Information Frictions, Nominal Shocks, and the Role of Inventories in Price-Setting Decisions*

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Abstract

Models with information frictions display output and inflation dynamics that are consistent with the empirical evidence. However, an assumption in the existing literature is that pricing managers do not interact with production managers within firms. If this assumption were relaxed, nominal shocks would not have real effects on the economy. In this paper, I present a model with perfect communication within firms in which nominal shocks have real effects. In this model, intermediate goods firms accumulate output inventories, observe aggregate variables with one period lag, and observe their nominal input prices and demand at all times. Firms face idiosyncratic shocks and cannot perfectly infer the state of nature. After a contractionary nominal shock, nominal input prices go down, and firms accumulate inventories because they perceive some positive probability that the nominal price decline is due to a good productivity shock. This prevents firms' prices from decreasing and makes current profits, households' income, and aggregate demand go down. According to my model simulations, a 1% decrease in the money growth rate causes output to decline 0.17% in the first quarter and 0.38% in the second followed by a slow recovery to the steady state. Contractionary nominal shocks also have significant effects on total investment, which remains 1% below the steady state for the first 6 quarters. I show that if firms make investment decisions and if their nominal input prices and demand do not perfectly reveal the state of nature, the economy exhibits money non-neutrality even under flexible prices and perfect communication within firms.

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1 Introduction

In the past decade, much progress has been made on models studying the impact of information frictions on aggregate supply. Models with sticky information, rational inattention, or dispersed information display output and inflation dynamics that are consistent with the empirical evidence: inflation exhibits inertia, responses to monetary shocks are delayed and persistent, and anticipated disinflations do not result in booms (Ball, Mankiew & Reis, 2005; Klenow & Willis, 2007; Mankiew & Reis 2002, 2010; Nimark, 2008; Woodford, 2002).

However, an assumption in the existing literature is that pricing managers do not interact with production managers within firms. Pricing managers set firms' prices with limited or noisy information regarding not only aggregate variables but also their own input prices and demand, while production managers hire all the labor and capital that is necessary to produce the quantity demanded at given prices. As stated by Hellwig and Venkateswaran (2012), if this assumption were relaxed, nominal shocks would not have real effects on the economy because firm's input prices and demand contain all the information that is relevant to infer the firm's best responses in the standard framework used in existing literature. Hence, it remains unclear why nominal shocks have real effects when prices are flexible and there is perfect communication within firms.

This paper contributes to the literature by presenting a model with perfect communication within firms, flexible prices, output inventories, and real information frictions in which nominal shocks have real effects.[1] This model is close in spirit to the islands model of Lucas (1972) and incorporates features from the inventory model of Khan and Thomas (2007). Intermediate producers observe aggregate variables with a lag but receive information on their nominal input prices and demand in real time. Intermediate goods firms face idiosyncratic shocks, and as a consequence cannot perfectly infer the aggregate state of the economy. Intermediate producers set their output prices, determine production, and make inventories decisions based on their information set.

In this model, inventories are the link between information frictions, perfect communication within firms, and non-neutrality of nominal shocks. This is because inventories help smooth cost shocks and thus affect pricing and production decisions. The idea that inventories smooth cost shocks has been extensively explored in the literature (Bils & Kahn, 2000;

[1] Following the terminology of Angeletos and La'O (2012), if firms make certain production decisions based on noisy information (or limited attention), the information friction is consider *real*. In standard information friction models, firms set their nominal prices based on noisy or limited information, but real variables adjust freely to the true state of the nature, as if they were made under perfect information (Angeletos & La'0, 2012, p 2). In the model of this paper, production and inventory decisions are taken based on noisy information about aggregate variables, which makes the information friction *real*.

Khan & Thomas, 2007; Ramsey & West, 1999). In almost every model with inventories, firms accumulate inventories when marginal cost goes down, increasing current marginal cost and smoothing marginal cost through time. In this model, I show that this also implies that firms' prices are smoothed through time under monopolistic competition.

In this model, the cost-smoothing role of inventories helps to explain the non-neutrality of nominal shocks for the following reason: given that firms only observe their nominal input prices and demand, they will accumulate inventories (by producing more) as long as they think that they are facing temporarily low real input prices. After a contractionary nominal shock, firms observe lower nominal input prices. They do not know what the source of this change is, but they know that it could be due to a positive productivity innovation or due to a nominal shock. Since positive productivity shocks have a positive probability, firms will increase their stock of inventories. This will prevent firms' current prices from decreasing, which will distort relative prices, and will make current profits and households' income go down. As a consequence, aggregate demand falls.

I study a quantitative version of my model and find that a one-percent decrease in the money growth rate causes output to decline 0.17% in the first quarter and 0.38% in the second quarter, followed by a slow recovery to the steady state. I also find that contractionary nominal shocks have significant effects on total investment, which remains 1% below the steady state for the first 6 quarters. The investment response to an aggregate nominal perturbation is -0.67% in the first quarter and reaches its trough response of -2.26% in the second quarter.

