be faced with a different set of adjustments, and a different compensating trend ρ_t, but in expectation they have a zero effect on her utility. Thus her effort choice is not affected by these adjustments. Consequently, incentive compatibility of the contract only depends on the exposure of the manager to the realization of cash-flows, and follows the same intuition as before.

Let $F(W_t, \mu_0) = F(W_t)$ denote the shareholders value function prior to the shock. Applying Ito's lemma to $F(W_t)$ using the dynamics of W_t given by 4.5 we find that the shareholder's expected cash-flow net of the cost of investment plus the expected appreciation in the value of the firm is given by

$$\begin{aligned} E_t\left[dY_t + dF(W_t) - \frac{1}{2}\theta\alpha^2 dt\right] &= \Big\{\mu - (1-\psi)C + F'(W)(\gamma W + \rho_t) + \frac{1}{2}F''(W)\sigma^2\beta_t^2 \\ &\quad +\alpha\left(\int_{\mathbb{R}}(\hat{F}(W + \Delta W_{\hat{\mu}}, \hat{\mu}) - F(W))dN(\hat{\mu})\right) - \frac{1}{2}\theta\alpha^2\Big\} \end{aligned}$$

Shareholders want to maximize the RHS of (**??**) by choice of $\beta, \alpha, \Delta W_{\mu_i}$ provided that the contract is incentive compatible, and satisfies the promise keeping constraint. Assuming that the value function is concave then it is optimal to set $\beta = \lambda$, as before. The inefficiency of liquidation provides the intuition why the manager should bear the minimum amount of risk required for her to choose work. A higher volatility would increase the probability of default with no extra benefit to the shareholders. Moreover, since the shareholders can always make a lump-sum payment to the manager it must be the case that $F'(W) \geq -1$ for all W. Let the reflecting boundary $\bar{W}$ be the lowest value such that $F'\left(\bar{W}\right) = -1$. We now would like to characterize the optimal choices of $\{\Delta W_{\hat{\mu}}\}_{\hat{\mu}\in\mathbb{R}}$. By concavity of $F(W)$ the optimal choices are given by:

$$F'(W_{\tau_S}) = \hat{F}'(W_{\tau_S} + \Delta W_{\hat{\mu}}, \hat{\mu}), \quad \text{if} \quad W_{\tau_S} + \Delta W_{\hat{\mu}} > 0 \tag{4.6}$$

$$\Delta W_{\hat{\mu}} = -W_{\tau_S} \quad \text{otherwise}$$

The optimal adjustments $\{\Delta W_{\hat{\mu}}\}$ to the manager's continuation value, which are applicable when there is a change in the mean cash-flow of the project , are such that the sensitivity of increasing the manager's continuation value by one unit are equalized before and after the shock. Because the shareholder has to deliver the manager her expected utility, the choice of adjustments $\{\Delta W_{\hat{\mu}}\}$ have to be offset by the compensating trend ρ_t. Thus, the shareholders find it optimal to compensate the manager in the states in which it is cheapest for them, to the point in which the cost of compensation is equated across states. Intuitively, the adjustments are such that the continuation value of the manager is increased when the project is good (high $\hat{\mu}$), and it is decreased when the project is bad (low $\hat{\mu}$). If shareholders draw a good project it is important for them to make sure that they can profit from this project for a long time. Thus, they need a manager that has a large continuation value, and is far away from her liquidation boundary. On the other hand, the benefit for shareholders of running a project with low cash-flows is small, thus it is not critical to have a manager with a large continuation value. We interpret this feature of the optimal contract as an internal hedging mechanism that allows the shareholders to avoid default when they have a good project, and thus benefit from the high cash-flows for an extended period of time.

Finally we turn to the choice of the optimal amount of risk-shifting. The first order condition with respect to α yields:

$$\alpha = \frac{1}{\theta}\left[\left(\int_{\mathbb{R}}(\hat{F}(W + \Delta W_{\hat{\mu}}, \hat{\mu}) - F(W))dN(\hat{\mu})\right) - \int_{\mathbb{R}} F'(W)\Delta W_{\hat{\mu}} dN(\hat{\mu})\right] \tag{4.7}$$

Similar to what we found in the benchmark case without moral hazard (equation (3.1)) the amount of risk-shifting is proportional to the expected capital gain upon arrival of the shock. We notice that the capital gain depends directly on the realized value of the new project $\hat{\mu}$, but also on the amount by which the continuation value is adjusted $\Delta W_{\hat{\mu}}$. In the next section we dissect in detail the role that the optimal choice of $\Delta W_{\hat{\mu}}$ has in the choice of α.

The following Verification Theorem summarizes the optimal contract before the shock:

Proposition 6. *Suppose there exists a concave unique twice continuously differentiable solution $F(W)$ to the ODE*

$$\begin{aligned} rF(W) &= \max_{\beta \geq \lambda, \alpha, \Delta W_{\mu_i}} \Big\{ \mu - (1-\psi)C + F'(W)(\gamma W + \rho_t) + \frac{1}{2} F''(W)\sigma^2\beta^2 \\ &+\alpha \left(\int_{\mathbb{R}} (\hat{F}(W + \Delta W_{\hat{\mu}}, \hat{\mu}) - F(W)) dN(\hat{\mu}) \right) - \frac{1}{2}\theta\alpha^2 \Big\} \end{aligned} \tag{4.8}$$

with boundary conditions:

$$F(0) = \max\{\max_{W_{reset}} F(W_{reset}) - M, 0\}, \quad F'(\bar{W}) = -1, \quad F''(\bar{W}) = 0.$$

Then $F(W)$ is the value function for the shareholders optimization problem (2.5). The optimal amount of risk-shifting $\alpha(W)$ is given by 8, the optimal adjustments to the manager continuation value after a shock $\Delta W_{\hat{\mu}}$are given by (4.6), and the optimal volatility to the manager's continuation value is given by $\beta(W) = \lambda$ by concavity of the value function. The continuation value of the manager follows (5) and the optimal payments to the manager are given by

$$P_t = \left(W_0 - \bar{W}\right)^+ + \int_0^t \mathbf{1}_{\left\{W_s = \bar{W}\right\}} dP_s.$$

such that the process W_t reflects on the boundary $\bar{W}$. Once W_t hits 0 *for the first time, the contract is terminated. At this point the shareholders can choose to default and get* 0 *or hire a new manager and optimally restart the contract. If $\tau_S > \tau_T$then the remaining part of contract will be given by Proposition 6 starting at $W_{\tau_S} = W_{\tau_{S-}} + \Delta W_{\mu_{\tau_S}}$.*

Similar to the case in Proposition 3 the flow value of shareholder's value function is equal to the firm's cash-flows, plus the expected capital gain of the firm. However, the last two terms in (4.8) are new. They correspond to the expected capital gain resulting from operating the firm under a new value of μ and optimally resetting the continuation value of the manager, net of the operating cost of the risky investment.

Panel A of Figure 4.1 shows an example of the value function $F(W)$, and two value functions after the shock: One in which the value of the mean cash-flow $\hat{\mu}$ is high and another for which it is low. The arrows show the adjustments $\Delta W_{\hat{\mu}}$ to the continuation value in these two cases. As it can be seen for this example, the continuation value of the

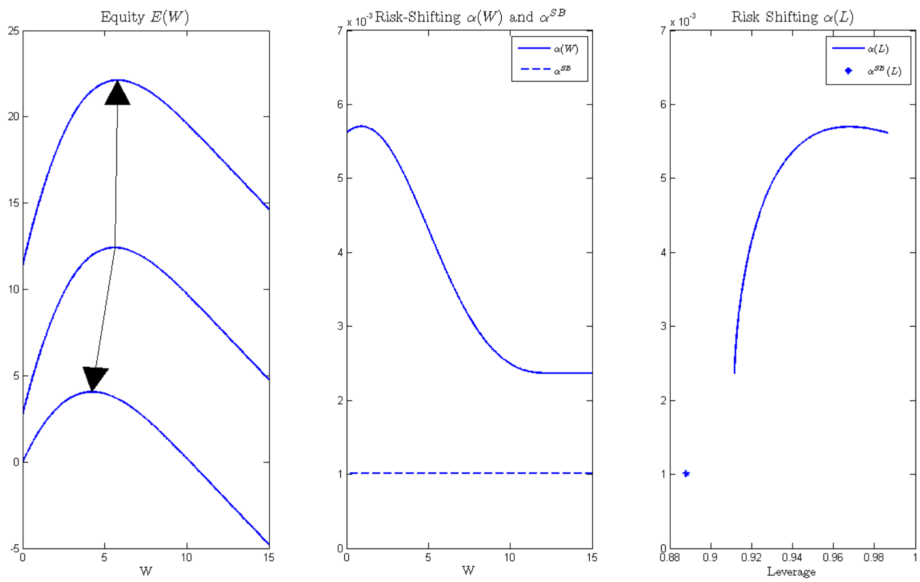

Figure 4.1: **Shareholder's value function, Risk-Shifting as a function of** W, **Risk-Shifting as a function of Leverage** The parameter values are $\mu_0 = 20, r = 0.1, \gamma = 0.15, \theta = 50, \psi = 0.2, \phi = 0.5, \sigma = 5, \sigma_\mu = 9.5, \kappa = 1, C = 12$

manager is increased after a good realization, and it is reduced after a bad realization. Panel B of Figure 4.1 depicts the optimal amount of risk-shifting $\alpha(W)$ as a function of the continuation value of the manager W, and compares it to the amount of risk-shifting in the case without moral hazard α^{SB}. Panel C of Figure 4.1 plots risk-shifting as a function of leverage $L = D(W)/(F(W) + E(W) + W)$, and compares to the leverage and risk-shifting in the case without moral hazard. In the next section we will carefully discuss these results and provide intuition for the interaction between risk-shifting and moral hazard.

The following proposition shows that the shareholder's value function $F(W;\theta)$ decreases if it is more costly for the firm to engage in risk-shifting, i.e, θ increases. Intuitively, once debt is in place, a lower value of θ makes it cheaper for the shareholders to implement higher risk-shifting, thus raising the option value of the equity as a result of the higher risk.

Proposition 7. *The shareholder's value function $F(W;\theta)$ is decreasing in θ for all $W \in [0, \bar{W}]$:*

$$\frac{\partial F(W;\theta)}{\partial \theta} = E\left[\int_t^{\tau_{T_0 I}} -e^{-r(s-t)}\frac{\alpha_s^2}{2}ds|W_t = W\right] \leq 0$$

We end this subsection by providing a necessary and sufficient condition for the manager's high effort to be optimal for any $t \in [0, \tau_S]$. The parameter values used in our numerical examples satisfy this condition.

Proposition 8. *Implementing high effort is optimal at any time before the shock $t \in [0, \tau_S]$ if and only if:*

$$F(W) \geq \max_{\alpha, \Delta W_{\mu_i}} \left\{ (1-\psi)\frac{C}{r} + \frac{\gamma}{r}F'(W)(W + \frac{r\rho_t}{\gamma} - W^S) + \frac{\alpha}{r}(\int_{\mathbb{R}} (\hat{F}(W + \Delta W_{\hat{\mu}}, \hat{\mu}) - F(W))dN(\hat{\mu})) - \frac{1}{2r}\theta\alpha^2 \right\}$$

for all $W \in [0, \bar{W}]$, where $W^S = \frac{\lambda\mu}{\gamma}$ represents the utility of the manager if she shirks forever, and $-\int_{\mathbb{R}} \alpha_t \Delta W_{\hat{\mu}} dN(\hat{\mu}) = \rho_t$.

4.3 Optimal Capital Structure

In this section we solve for the optimal coupon payment C. The initial shareholders need to optimally split the firm into debt and equity as to get the largest possible value from the initial issuance. The value of equity post-issuance is given by (4.4) . The value of debt satisfies (2.3), which implies that debt is fairly priced. The value of debt is calculated in the appendix, and can be calculated numerically as the solution to an ODE. At this point it is important to notice that the value of debt and of equity depend on W_0 as well as in C. Recall, that as to simplify our analysis we assume shareholders have full bargaining power when negotiating the manager's initial compensation W_0. Therefore, they will choose the manager's initial continuation value as to maximize their profits. The first order condition for this maximization is $F'(W_0) = 0$. Formally the initial shareholder's problem is to maximize the initial value of the firm subject to the assets being fairly priced:

$$\max_C D(\mu_0, W_0; C) + F(\mu_0, W_0; C)$$

s.t (3.2) and (2.3)

As in the case without moral hazard the optimal coupon will tradeoff the tax benefit of debt and the cost of bankruptcy. In general, the amount of risk-shifting is larger (see next section) than in the case without moral hazard, and thus the value of debt will reflect this increased probability of default. Hence the optimal ratio of debt to equity will be indirectly affected by the magnitude of the moral hazard problem.

5 Implementation

In this section we show how the optimal contract can be implemented using equity, debt, cash reserves, and insurance. At time 0 the initial shareholders will split the firm into debt (which we have assumed takes the form of a perpetuity) and equity which will specify contingent payments that implement the optimal contract characterized in proposition 6. A fraction of the money raised will be allocated to start the cash reserves of the firm. We begin by describing each of the components used in the implementation:

- *Equity:* Equity holders will receive dividend payments from the firm. Dividends are of two kinds: regular dividends and special dividends. Regular dividends are paid continuously throughout the life of the firm. Special dividends are paid occasionally, and only after the cash reserves reach a sufficiently high level. Dividends are paid from firm's cash reserves at the discretion of the manager.

- *Debt:* Debt is a perpetuity that makes coupon payments C. The face value of debt is given by C/r. Upon default bondholders collect the defaulted value of the firm $\frac{(1-\phi)\mu_{\tau_S}}{r}$. If the firm misses a coupon payment, debt holders force the firm into default.

- *Insurance:* The firm will enter an insurance contract that will entail premium payments to the insurance company until the time of the shock. The premium payments will be made at rate $\int_{\mathbb{R}} \alpha_t \Delta W_{\hat{\mu}}/\lambda dN(\hat{\mu})$. The insurance company will disburse a one time cover payment $\Delta W_{\hat{\mu}}/\lambda$ upon the realization of the shock. The insurance is actuarially fair, and will have a value of zero.