I compare the model with information frictions to a model with perfect and complete information, and I find that information frictions makes the model more consistent with the empirical evidence. In a model with complete and perfect information, inventory investment is pro-cyclical, and the standard deviation of inventory investment is small. In contrast, in the model with information frictions, inventory investment is counter-cyclical, and its standard deviation is closer to the data. Also, given the role of inventories, prices are more stable in absolute terms and relative to output in the model with information frictions.

This paper also contributes to the literature by showing that if firms make investment decisions (such as capital accumulation or inventory decisions) and if their nominal input prices and demand do not perfectly reveal the state of nature, the economy exhibits money non-neutrality even under flexible prices and perfect communication within firms (Proposition 3). This non-neutrality occurs because firms need to forecast future aggregate conditions in order to make their investment decisions. Hence, when current input prices and demand do not perfectly reveal aggregate conditions, firms make forecast errors because their inference about the state of the nature is wrong, and their real decisions deviate from the decision that would have been taken under perfect information. Thus, investment is key for money non-neutrality. Similarly, these results point out that firms input prices and demand contain noisy but important information about aggregate conditions, implying that how firms process information is key for understanding real responses to monetary shocks. The existing literature abstracts from this issue.

I solve the model by combining the Kalman-Filter and the solution method for heterogenous agents models proposed by Reiter (2009). The idea behind my solution method is to guess a linear law of motion for the aggregate variables and find the steady state of the economy using the Kalman Filter. Then, the economy is linearized around this steady state following the methodology of Reiter (2009), which generates a new law of motion for the economy. The law of motion is updated until a fixed point is reached.

This paper is related to the literature on information frictions and aggregate supply. In this paper, nominal shocks have real effects mainly because firms have imperfect information, not because prices are sticky. As argued by Ball, Mankiw and Reis (2005), models with information frictions may be able to solve the implausible inflation-output dynamics of the new Keynesian models. Mankiw and Reis (2002) assume that *pricing managers* update their information set every period with an exogenous probability and show that nominal disturbances can produce persistent real responses. Klenow and Willis (2007) assume that firms receive information regarding macro state variables every A_T periods in a staggered fashion and find that greater values for A_T lead to a delayed, hump shaped response of inflation and a stronger output response to nominal shocks. The assumption that agents receive information about macro state variables with a lag has microfundation in the papers of Reis (2006) and Acharya (2012). Reis (2006) shows that producers optimally do not process current news about aggregate variables when firms have to pay a cost of acquiring new information. Similarly, Acharya (2012) shows that firms optimally update their information about idiosyncratic shocks more often than their information about aggregate shocks when the cost of updating both types of information is the same but the standard deviation of the idiosyncratic disturbances is greater. Unlike this paper, these articles implicitly assume imperfect communication within firms. Namely, pricing managers do not observe firm's input prices and demand at all times.

A key assumption of the model presented in this paper is that firms face a signal extraction problem. Firms need to form expectations about aggregate conditions based on *perfectly* observed input prices and demand, which contained important but noisy information about the state of the nature. This assumption follows Lucas (1972), who assumes that producers face real idiosyncratic shocks and aggregate nominal shocks, and need to form beliefs about the idiosyncratic and aggregate part of their demand in order to make production decisions. Hence, nominal innovations have real effects on the economy because firms make forecast errors by misinterpreting price changes. A signal extraction problem also appears in Nimark (2008), who studies a model with sticky prices and information frictions. Nimark assumes that firms face Calvo-Type nominal regidities and observe their idiosyncratic marginal cost, but do not have perfect information regarding the economy-wide average marginal cost, which is needed in order to set firms' prices optimally. Nimark shows that these assumptions help explain a gradual and persistent inflation response to nominal shocks. Similarly, recent literature on dispersed information assumes that producers face a signal extraction problem. For example, Woodford (2001) and Paciello and Wiederholt (2011) assume that pricing managers observe some aggregate variables such as productivity and markups with noise. In contrast to these articles, this paper assumes that the person making the pricing decision perfectly observes everything that happens inside the firm; including input prices, input quantities and quantity sold at given prices.

This paper also builds on the work of Angeletos and La'O (2012), who make a clear distinction between *real* and *nominal* information frictions. According to their terminology, an information friction is considered real if it affects the firm's decision of a real variable. For example, Angeletos and La'O assume that firms make capital decisions based on the same limited or noisy information used to set firm's prices. In this paper, the information friction is real because it affects inventory decisions.

This work is also part of a recent literature studying monetary models with inventories. Jung and Yun (2013) show that the relationship between current inflation and the marginal cost of production weakens in a model with inventories and Calvo-type nominal rigidities. Krytsov and Midrigan (2013) point out that countercyclical markups produced by inventories, rather than nominal rigidities, can account for much of the real effects of monetary policy. Even though I do not study markups per se in this paper, I also find that the relationship between prices and the marginal cost of production breaks down when firms can accumulate inventories. When a firm's cost increases drastically, the firm reduces production and sells a fraction of its inventory holdings. This reduction in the stock of inventories prevents the firm's price from rising as much as it would in a model without inventories. In contrast to previous work, inventories in my model are crucial to explaining why there are real responses to monetary shocks. This is not true in Jung and Yun (2013) and Krytsov and Midrigan (2013), which both assume some type of nominal price rigidity, so that monetary policy is effective even without inventories.

Finally, this work is related to previous studies exploring the implications of the cost smoothing motive of inventory investment (e.g Bills & Kahn, 2000; Eichenbaum, 1989; Khan & Thomas, 2007a, 2007b). In contrast to the existing literature, my work studies the role of inventories in pricing decisions in a setting with monopolistically competitive firms. This will be relevant to understanding what makes prices more or less responsive to monetary shocks.

This paper is divided into five sections. In section two, I present the model setup and discuss some properties of the decision rules. In section three, I solve the model when all agents have perfect information. in section four, I solve the model when a particular information friction is assumed. Section five concludes.

2 Model

The model is close in spirit to Lucas (1972) and incorporates features from the inventory model of Khan and Thomas (2007). There are three agents in this economy: a representative household, a representative final good producer, and a continuum of intermediate goods firms. Households supply labor and capital to the intermediate goods firms, and they purchase a final good that can be used for consumption and investment. The final good producer aggregates the intermediate goods of the economy through a constant returns to scale production function, sells its output in a competitive market to the household, and cannot accumulate inventories. Intermediate goods producers sell their product in a monopolistic market to the representative final good firm and can accumulate output inventories.

Households derive utility from consumption and leisure and discount future utility by β. Households supply labor and capital to the intermediate goods producers in perfectly competitive and *sector specific* markets, and they own all intermediate and final goods firms. Capital depreciates at rate δ_K and can be augmented by using the final good as investment: $K_t = (1 - \delta_K)K_{t-1} + X_t$.

I assume a continuum of differentiated industries with measure one and indexed by j. Each industry is represented by an intermediate goods firm that produces with capital, k, and labor, h, through a concave production function. Each intermediate goods firm can accumulate output inventories, and its output is denoted by $y = (k^{\alpha}h^{1-\alpha})^{\gamma}$; where $\gamma < 1$. I provide an explicit motive for inventory accumulation by assuming that intermediate goods firms face idiosyncratic shocks to their demand and input prices. At the beginning of each period, an intermediate good firm is identified by its inventory holdings, I, its current demand, d, and its current input prices, q. An intermediate goods firm sets its output price and determines current production, which is devoted to sales and inventory investment.

I assume that intermediate goods firms always observe their nominal input prices and demand but do not observe current aggregate variables. Firms observe the nominal wage and rental rate of capital of their sector. As a consequence, firm know how much it costs to produce y units and how many units of their product they can sell at price p for any $y, p > 0$.

Finally, I follow the literature and assume a *cash-in-advance* constraint for the nominal output: $P_t Y_t = M_t$, where Y_t denotes total aggregate production. The productivity of the final good firm (aggregate total factor productivity), A, and money balances, M, follow AR(1) processes in logs, and these are the only sources of aggregate uncertainty in the model.

2.1 Household's Problem

The representative household owns all the economy's firms, and supplies labor and capital to the intermediate goods producers in competitive and sector-specific markets. Each period, the household allocates its total income between money holdings, consumption and investment, in order to maximize its expected discounted lifetime utility. The monetary authority is assumed to pay interest on money holdings and as a consequence there is not revenue from seigniorage. Hence, the household's problem reads:

$$U = \max_{\{C_t, h_{jt}, k_{jt}, K_{t+1}, M_{t+1}, X_t\}} \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \Psi \frac{\left(\int_0^1 \phi_{w,jt} h_{jt} dj\right)^{1+\eta}}{1+\eta} \right) \tag{1}$$

s.t.