- *Cash:* The firm starts with an initial amount of cash reserves that will earn interest rate r at the bank. The cash reserves receive the cash-flows from the project, and are used to make coupon and dividend payments. Once the firm runs out of cash, the manager is fired, and either a new manager is hired or the firm defaults.

The cash reserves follow dynamics

$$dM_t = rM_t + dY_t - dP_t - d\Psi_t - c(\alpha_t)dt - (1-\psi)Cdt - dI_t \qquad M_0 = W_0/\lambda \quad (5.1)$$

for $0 \leq t \leq \tau$ where

$$d\Psi_t = \underbrace{\left[\mu - (1-\psi)C - (\gamma - r)\, M_t - c(\alpha_t)\right] dt}_{\text{regular dividends}} + \underbrace{\frac{1-\lambda}{\lambda} dP_t}_{\text{special dividends}}$$

$$dI_t = \underbrace{\left[\int_{\mathbb{R}} \alpha_t \Delta W_x dN(x)\right] dt}_{\text{insurance premium}} - \underbrace{\int 1_{\{\mu_{t_+} = \hat{\mu}\}} \Delta W_{\hat{\mu}}/\lambda d\hat{\mu} dJ_t}_{\text{insurance cover}}$$

The manager will hold a fraction λ of non-tradeable inside equity in the firm. Inside equity only pays "special dividends". In contrast the fraction $(1-\lambda)$ of traded equity held by the shareholders pays special and regular dividends. Thus, the manager will receive special dividends dP_t. Shareholders will receive special dividends $\frac{1-\lambda}{\lambda} dP_t$ and regular dividends $\left[\mu - (1-\psi)C - (\gamma - r)\, M_t - c(\alpha_t)\right] dt$. Special dividends dP_t/λ are distributed once the cash reserves reach threshold $\bar{M} = \bar{W}/\lambda$. Once the firm runs out of cash (i.e. $M_t = W_t/\lambda = 0$) the manager is fired, and the firm defaults or the manager is replaced. Whichever is optimal from the perspective of the shareholders. [7] Noting that $dI_t = 0$ and $c(\alpha_t)dt = 0$ for $t > \tau_S$ and using equations (4.5) and (4.2) one can show that $M_t = W_t/\lambda$, for $\beta_t = \lambda$ and $a_t = 1$. Moreover, $W_t = E[\int_0^\tau e^{-\gamma(s-t)} dP_t]$, therefore the implementation above is incentive compatible.

The market price of outside equity satisfies:

$$S_t = E_t \left[\int_t^\tau e^{-r(s-t)} \frac{1}{\lambda} dP_s + \frac{1}{1-\lambda} \left[\mu - (1-\psi)C - (\gamma - r)\, M_t - c(\alpha_t) dt\right] dt\right] \quad (5.2)$$

Moreover, we recall the bond price is given by

$$D_t = E_t \left[\int_t^\tau e^{-r(s-t)} C ds + e^{-r(\tau - t)} (1-\phi) \frac{\mu_\tau}{r}\right] \quad (5.3)$$

Finally, the price of the insurance contract is zero, since the cash-flows from the

[7] If the manager is replaced shareholders will have to recapitalize the firm as to replenish the cash reserves to the level $M_0$and pay for the replacement cost of the manager N.

insurance contract are equal to zero in expectation (i.e $E_t[dI_t] = 0$)

Similar to Bias et al (2007) the history dependence of the optimal contract is implemented through the cash reserves. Cash-reserves act as the "memory" device that tracks the continuation value of the manager W_t, and summarizes all the relevant information from the history of output. As in Bias et al (2007) cash-reserves increase after positive cash-flow surprises, and shrink after negative ones. However, in our model cash reserves also react to the realization of the Poisson shock. Importantly, the insurance contract allows the firm to adjust the amount of cash in the firm in response to the quality of the project drawn. The firm enters this contract with the insurance company in such a way that it allows it to relax the financing constraint of the firm precisely when it is most valuable to do so. The intuition being that it is very valuable to have enough cash when one has a very profitable project, in which case running out of cash would be very costly. Therefore, the optimal adjustments made to the cash reserves in response to the shock result from the precautionary effect associated with running out of cash. In that sense our model is consistent with both empirical and survey evidence in Opler, Pinkowitz, Stulz, and Williamson (1999), and Lins, Servaes, and Tufano (2010) who show that the main reason for corporate cash holdings is precautionary.

We conclude this section by clarifying that the value function of the shareholders $F(W)$ is not the same thing as the price of equity S_t obtained under this particular implementation. Because our implementation uses cash reserves as another asset in the firm, the cash-flows accrued to equity in this implementation are larger than those accrued by the shareholders in the optimal contract in its abstract form. Thus, the portion of equity held by the outside investors has to equal the sum of the shareholders value function and the cash inside the firm. The following proposition is similar to Proposition 6 in Bias et al. (2007) and clarifies this point.

Proposition 9. *At any time $t \geq 0$, the following holds:*

$$(1-\lambda)\, S_t = F\left(W_t\right) + M_t \tag{5.4}$$

6 Empirical Implications

In this section we explore the empirical implications of the model. First, we explore how the presence of managerial moral hazard influences the amount of risk-shifting. We show that firms in which managerial moral hazard is larger also engage in higher risk-shifting. Second, we study the optimal capital structure of firms with different levels of moral hazard. Firms in which moral hazard is prevalent issue less debt, and have lower leverage. Since moral hazard leads to more risk-shifting, bankruptcy costs will be greater in expectation. Thus, it is optimal for firms to reduce leverage as a way to lower expected bankruptcy costs. Third, we study firm survival probability and age effects of our model. Our model implies that younger firms engage in more risk-shifting, and have lower survival probabilities that older firms. Finally, we discuss the property of counter-cyclical risk-shifting implied by our model, and how it contributes to the amplification and propagation of shocks in the economy.

6.1 Risk-Shifting with and without Moral Hazard

In this section we discuss how managerial moral hazard influences the risk-shifting problem. We recall that λ represents the manager's cost of effort. Thus, we will interpret λ as the parameter that will capture the severity of the moral hazard problem. In order to study the relationship with the risk-shifting problem we consider $\alpha(W;\lambda)$. In contrast to the case without moral hazard, risk-shifting depends on the state W. Therefore, we will explore this problem in two steps: First, we will focus on the amount of risk-shifting at the payout boundary $\bar{W}$. [8] Under mild conditions, we show that $\alpha(\bar{W};\lambda)$ converges from above to α^{SB} as λ goes to 0. Second, we will fix λ and show that $\alpha(W)$ is greater than $\alpha^{SB.}$ Moreover, we will show that the greater incidence of risk-shifting in the presence of moral hazard is not entirely explained by higher leverage, and we will elaborate on the role that the internal hedging channel plays.

6.1.1 Risk-shifting at the payout boundary

The model in section 4 converges to the model without moral hazard when the cost of moral hazard λ goes to 0. Therefore, it is intuitive that the amount of risk-shifting in the case with moral hazard will also converge to the amount of risk-shifting in the case without moral hazard as the cost of effort goes to 0. Proposition 10 below formalizes

[8] As we will show below $\bar{W}$ is the attractive point of the system. Thus, focusing on the amount of risk-shifting near the payout boundary captures the risk-shifting that will be implemented a big proportion of the time.

Figure 6.1: **Risk-Shifting as a function of** $1/\lambda$ The parameter values are $\mu_0 = 20, r = 0.1, \gamma = 0.15, \theta = 50, \psi = 0.2, \phi = 0.5, \sigma = 5, \sigma_\mu = 9.5, \kappa = 1, C = 12$

this intuition by showing that risk-shifting at the payout boundary in the presence of moral hazard converges to the risk-shifting without moral hazard. Moreover, we show that when the cost of replacing the manager is small, risk-shifting in the presence of moral hazard is greater than without moral hazard, and thus that the convergence is from above.

Proposition 10. *Let α^{SB} denote the amount of risk-shifting in the absence of moral hazard, and $\alpha(\bar{W};\lambda)$ the amount of risk-shifting at the payout boundary for a given cost of effort λ. Then:*

1. $\alpha(\bar{W};\lambda) \longrightarrow \alpha^{SB}$ *as* $\lambda \longrightarrow 0$
2. $\alpha(\bar{W};\lambda) \geq \alpha^{SB}$ *if* $\int \Delta W_{\hat{\mu}} dN(\hat{\mu}) \leq 0$

Figure 6.1 shows an example in which $\alpha(\bar{W})$ converges from above to α^{SB} as the manager's cost of effort goes to 0. Hence, we can conclude that at least at the payout boundary $\bar{W}$ the presence of managerial moral hazard compounds the magnitude of the risk-shifting problem. We turn next to the study of what drives this result, and how risk-shifting varies away from the payout boundary.

6.1.2 Risk-shifting: leverage and internal hedging channels

We had already seen from figure 4.1 that the amount of risk-shifting in the presence of moral hazard depended on the continuation value of the manager W. Importantly, risk-shifting $\alpha(W)$ is greater that in the case without moral hazard α^{SB}, and it does not vary monotonically with either either the continuation value or leverage. This has important empirical implications because many models that try to estimate the magnitude of the risk-shifting problem often assume a linear and monotonic relation between leverage and risk-shifting. Thus our model suggests that such models are misspecified. Moreover, Rauh (2009) finds that firms pension funds tend to take on less risk when they are financially distressed, while Eisdorfer (2009) finds indirect evidence that risk-shifting is higher for firms that are financially distressed. Our model has

Figure 6.2: **Shareholder's value function, Risk-Shifting as a function of W, Risk-Shifting as a function of Leverage (Without Internal Hedging)** The parameter values are $\mu_0 = 20, r = 0.1, \gamma = 0.15, \theta = 50, \psi = 0.2, \phi = 0.5, \sigma = 5, \sigma_\mu = 9.5, \kappa = 1, C = 12$

the potential to reconcile this seemingly contradictory evidence: risk-shifting initially increases as the firms become financially distressed (and leverage grows), but it tapers off and decreases for higher levels of financial distress (higher levels of leverage).

We will now try to understand why risk-shifting is greater in the presence of moral hazard and why it is non-monotonic in leverage. We first notice from panel C of figure 4.1 that leverage in the case with moral hazard is greater than in the case without it (for all values of W). This is not surprising as moral hazard decreases the overall value of the firm. Since we are holding the coupon payment C constant, the value of debt stays approximately unchanged. Thus, leverage will increase. Therefore, the leverage channel discussed in section 2 will induce shareholders to engage in more risk-shifting.

However, the relationship between leverage and risk-shifting is non-monotonic. Risk-shifting is increasing in the expected capital gain for shareholders after the shock. Firms closer to default (highly leveraged) benefit from having limited liability and thus profit from the convexity of the payoffs induced by the option to default (standard leverage channel). However, in the case with moral hazard the optimal contract allows shareholders to make adjustments to the continuation value of the manager in response to the profitability of the project drawn after the shock. Thus, the expected capital gain also depends on the flexibility of the contract to allow shareholders to benefit from good projects by having a highly incentivized manager (high W). The flexibility of the contract to make this adjustments is what we call the internal hedging channel, because this adjustments allow the firm to internally hedge its response to the quality of the project drawn in an optimal way.

In order to enhance our previous intuition in a more systematic manner we will shut down the internal hedging channel. To do so, we will preclude the shareholders from making adjustments to the continuation value in response to the quality of the shock. That is, we will impose the constraint $\Delta W_{\hat{\mu}} = 0$. Let us denote the solution to this problem $F_{NIH}(W)$, and the respective amount of risk shifting $\alpha_{NIH}(W)$, where the subscript NIH stands for no internal hedging. Formally $F_{NIH}(W)$ satisfies:

$$rF_{NIH}(W) = \max_{\beta \geq \lambda, \alpha} \Big\{ \mu - (1-\psi)C + F'_{NIH}(W)\gamma W + \frac{1}{2}F''_{NIH}(W)\sigma^2\beta^2 \tag{6.1}$$

$$+\alpha \left(\int_{\mathbb{R}} (\hat{F}(W, \hat{\mu}) - F_{NIH}(W)) dN(\hat{\mu}) \right) - \frac{1}{2}\theta\alpha^2 \Big\}$$

with boundary conditions:

$$F_{NIH}(0) = \max\{\max_{W_{reset}} F_{NIH}(W_{reset}) - M, 0\}, \quad F'_{NIH}(\bar{W}) = -1, \quad F''_{NIH}(\bar{W}) = 0.$$

Panel A of Figure 6.2 shows an example of the value function $F_{NIH}(W)$, and two value functions after the shock: One in which the value of the mean cash-flow $\hat{\mu}$ is high and another for which it is low. The arrows in this case indicate that the continuation value of the manager will not change in response to the shock. Suppressing the flexibility of the contract to hedge against the shock by adjusting the continuation value shuts down the internal hedging channel. As can be seen from Panels B and C of Figure 6.2, risk-shifting $\alpha_{NIH}(W)$ is monotonically related to the continuation value of the manager, and to leverage. By suppressing the internal hedging channel our standard intuition that higher leverage should induce higher risk-shifting is restored. In other words, firms that are closer to default engage more in gambling for resurrection. However, as have seen that intuition is incomplete when the dynamic features of the optimal contract allow the shareholders to make adjustments to the continuation value of the manager. Hence, endowing the contract with this flexibility can reverse the usual relation between leverage and risk-shifting.

Importantly, our analysis suggests that the compensation package of the manager is an important determinant of risk-shifting. Thus, attempts to regulate risk-shifting by means of restricting leverage are not optimal. Policies aimed at reducing risky activities need to look jointly at the leverage of the firms and the contract that binds the management and the shareholders of the firm.