$$M_{t+1} + P_t C_t + P_t X_t \leq \int_0^1 W_{jt} h_{jt} dj + \int_0^1 R_{jt} k_{jt} dj + \Pi_t^F + i_t M_t \tag{2}$$

$$K_t = \int_0^1 \phi_{r,jt} k_{jt} dj \tag{3}$$

$$K_{t+1} = (1-\delta_K) K_t + X_t \tag{4}$$

Where C_t is consumption, M_t is money balances, X_t represents fixed capital investment, K_t is the stock of capital at the beginning of period t, i_t is the nominal interest rate, and Π^F stands for aggregate nominal dividends from the economy's firms. h_{jt} is the labor supply to sector j, and W_{jt} is the nominal wage in that sector. $\phi_{w,jt}$ is a sector-specific preference shock that is i.i.d. across sectors and independent of all other shocks. $log(\phi_{w,jt})$ is distributed normal with zero mean and variance σ_w^2. R_{jt} is the nominal rental rate of capital in sector j at time t, and k_{jt} is the supply of capital to that sector at time t. I assume that at the beginning of each period, each unit of "general" capital can be *"transformed"* into $\frac{1}{\phi_{r,jt}}$ units of type-j capital.[2] $\phi_{r,jt}$ is a sector-specific shock that is i.i.d. across sectors and independent of other shocks, and $log(\phi_{r,jt})$ is distributed normal with zero mean and variance σ_r^2.

From the first order conditions, the supplies of type-j labor and capital are given by:

$$\phi_{w,jt}\left(\int_0^1 \phi_{w,jt}h_{jt}dj\right)^{\eta} = \frac{W_{jt}}{P_t}C_t^{-\sigma} \tag{5}$$

$$\frac{R_{jt}}{\phi_{r,jt}} = \frac{R_{it}}{\phi_{r,it}} \qquad \forall i,j \tag{6}$$

Hence in equilibrium:

$$W_{jt} = \phi_{w,jt}W_t \tag{7}$$

$$R_{jt} = \phi_{r,jt}R_t \tag{8}$$

Where W_t and R_t are the aggregate nominal wage and rental rate of capital.[3] After substituting equations (7) and (8) in the household's problem, we have:

$$U = \max_{\{C_t,H_t,K_{t+1},M_{t+1},X_t\}} \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \Psi\frac{H_t^{1+\eta}}{1+\eta}\right) \tag{9}$$

s.t.

$$M_{t+1} + P_tC_t + P_tX_t \leq W_tH_t + R_tK_t + \Pi_t^F + i_tM_t \tag{10}$$

$$K_{t+1} = (1-\delta_K)K_t + X_t \tag{11}$$

[2]For instance, one can think of computers as being the capital good. Every sector needs computers in order to produce, but each sector needs some specific programs that should be installed or updated before they can be used in the production process.

[3]In other words, the nominal wage and rental rate of capital in a sector with no idiosyncratic shocks ($\phi_{w,jt} = \phi_{r,jt} = 1$)

Where $H_t \equiv \int_0^1 \phi_{w,jt} h_{jt} dj$. Then, the optimality conditions are given by:

$$C_t^{-\sigma} = \beta \mathbb{E}\left[\frac{i_{t+1}}{P_{t+1}/P_t} C_{t+1}^{-\sigma}\right] \tag{12}$$

$$C_t^{-\sigma} = \beta \mathbb{E}\left[\left(\frac{R_{t+1}}{P_{t+1}} + (1-\delta_K)\right) C_{t+1}^{-\sigma}\right] \tag{13}$$

$$\Psi H_t^{\eta} = \frac{W_t}{P_t} C_t^{-\sigma} \tag{14}$$

2.2 Final Good Firm Problem

There is a representative final good firm that sells its product, S_t, to the household in a competitive market. This firm produces using the intermediate goods of the economy through a constant returns production function. Hence, the problem for the final good firm reads:

$$\pi_t^f = \max_{s_{jt}} \left\{ P_t S_t - \int_0^1 p_{jt} s_{jt} dj \right\} \tag{15}$$

s.t.

$$S_t = A_t \left(\int_0^1 \chi_{jt}^{\frac{1}{\epsilon}} s_{jt}^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}} \tag{16}$$

Where π_t^f stands for nominal profits, A_t is aggregate total factor productivity, s_{jt} is the amount of the intermediate good j used in the production of the final good, and χ_{jt} is a good-specific technology shock that is i.i.d. across sectors and independent of all other shocks. $log(\chi_j)$ is distributed normal with zero mean and variance σ_χ^2. Therefore, by cost minimization, the demand for intermediate good j is given by:

$$s_{jt} = \chi_{jt}\left[A_t^{\epsilon-1} S_t \left(\frac{P_t}{p_{jt}}\right)^{\epsilon}\right] \tag{17}$$

Below I will assume that intermediate firm j takes $d_{jt} \equiv \chi_{jt}\left[A_t^{\epsilon-1} S_t P_t^{\epsilon}\right]$ as given. Throughout this paper, I define d_{jt} as firm j's nominal demand in period t. Therefore, it is convenient to re-write s_{jt} as follows:

$$s_{jt} = d_{jt} p_{jt}^{-\epsilon} \tag{18}$$

I assume that intermediate goods firms always observe d_{jt}, which means that they know how many units of their output they can sell at different prices. Intermediate goods firms know that their nominal demand depends on aggregate (S_t, P_t, A_t) and idiosyncratic (χ_{jt})

variables, but they cannot infer these components separately by observing d_{jt}. In equilibrium the profits of the final good firm are zero, $S_t \equiv Ct + X_t$, and the price of the final good is given by:

$$P_t = \frac{1}{A_t}\left(\int_0^1 \chi_{jt} p_{jt}^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}} \tag{19}$$

Finally, I assume that total aggregate factor productivity, A_t, follows an AR(1) process in logs:

$$log(A_t) = \rho_A log(A_{t-1}) + \varepsilon_{A,t} \tag{20}$$

$$\varepsilon_{A,t} \sim N(0, \sigma_A^2) \tag{21}$$

2.3 Intermediate Goods Firms Problem

In each industry j, there is a single intermediate producer that supplies its product in a monopolistic market to the final good firm. Each intermediate producer chooses employment, capital, the price of its product, and the stock of inventories for the next period. The cost of borrowing one unit of type-j capital in period t is given by the nominal rental rate R_{jt}, and the nominal wage of type-j labor is given by W_{jt}. Hence the problem for the intermediate good firm in sector j is given by:

$$V(I_{0j}, d_{0j}, R_{0j}, W_{0j})_0 = \max_{\{p_{jt}, s_{jt}, y_{jt}, k_{jt}, h_{jt}, I_{jt+1}\}} E_0 \sum_{t=0}^{\infty} Q_{0,t}\pi_{jt} \tag{22}$$

s.t.

$$\pi_{jt} = p_{jt}s_{jt} - R_{jt}k_{jt} - W_{jt}h_{jt} \tag{23}$$

$$s_{jt} = d_{jt}p_{jt}^{-\epsilon} \tag{24}$$

$$y_{jt} = s_{jt} + I_{jt+1} - I_{jt} \tag{25}$$

$$y_{jt} = \left(k_{jt}^{\alpha} h_{jt}^{1-\alpha}\right)^{\gamma} \tag{26}$$

$$I_{jt+1} \geq 0 \tag{27}$$

π_{jt} is the current nominal profit, p_{jt} is the price of good j, and $Q_{0,t}$ is the stochastic discount factor for the economy's firms: $Q_{0,t} = \beta \frac{u'(C_t)/P_t}{u'(C_0)/P_0}$. Equation (24) is the firm's demand, which was defined in equations (17) and (18). Now, by cost minimization, we can re-write this problem as follows:

$$V(I_{0j}, d_{0j}, q_{0j})_0 = \max_{\{p_{jt}, s_{jt}, y_{jt}, I_{jt+1}\}} E_0 \sum_{t=0}^{\infty} Q_{0,t}\pi_{jt} \tag{28}$$

$$\text{s.t.}$$

$$\pi_{jt} = p_{jt}s_{jt} - q_{jt}y_{jt}^{\frac{1}{\gamma}} \tag{29}$$

$$s_{jt} = d_{jt}p_{jt}^{-\epsilon} \tag{30}$$

$$y_{jt} = s_{jt} + I_{jt+1} - I_{jt} \tag{31}$$

$$I_{jt+1} \geq 0 \tag{32}$$

Where $q_{jt} \equiv \left(\frac{R_{jt}}{\alpha}\right)^{\alpha} \left(\frac{W_{jt}}{1-\alpha}\right)^{1-\alpha}$ is the nominal price of the firm's inputs. Notice that q_{jt} can be decomposed as follows:

$$q_{tj} = \varphi_{jt}\bar{q}_t \tag{33}$$

$$\varphi_{jt} = \phi_{r,jt}^{\alpha}\phi_{w,jt}^{1-\alpha} \tag{34}$$

$$\bar{q}_t = \left(\frac{R_t}{\alpha}\right)^{\alpha} \left(\frac{W_t}{1-\alpha}\right)^{1-\alpha} \tag{35}$$

Where $\bar{q}_t$ is the *"aggregate"* nominal input price, and ϕ_{jt} is an idiosyncratic shock that is i.i.d. across sectors and is distributed log-normal with zero mean and variance $\sigma_{\varphi}^2 \equiv \alpha\sigma_r^2 + (1-\alpha)\sigma_w^2$. The above problem is strictly concave, and the following first-order conditions pin down the firm's optimal decisions:[4]

$$p_{jt} = \left(\frac{\epsilon}{\epsilon - 1}\right) \left(\frac{q_{jt}}{\gamma}\right) y_{jt}^{\frac{1-\gamma}{\gamma}} \tag{36}$$

$$\left(\frac{q_{jt}}{\gamma}\right) y_{jt}^{\frac{1-\gamma}{\gamma}} \geq E\left[Q_{t,t+1}\left(\frac{q_{jt+1}}{\gamma}\right) y_{jt+1}^{\frac{1-\gamma}{\gamma}}\right] \tag{37}$$