6.2 Capital Structure and Leverage

In this section we discuss how managerial moral hazard influences the optimal capital structure of the firm, and the optimal leverage. Panel A of Figure 6.3 plots the initial value of the firm for different values of C for three different values of λ. As expected the value of the firm decreases as the incidence of moral hazard increases (higher λ).

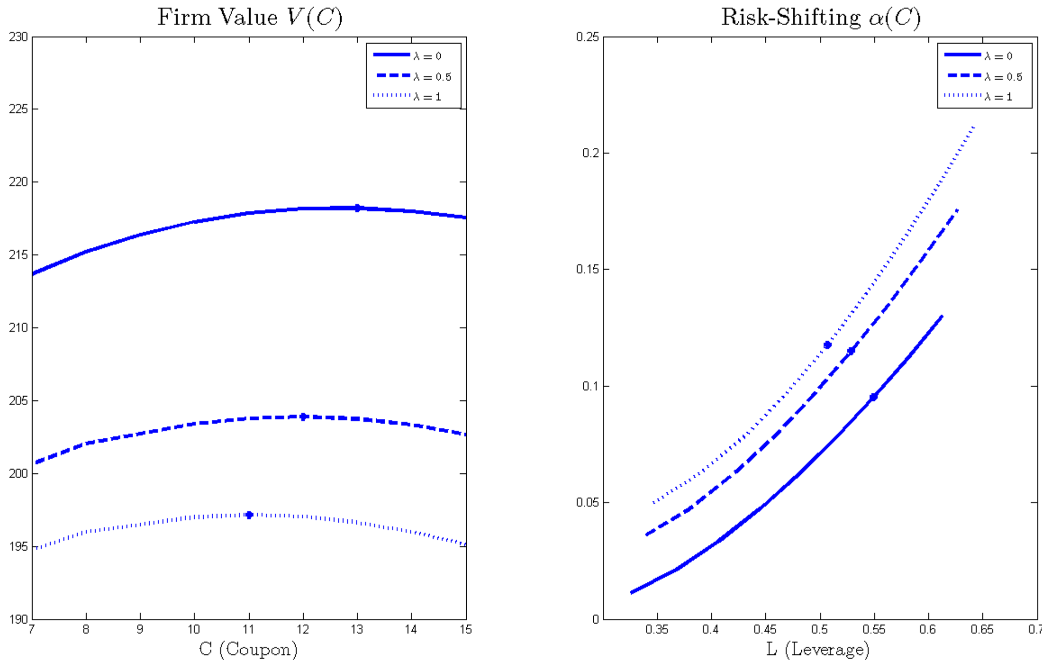

Figure 6.3: **Initial firm value, and risk-shifting .** The parameter values are $\mu_0 = 20, r = 0.1, \gamma = 0.15, \theta = 50, \psi = 0.2, \phi = 0.5, \sigma = 5, \sigma_\mu = 9.5, \kappa = 1$

Moreover, the optimal coupon chosen is decreasing in λ. The intuition for this result is that higher moral hazard induces higher risk-shifting. Bondholders anticipate higher rates of bankruptcy as a result, and thus will only buy this debt at a discount. At time 0 the initial shareholders internalize the costs associated with higher bankruptcy, and decide to reduce the amount of debt issued. Moreover, the reduction in the initial issuance of debt dominates the reduction in firm value associated with higher moral hazard and leads to lower initial leverage $L_0 = \frac{D(W_0)}{D(W_0)+F(W_0)+W_0}$. Thus, our model implies that firms in which there is more prevalence of managerial moral hazard will choose a lower initial amount of leverage. Table 1 reports comparative statics for various values of λ.

Panel B of Figure 6.2 shows the relation between initial leverage $L_0$and $\alpha(W_0)$. As expected, risk-shifting and leverage are positively related for a given value of λ.However, higher values of λ imply higher risk-shifting for the same value of leverage. This is consistent with our previous result that in a model with moral hazard, leverage is not the only determinant of the amount of risk-shifting undertaken by the firm.

Table 1: **Comparative statics: moral hazard.** This table reports the results from comparative statics on the parameter λ that captures the magnitude of the moral hazard problem. Other parameter values are $\mu_0 = 20, r = 0.1, \gamma = 0.15, \theta = 50, \psi = 0.2, \phi = 0.5, \sigma = 5, \sigma_\mu = 9.5, \kappa = 1, C = 12$

	coupon rate C	initial leverage L_0	initial firm value $F(W_0) + D(W_0)$	initial risk- shifting $\alpha(W_0)$	change in risk-shifting $\alpha(W_0) - \alpha(\bar{W})$	initial credit spread	change in credit spread
$\lambda = 0$	13	0.5497	218.20	0.0952	—	0.84%	—
$\lambda = 0.5$	12	0.5290	203.88	0.1148	0.0252	0.85%	0.04%
$\lambda = 1$	11	0.5072	197.16	0.1176	0.0361	0.72%	0.05%

6.3 Business Cycle Implications

In this section we discuss the model's implication for business cycle fluctuations. We recall that under the optimal contract the continuation value of the manager $W \in [0, \bar{W}]$ follows

$$dW_t = \gamma W_t dt + \sigma\lambda dB_t + \left\{ \int 1_{\{\mu_{t_+} = \hat{\mu}\}} \Delta W_{\hat{\mu}} d\hat{\mu} dJ_t + \rho_t dt \right\}$$

where $-\int_{\mathbb{R}} \alpha_t \Delta W_{\hat{\mu}} dN(\hat{\mu}) = \rho_t$. Since the compensation to the manager is deferred until $\bar{W}$, the evolution of W_t needs to appreciate at rate γ (first term), and from the martingale representation theorem expose the manager to the brownian shock (second term) and the Poisson shock (third term). These last two terms are zero in expectation. Therefore, on average the continuation value is drifting upwards towards the attractive point of the system $\bar{W}$.

Hence, on average the continuation value of the manager stays near the payout boundary, in which firms face low financial distress, and engage in little risk-shifting. [9] However, a sufficiently bad sequence of negative output shocks erodes the continuation value of the manager and brings the firm into financial distress. Thus, shareholders find it opportune to engage in higher risk-shifting activities, which in turn raise the probability of bankruptcy. As a result, the initial negative shock is amplified by the aggregate deadweight cost of bankruptcy.

Importantly, in the benchmark model without moral hazard presented in section 3 output shocks have no persistent effect on the dynamics of the firm. In that case the

[9] See Figure 4.1

shock is fully absorbed by the shareholders, but has no impact on the amount of risk-shifting of the firm, nor on its expected probability of bankruptcy. Therefore, it is the counter-cyclical nature of risk-shifting induced by modeling jointly the moral hazard and the risk-shifting problem that underpins this amplification mechanism.

6.4 Firm Survival: Age Effects

Multiple studies have documented that young firms experience higher turnover rates than older firms. In this section we study the implications of our model for firm survival. We recall we have assumed the initial outside option of the manager is sufficiently low that the initial continuation value of the manager W_0 is set to maximize shareholder value.[10] The first order condition for this maximization is

$$F'(W_0; C) = 0 \tag{6.2}$$

The optimal choice of W_0 implies a tradeoff for the shareholders. On the one hand, a high continuation value minimizes the costs of financial distress associated with liquidating the firm or costly replacement of the manager. On the other hand, a high continuation value implies greater payments to the manager in the future, which are costly to the shareholders. Intuitively, this will imply that the shareholders will choose a value $W_0 \in [0, \bar{W})$. More rigorously, combining the concavity of the value function, (6.2), and $F'(\bar{W}) = -1$ imply that $W_0 < \bar{W}$.