Equation (38) states that the firm's price is equal to a markup times the marginal cost of

[4] Notice that the firm's problem can be written as follows:

$$V(I_{0j}, d_{0j}, q_{0j})_0 = \max_{\{y_{jt}, I_{jt+1}\}} E_0 \sum_{t=0}^{\infty} Q_{0,t}\left(d_{jt}\left(y_{jt} + I_{jt} - I_{jt+1}\right)^{\frac{\epsilon-1}{\epsilon}} - q_{jt}y_{jt}^{\frac{1}{\gamma}}\right)$$

$$\text{s.t.}$$

$$I_{jt+1} \geq 0$$

Since $\epsilon > 1$ and $\gamma \leq 1$, the first term in the firm's objective is strictly concave, and the second term is convex. Hence, this problem is strictly concave.

production regardless of the production allocation. On the other hand, according to equation (39), inventories are used to smooth the marginal cost of production through time, and this equation holds with equality if $I_{jt+1} > 0$. Suppose, for example, that a firm expects its marginal cost to go up in future periods due to an increase in the price of its inputs, q_j. In anticipation, the firm could increase its production in the current period, in order to sell those additional units when q_j goes up. This would make the current and future marginal cost move in opposite directions, smoothing the firm's marginal cost. We have a similar story when a firm expects its demand, d_j, to increase. For the purposes of this work, the following lemmas will be useful.

Lemma 1. *p_{jt} is strictly decreasing in I_{jt}*

Proof. See appendix B.1 □

In order to understand Lemma 1, suppose that the stock of inventories of a firm increases unexpectedly. Therefore, given that the firm will eventually sell those additional units, the firm's price will have to decrease at some point in order to induce consumers to buy more.

Lemma 2. *Assuming that $\epsilon > 1$ and that $\gamma \leq 1$, the optimal decision rules for p_{jt} and I_{jt+1} have the following properties:*

- *The current optimal price (p^*_{jt}) is strictly increasing in the firm's current demand (d_{jt}) and input prices (q_{jt}).*
- *The current optimal price (p^*_{jt}) is weakly increasing in the firm's future demand (d_{jt+1}) and input prices q_{jt+1}.*
- *The optimal next period's stock of inventories (I^*_{jt+1}) is weakly decreasing in the firm's current demand (d_{jt}) and input prices q_{jt}. Moreover, if the initial stock of inventories is positive ($I_{jt} > 0$), I^*_{jt+1} is strictly decreasing in d_{jt} and q_{jt}.*
- *The optimal next period's stock of inventories (I^*_{jt+1}) is weakly increasing in the firm's future demand (d_{jt+1}) and input prices (q_{jt+1}).*

Proof. See appendix B.2. □

Intuitively, given that inventories are used to smooth cost shocks, a firm will sell inventories when its demand or input price increase. This will lower current marginal cost below what it would otherwise be in the absence of inventories. Similarly, if a firm expects its demand or input price to go up in the future, it will accumulate inventories by increasing its current production. This will make the current marginal cost, and thus the firm's current output price, increase relative to what it would otherwise be in the absence of inventories.

Lemma 3. *At the firm level, inventories impose an upper bound on the expected increase in the firm's price. In particular,*

$$1 \geq \mathbb{E}\left[Q_{t,t+1}\frac{p_{t+1}}{p_t}\right] \tag{38}$$

Proof. See appendix B.3 □

This lemma implies that, with monopolistic competition, inventories smooth not only the marginal cost of production but also firms' prices. Intuitively, suppose that a firm expects its price to go up in the following period and that $p_t < E\left[Q_{t,t+1}p_{t+1}\right]$ so that $E\left[Q_{t,t+1}\frac{p_{t+1}}{p_t}\right] > 1$. Notice that this firm could increase its profits by producing more today and selling those extra units in the next period. On the one hand, the increase in current production would make the current marginal cost go up, increasing p_t. On the other hand, according to lemma 1, the increase in the stock of next period's inventories will make p_{t+1} decrease. As a consequence, the firm will accumulate inventories up to the point where $p_t = E\left[Q_{t,t+1} \cdot p_{t+1}\right]$. In that situation, the marginal benefit of selling one extra unit today (p_t) will be equal to the marginal benefit of selling one extra unit in the next period ($E[Q_{t,t+1} \cdot p_{t+1}]$).

2.4 Money And Nominal Shocks

I sidestep the micro-foundations of money and impose a *cash-in-advance* constraint on nominal output:

$$P_t Y_t = M e^{\mu_t} \tag{39}$$

$$\mu_t = \rho_\mu \mu_{t-1} + \varepsilon_{\mu,t} \tag{40}$$

$$\varepsilon_{\mu,t} \sim N(0, \sigma_\mu^2) \tag{41}$$

This assumption is standard in the literature. For example, Angeletos and La'O (2012) impose a similar restriction on total aggregate expenditure. Given these assumptions, inflation is zero in the deterministic steady state, in which $\varepsilon_{\mu,t} = \varepsilon_{A,t} = 0$.