As discussed in the previous section, the manager's continuation value drifts upwards toward $\bar{W}$. Therefore, on average firms relax their financial constraints with the passage of time. Hence, as firms grow older they become less financially constrained, have lower risk-shifting, and higher survival rates. Column 5 in Table 1 calculates the difference in risk-shifting for new firms and for firms at the payout boundary. Similarly, column 7 compares the change in credit spreads, which reflect a higher probability of default for younger firms.

Our mechanism bears some resemblance to that in Albuquerque and Hopenhayn (2004). In their model leverage goes down over time as firms reduce their long-term debt, thereby reducing the instances in which shareholders find it optimal to default. The key difference with our model, is that higher survival rates for more mature firms results from lower risk-shifting, rather than from having reduced their debt obligations.

[10] Other specifications are possible, in which we would need to specify the bargaining power of the shareholders and the manager. Our results do not vary qualitatively as long as the initial continuation value of the manager $W_0 < \bar{W}$.

7 Concluding Remarks

This paper analyzes the interaction of the risk-shifting problem between shareholders and bondholders, and the moral hazard problem between shareholders and the manager. We show the presence of managerial moral hazard induces shareholders to engage in higher risk-shifting activities. We decompose this results into two effects: the leverage channel, and the internal hedging channel. The leverage channel is standard: highly leveraged firms are closer to default, thus they have greater incentives to increase risk-shifting. The internal hedging channel is novel: the dynamic contract allows shareholders to compensate the manager contingent on the quality of the project drawn. Thus, relaxing the incentive constrain of the manager in the event that a good project is drawn. As a results shareholders benefit more from the upside, hence choosing higher risk-shifting.

Moreover, the internal hedging channel induces a non-monotonic relation between risk-shifting and leverage. Thus, potentially reconciling seemingly contradictory empirical evidence on the sign of this relation. Importantly, policies aimed at regulating excessive risk-taking via capital requirements (effectively setting an upper bound on leverage) are incomplete without looking at the structure of managerial compensation. In particular, regulating contracts that reward managers for luck can be a good complement to capital requirements.

An obvious shortcoming of our work is the a priori structured assumed of the debt contract. Further insights could be gained by endogenizing the form of the debt contract. Specifically, it would be interesting to study the role of the maturity structure of debt, and of performance sensitive debt in addressing the risk-shifting and moral hazard problems. It would also be interesting to consider the case in which the manager is risk-averse. I this case the moral hazard problem will be compounded as it is costlier to expose the agent to risk. However, this may dampen the risk-shifting problem. We leave these questions for future work.

Appendices

A Proofs

Proof of Proposition 2:

Fix an arbitrary contract (P, τ_T). Define

$$W_t = E_t\left[\int_t^{\tau_T} e^{-\gamma(s-t)}(\mu_s\lambda(1-a_s)ds + dP_s)\right]$$

as the manager's utility when she follows action a under this contract.

Let

$$M_t = E_t\left[\int_0^{\tau_T} e^{-\gamma s}(\mu_s\lambda(1-a_s)ds + dP_s)\right] = \int_0^{t} e^{-\gamma s}(\mu_s\lambda(1-a_s)ds + dP_s) + e^{-\gamma t}W_t$$

which by construction is a martingale.

By the martingale representation theorem there exits measurable β_t such that $dM_t = \beta_t e^{-\gamma t}dB_t$. But we also know that

$$dM_t = e^{-\gamma t}(\mu_t\lambda(1-a_t)dt + dP_t) - \gamma e^{-\gamma t}W_t + e^{\gamma t}dW_t$$

Rearranging yields (4.2).

Moreover, since the manager is risk-neutral, if she shirks she receives λdt, but she loses $\beta_t dt$ via a lower continuation value. Applying the proofs of Propositions 1 and 2 in the Appendix in Sannikov (2008) completes the proof.

Proof of Proposition 3:

Our contract after the shock is identical to the hidden effort model of DeMarzo and Sannikov (2006, Section III). We prove this proposition by a similar procedure to the one in DeMarzo and Sannikov (2006).

Proof of Proposition 4:

If the shareholders induce the manager to shirk, her continuation would evolve according to

$$dW_t = \gamma W_t dt - \lambda\mu dt - dP_t$$

Optimality of implementing work between $t \in [\tau_S, \tau_T]$ implies that the expected gain for shareholders from letting the manager shirk is lower than under the existing contract for all $W \in [0, \bar{W}]$:

$$r\hat{F}(W) \geq -(1-\psi)C + \hat{F}'(W)(\gamma W - \lambda\mu)$$

Defining $W^S = \frac{\lambda\mu}{\gamma}$ gives the result.

Proof of Proposition 5:

Applying the martingale representation theorem in a similar manner to that in the proof of Proposition 2 shows (4.5). Moreover, incentive compatibility is proven applying the proofs of Propositions 1 and 2 in the Appendix in Sannikov (2008). This completes the proof.

Proof of Proposition 6:

We verify that the shareholder value function and policy given are indeed optimal. Consider the case in which the manager cannot be replaced. It is not difficult to add replacement, but complicates notation substantially. Let $\Gamma = \Gamma(\alpha, P, \tau_T)$ be an arbitrary contract that implements high effort at all times, and define the shareholders objective function $J(W, \Gamma)$ as

$$J(W,\Gamma) \;\; = \;\; E\left[\int_0^{\min\{\tau_S,\tau_T\}} e^{-rt}(dY_t - c(\alpha)dt - (1-\psi)C_t dt - dP_t) + 1_{\{\tau_S \leq \tau_T\}} e^{-r\tau_S}\hat{F}(W_{\tau_S})\right]$$

Step 1. Define G_t^Γ:

$$G_t^\Gamma = \int_0^t e^{-rs}(dY_s - c(\alpha_s)ds - (1-\psi)C_s ds - dP_s) + e^{-rt}F\left(W_t\right)$$

where W_t follows (4.5). Applying Ito's lemma and its generalization for point processes we obtain

$$
\begin{aligned}
e^{rt}dG_t^{\Gamma} &= \Big\{\mu_0 - c(\alpha_t) - (1-\psi)C_t + F'(W_t)\left(\gamma W_t + \int_{\mathbb{R}} \alpha_t \Delta W_{\hat{\mu}} dN(\hat{\mu})\right) \\
&\quad + \frac{1}{2}F''(W_t)\beta_t^2 + \alpha_t\left(\int_{\mathbb{R}} (\hat{F}(W_t + \Delta W_{\hat{\mu}}, \hat{\mu}) - F(W))dN(\hat{\mu})\right) - rF(W_t)\Big\}dt \\
&\quad + \{F'(W_t) - 1\}dP_t + \{\sigma + F'(W_t)\beta_t\}dB_t \\
&\quad + \left\{\int 1_{\{\mu_{t_+}=\hat{\mu}\}}\hat{F}(W_t + \Delta W_{\hat{\mu}}, \hat{\mu})d\hat{\mu}dJ_t - \alpha_t\left(\int_{\mathbb{R}} (F(W_t + \Delta W_{\hat{\mu}}, \hat{\mu})dN(\hat{\mu})\right)dt\right\}
\end{aligned} \tag{A.1}
$$

The first term is less than or equal to zero by (4.4), the second term is less than or equal to zero since $F'(W) \geq -1$, and finally the last two terms are martingales and vanish in expectations. Thus

$$
G_0^{\Gamma} = F(W_t) \geq E\Big[\int_0^{\min\{\tau_S,\tau_T\}} e^{-rt}(dY_t - c(\alpha)dt - (1-\psi)C_t dt - dP_t) + 1_{\{\tau_S \leq \tau_T\}}e^{-r\tau_S}\hat{F}(W_{\tau_S})\Big]
$$

Since Γ was arbitrary we conclude that $F(W)$ is an upper bound for the shareholder value function.

Step 2. Since the inequalities in **(A.1)** hold with equality for the policies in the proposition, we conclude that $F(W)$ is attained. Thus, $F(W)$ is the shareholders value function.

Proof of Proposition 7:

We prove this proposition by adapting Lemma 6 in DeMarzo and Sannikov (2006). We rewrite (4.8) as:

$$
rF(W) = \mu - (1-\psi)C + F'(W)\gamma W + \frac{1}{2}F''(W)\sigma^2\lambda^2 + \frac{1}{2}\theta\alpha(W)^2 \tag{A.2}
$$

where $\alpha(W)$, $\Delta W_{\hat{\mu}}(W)$, and $\beta(W) = \lambda$ represent the optimal policies from the maximization problem given by (4.6) and (4.7) . Differentiating (A.2) with respect to θ yields

$$
r\frac{\partial F(W)}{\partial\theta} = \frac{\partial F'(W)}{\partial\theta}\gamma W + \frac{\partial F''(W)}{\partial\theta}\frac{\sigma^2\lambda^2}{2} + \sigma\alpha(W)\frac{\partial\alpha(W)}{\partial\theta} + \frac{\alpha(W)^2}{2}
$$

where

$$\frac{\partial \alpha(W)}{\partial \theta} = -\frac{\alpha(W)}{\theta} + \frac{1}{\theta}\left[-\frac{\partial F(W)}{\partial \theta} - \frac{\partial F'(W)}{\partial \theta}\int \Delta W_{\hat{\mu}} dN(\hat{\mu})\right]$$

Hence

$$r\frac{\partial F(W)}{\partial \theta} = -\frac{\alpha(W)^2}{2} + \frac{\partial F'(W)}{\partial \theta}(\gamma W - \rho_t) + \frac{\partial F''(W)}{\partial \theta}\frac{\sigma^2\lambda^2}{2} + \alpha(W)\left[r\frac{\partial F(W)}{\partial \theta}\right]$$

with boundary conditions:

$$\frac{\partial F(W)}{\partial \theta} = \frac{\partial F(W_{reset})}{\partial \theta}, \quad \frac{\partial F'(\bar{W})}{\partial \theta} = 0 \tag{A.3}$$

Applying the Feynman-Kac formula we obtain that

$$\frac{\partial F(W;\theta)}{\partial \theta} = E\left[\int_t^{\tau_S} -e^{-r(s-t)}\frac{\alpha_s^2}{2}ds|W_t = W\right] \leq 0$$

The boundary conditions in (A.3) have assumed that the manager is replaced when it's continuation value runs out. For the case in which termination equals default we would have obtained

$$\frac{\partial F(W;\theta)}{\partial \theta} = E\left[\int_t^{\tau_T} -e^{-r(s-t)}\frac{\alpha_s^2}{2}ds|W_t = W\right] \leq 0$$

which completes the proof since $\tau_{T_0 I} = \min(\tau_{T_0}, \tau_S)$.

Proof of Proposition 8:

We proceed in a similar fashion as in the proof of Proposition 4. If the shareholders induce the manager to shirk, her continuation would evolve according to

$$dW_t = \gamma W_t dt - \lambda\mu dt - dP_t + \int 1_{\{\mu_{t_+}=\hat{\mu}\}}\Delta W_{\hat{\mu}} d\hat{\mu} dJ_t + \rho_t dt$$

Optimality of implementing work between $t \in [0, \tau_{S,})$ implies that the expected gain for the shareholders from letting the manager shirk is lower than under the existing contract for all $W \in [0, \bar{W}]$:

$$\begin{aligned} rF(W) \quad \geq \quad & \max_{\alpha, \Delta W_{\mu_i}} \{(1-\psi)C + F'(W)(\gamma W + \rho_t - \lambda\mu) \\ & + \alpha \left(\int_{\mathbb{R}} (\hat{F}(W + \Delta W_{\hat{\mu}}, \hat{\mu}) - F(W)) dN(\hat{\mu}) \right) - \frac{1}{2r}\theta\alpha^2 \right\} \end{aligned}$$

Using $W^S = \frac{\lambda\mu}{\gamma}$ and reorganizing the terms gives the result.

Proof of Proposition 9: As to keep the exposition simple consider the case in which the manager is not replaced. We recall that under the optimal contract:

$$dW_t = \gamma W_t - dP_t + \sigma\lambda dB_t + \int 1_{\{\mu_{t_+} = \hat{\mu}\}} \Delta W_{\hat{\mu}} d\hat{\mu} dJ_t + \rho_t dt$$

where $\int_{\mathbb{R}} \alpha_t \Delta W_{\hat{\mu}} / dN(\hat{\mu}) = -\rho_t$ By Ito's Lemma,

$$\begin{aligned} e^{-r\min\{T,\tau_T\}} W_{\min\{T,\tau_T\}} \quad = \quad & e^{-rt} W_t + \int_t^{\min\{T,\tau_T\}} e^{-rs} (\gamma - r) W_s ds + \int_t^{\min\{T,\tau_T\}} e^{-rs} \lambda dB_s \\ & - \int_t^{\min\{T,\tau_T\}} e^{-rs} dP_s + \int_t^{\min\{T,\tau_T\}} e^{-rs} \left[\int 1_{\{\mu_{t_+} = \hat{\mu}\}} \Delta W_{\hat{\mu}} d\hat{\mu} dJ_s + \rho_s ds \right] \end{aligned}$$

for any $T > t$, where $\tau_T = \inf\{t \geq 0 : W_t = 0\}$. Taking expectations and Letting $T \to \infty$, and taking expectations using $M_t = W_t/\lambda$ and $W_{\tau_T} = 0$ we derive

$$M_t = E_t \left[\int_t^{\tau_T} e^{-r(s-t)} \left[\frac{1}{\lambda} dP_s - (\gamma - r) M_s ds - \int_t^{\tau_T} e^{-rs} \left(\int 1_{\{\mu_{t_+} = \hat{\mu}\}} \Delta W_{\hat{\mu}} / \lambda d\hat{\mu} dJ_s + \rho_s / \lambda ds \right) \right] \right]$$

$$M_t = E_t \left[\int_t^{\tau_T} e^{-r(s-t)} \left[\frac{1}{\lambda} dP_s - (\gamma - r) M_s ds \right] \right]$$