3 Solving the Model With Perfect Information

In this section, I solve this model assuming perfect information. As I will show, nominal shocks do not have real effects on this economy. However, the optimal decision rules depicted in this subsection will help to explain why nominal shocks have real effects when a particular information friction is introduced. I start by defining the competitive equilibrium of this economy and establishing that this economy exhibits the classical dichotomy. Next, I report the impulse response functions to a productivity shock and compare them with those generated by two alternative models: (*i*) one in which there is no heterogeneity across sectors and firms cannot accumulate inventories, and (*ii*) one model in which there is heterogeneity across firms but firms cannot accumulate inventories.

3.1 Competitive Equilibrium with Perfect and Complete Information

Definition: A competitive equilibrium with perfect and complete information in this economy is a sequence of prices $\left\{P_t,\, W_t, R_t,\, i_t,\, p_{jt}\right\}$, allocations $\left\{C_t,\, K_t,\, I_t,\, Y_t,\, X_t,\, H_t,\, y_{jt},\, h_{jt},\, k_{jt}\right\}$, a distribution of intermediate goods firms $\{\lambda(I,q,d)_t\}$, and exogenous variables $\{\mu_t, A_t\}$, such that given the initial conditions K_0, $\lambda(I,q,d)_0$:

1. *Households optimize taking prices, exogenous variables, the distribution of intermediate goods firms and initial conditions as given.* The sequence $\{C_t, K_{t+1}, M_{t+1}, X_t, H_t\}$ satisfies equations (**??**), (12), (13), (10), and (11) along with the transversality condition:

$$\lim_{t\to\infty} \beta^t u'(C_t) K_t = 0. \tag{42}$$

$$\lim_{t\to\infty} \beta^t u'(C_t) M_t = 0. \tag{43}$$

2. *The final good producer optimize taking prices, exogenous variables, the distribution of intermediate goods firms and initial conditions as given*:

$$P_t = \frac{1}{A_t}\left(\int_0^1 \chi_{jt} p_{jt}^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}} \tag{44}$$

$$C_t + X_t = A_t\left(\int_0^1 \chi_{jt}^{\frac{1}{\epsilon}} s_{jt}^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}} \tag{45}$$

3. *Intermediate goods producers optimize taking* $\left\{P_t, W_t, R_t, i_t, q_{jt}, \{p_{zt}\}_{z\neq j}\right\}$, *exogenous*

variables, the distribution of intermediate goods firms, and initial conditions as given. The sequence $\{y_{jt}, I_{jt+1}, p_{jt}\}$ satisfies equations (38), (39), (25), and (26) along with the transversality condition:

$$\lim_{t\to\infty} \beta^t u'(C_t) I_t = 0. \tag{46}$$

4. The distribution of intermediate goods firms evolves according to

$$\lambda(I', q', d')_{t+1} = \int \mathbf{1}_{\{I(I,q,d)=I'\}} \cdot pr(q' \wedge d'|q, d) \cdot d\lambda(I, q, d)_t \tag{47}$$

Where $\mathbf{1}_{\{I(I,q,d)=I'\}}$ is an indicator function that is equal to 1 if a firm with initial stock of inventories I, input price q, and demand d, chooses a stock of inventories for the next period equal to I'.

5. *Markets Clear*

$$H_t = \int_0^1 \phi_{w,jt} h_{jt} dj \tag{48}$$

$$K_t = \int_0^1 \phi_{r,jt} k_{jt} dj \tag{49}$$

$$Y_t = C_t + X_t + I_{t+1} - I_t \tag{50}$$

6. The money growth rate and log total factor productivity follow AR(1) processes:

$$\mu_t = \rho_\mu \mu_{t-1} + \varepsilon_{\mu,t} \tag{51}$$

$$log(A_t) = \rho_A log(A_{t-1}) + \varepsilon_{A,t} \tag{52}$$

Proposition 1. *The set of real allocations* $\{C_t,\ K_t,\ I_t,\ Y_t,\ X_t,\ H_t,\ y_{jt},\ h_{jt},\ k_{jt}\}$ *and distribution of intermediate goods firms* $\{\lambda(I, q, d)_t\}$ *that are consistent with the existence of a competitive equilibrium is independent of the path for money.*

Proof. See appendix B.4 □

Hence, this economy exhibits the classical dichotomy. As long as prices are flexible and all agents in this economy have perfect and complete information, real and nominal variables can be analyzed separately.