It follows that

$$\begin{aligned} & (1-\lambda) S_t \\ = \quad & E_t \left[\int_t^{\tau_T} e^{-r(s-t)} \frac{1-\lambda}{\lambda} dP_s + [\mu - (1-\psi)C - (\gamma - r) M_t - c(\alpha_t)] dt \right] \\ = \quad & E_t \left[\int_t^{\tau_T} e^{-r(s-t)} (dY_t - dP_s - (1-\psi)C - c(\alpha_s) ds) \right] \\ + \quad & E_t \left[\int_t^{\tau_T} e^{-r(s-t)} \left[\frac{1}{\lambda} dP_s - (\gamma - r) M_s ds \right] \right] \\ = \quad & F(W_t) + M_t \end{aligned}$$

as desired.

Proof of Proposition 10:

Evaluating (3.1) at $W = \bar{W}$ we obtain

$$\alpha(\bar{W}) = \frac{1}{\theta}\left[\left(\int_{\mathbb{R}}(\hat{F}(\bar{W}+\Delta W_{\hat{\mu}},\hat{\mu}) - F(\bar{W}))dN(\hat{\mu})\right) - \int_{\mathbb{R}} F'(\bar{W})\Delta W_{\hat{\mu}} dN(\hat{\mu})\right]$$

substituting back into (4.4) we solve for $F(\bar{W})$, and plugging back in the above expression for we obtain

$$\begin{aligned}\alpha(\bar{W}) &= \frac{1}{\theta}\Bigg[-\theta r + \Bigg(\theta^2 r^2 + 2\theta r(\int_{\mathbb{R}} \hat{F}(\bar{W}+\Delta W_{\hat{\mu}},\hat{\mu})dN(\hat{\mu}) \\ &\quad + \int \Delta W_{\hat{\mu}} dN(\hat{\mu}) - 2\theta(\mu_0 - (1-\psi)C) + 2\theta\gamma\bar{W})\Bigg)^{1/2}\Bigg]\end{aligned}$$

Recalling that

$$\alpha^{SB} = \frac{1}{\theta}\left[-\theta r + \sqrt[2]{\theta^2 r^2 + 2\theta(r\int_{\mathbb{R}} \hat{F}(\hat{\mu})]dN(\hat{\mu}) - (\mu_0 - (1-\psi)C))}\right]$$

and that $\hat{F}(\bar{W}_{\hat{\mu}},\hat{\mu}) = \hat{\mu}/r - \gamma\bar{W}_{\hat{\mu}}/r$ we notice that as $\lambda \longrightarrow 0$ then $\bar{W}_{\hat{\mu}} \longrightarrow 0$ and thus $\int \Delta W_{\hat{\mu}} dN(\hat{\mu}) \longrightarrow 0$. Moreover, when $\int \Delta W_{\hat{\mu}} dN(\hat{\mu}) \leq 0$ then $\alpha(\bar{W};\lambda) \geq \alpha^{SB}$. Finally, when the cost of replacing the manager is constant one can show that the value functions are parallel shifts of each other [11] for the cases in which the manager is replaced, and for the cases in which the firm defaults the payout boundary is lower. Therefore, the condition that $\int \Delta W_{\hat{\mu}} dN(\hat{\mu}) \leq 0$ will be satisfied.

B Pricing Formulas

In this appendix we calculate the pricing formulas for debt in the cases with and without moral hazard.

B.0.1 Debt price without moral hazard

The value of debt after the shock $\hat{D}(\mu)$ is given by:

[11] See Hoffmann and Pfeil (2012)

$$\hat{D}(\mu) = \begin{cases} \frac{C}{r}, & (1-\psi)C > \mu \\ \frac{(1-\phi)\mu}{r}, & (1-\psi)C \leq \mu \end{cases}$$

The value of debt before the shock $D(\mu_0)$ satisfies

$$rD(\mu_0) = C + \alpha \left[\int_{\mathbb{R}} \hat{D}(\hat{\mu}) dN(\hat{\mu}) - D(\mu_0) \right]$$

solving yields

$$D(\mu_0) = \frac{C}{r+\alpha} + \frac{\alpha \int_{\mathbb{R}} \hat{D}(\hat{\mu}) dN(\hat{\mu})}{r+\alpha}$$

B.0.2 Debt price with moral hazard

The value of debt after the shock $\hat{D}(W;\mu)$ satisfies

$$r\hat{D}(W;\mu) = C + \hat{D}'(W;\mu)\gamma W + \frac{\hat{D}''(W;\mu)\sigma^2\lambda^2}{2}$$

with boundary conditions

$$\hat{D}(0) = \frac{C}{r} \quad \hat{D}'(\bar{W}) = 0$$

when the manager is replaced. And with boundary conditions

$$\hat{D}(0) = \frac{(1-\psi)\mu}{r} \quad \hat{D}'(\bar{W}) = 0$$

when the firm defaults upon termination of the contract.
The value of debt before the shock $D(W;\mu_0) = D(W)$ satisfies

$$\begin{aligned} rD(W) &= C + D'(W)(\gamma W + \rho_t) + \frac{1}{2}D''(W)\sigma^2\lambda^2 \\ &\quad +\alpha \left(\int_{\mathbb{R}} (\hat{D}(W + \Delta W_{\hat{\mu}}, \hat{\mu}) - D(W)) dN(\hat{\mu}) \right) \end{aligned}$$

with boundary conditions

$$D(0) = D(W_{Reset}) \quad D'(\bar{W}) = 0$$

when the manager is replaced. And with boundary conditions

$$D(0) = \frac{(1-\psi)\mu_0}{r} \quad D'(\bar{W}) = 0$$

when the firm defaults upon termination of the contract.

B.0.3 Debt price without internal hedging (NIH)

The price of debt after the shock $\hat{D}(W;\mu)$ is the same as in the previous section. The price of debt before the shock when we shut down the internal hedging channel $D_{NIH}(W;\mu_0) = D_{NIH}(W)$ satisfies

$$\begin{aligned} rD_{NIH}(W) &= C + D'_{NIH}(W)\gamma W + \frac{1}{2}D''(W)\sigma^2\lambda^2 \\ &\quad + \alpha\left(\int_{\mathbb{R}}(\hat{D}(W,\hat{\mu}) - D_{NIH}(W))dN(\hat{\mu})\right) \end{aligned}$$

with boundary conditions

$$D_{NIH}(0) = D_{NIH}(W_{Reset}) \quad D'_{NIH}(\bar{W}) = 0$$

when the manager is replaced, and with boundary conditions

$$D_{NIH}(0) = \frac{(1-\psi)\mu_0}{r} \quad D'_{NIH}(\bar{W}) = 0$$

when the firm defaults upon termination of the contract.

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