3.2 Numerical Analysis

I now examine impulse responses for a parameterized version of the model. The time period for this model is one quarter. I draw on existing literature for the values of σ, η, δ, and ϵ. The intertemporal elasticity of substitution (σ) is set to 2. The inverse of the Frisch elasticity (η) is equal to 0.4. The rate of capital depreciation δ is fixed to 0.017, and the elasticity of substitution (ϵ) is set to 5.

β is selected so that the model has a real interest rate of 6.5% per year in steady state. The preference parameter Ψ is calibrated to set the average hours worked in steady state to one-third of available time. The parameter associated with the capital share (α) is chosen so that the annual capital-output ratio in steady state is equal to 2.2, a value consistent with US data from 1960 to 2013.

In order to calibrate the persistence and standard deviation of the productivity shock (ρ_A and σ_A), I use the series for Total Factor Productivity from the Federal Reserve Bank of San Francisco for the period between 1960 and 2013. I detrending the series using the Hodrick-Prescott filter and estimating an AR(1) process to this data yields a value of 0.8 for ρ_A and 0.013 for σ_A.

I use the sweep-adjusted M1S series to calibrate the parameters associated with the money growth rate. Detrending the log series using the Hodrick-Prescott filter and estimating an AR(1) to this data yields a value of 0.9 for ρ_μ and 0.0084 for σ_μ.

Finally, I assume that the standard deviations of the idiosyncratic shocks are equal so firms' demand and input prices are equally informative about aggregate conditions. This standard deviation is calibrated so that the stock of inventories represents 13% of total GDP in the model with no information frictions. This is consistent with the inventories-output ratio for finished manufactured goods for the U.S. This implies a standard deviation of idiosyncratic shocks equal to 9%.

3.3 Model Dynamics with Perfect Information

Given this set of parameters, I find the deterministic steady state and report it in table 1.[5] Figure 1 displays the intermediate firms' decision rules for different levels of the nominal demand d_j and input prices q_j, and the first panel of Figure 2 shows the ergodic distribution of inventories for this model. As stated in Lemmas 1 and 2, the price decision rule is strictly decreasing in the initial stock of inventories. Also, notice that firms accumulate inventories

[5] In the deterministic steady state $\sigma_A = \sigma_\mu = 0$

when they face low input prices or demand because in those situations the marginal cost of production is low. Another feature of this figure is that the higher the initial stock of inventories, the smaller the impact of a cost or demand shock on the firm's price. For instance, when a firm's input price increases, the impact on the firm's price can be smoothed as long as the firm has a positive initial stock of inventories. According to the ergodic distribution, 45% of firms do not have inventories at a typical point in time, and 95% have an initial stock of inventories between zero and 0.5.

To compute the impulse responses of this model, I take a first order approximation of the economy around the deterministic steady state, following the methodology proposed by Reiter (2009).[6] This methodology allows a higher order representation of the cross-sectional distribution in the state vector and has the advantage that the solution is fully non-linear in the idiosyncratic (presumably large) shocks but linear in the aggregate (presumably small) shocks.[7]

Figure 3 plots the impulse response functions to a 1% increase in aggregate total factor productivity, A. The figure shows that inventories decline initially, then exhibits a hump shaped increase. These dynamics are the net results of several competing forces. First, the increase in productivity creates an incentive to accumulate more inventories for intermediate firms that are also facing a positive idiosyncratic productivity shock. In contrast, intermediate firms that are facing a negative idiosyncratic shock *know* that they will face a better shock with a high probability in the next period, and therefore they have an incentive to sell their stock of inventories in the current period. Second, firms expect total demand to keep increasing for another three quarters, which creates an incentive to accumulate inventories in the current period. Third, there is a big initial jump in total demand. Hence, firms have an incentive to use their stock of inventories in the current period in order to keep their prices relatively constant and take advantage of the increase in aggregate demand. As a result of these competing effects, most firms decide to sell a fraction of their inventories initially and wait until next period to accumulate inventories, making inventory investment countercyclical. However, inventory investment is procyclical in the data (e.g. Ramey & West, 1999; Bils & Kahn, 2000; Khan & Thomas, 2007). As I will discuss in the next section, one important assumption that drives the response for inventories is that firms *know* what is happening in the economy. Once I modify this assumption, inventory investment will become

[6]Appendix C discusses in detail how I solved the model.

[7]For the purposes of this work, this will be particularly useful when computing firm's expectations. Given the linearity of the solution in aggregate variables, firms can use a linear filter, such as the Kalman Filter, in order to compute their expectations.

procyclical.

4 Solving the Model with Information Frictions

I now introduce a particular information friction in this economy. I assume that final goods firms observe aggregate variables with a lag of $\mathcal{T}$ periods but receive information about their input prices and demand in real time. For simplicity, I set $\mathcal{T}$ equal to 1, which implies that firms do not observe the current level of the aggregate variables. As stated before, one contribution of this paper is to provide a model with perfect communication within firms in which nominal shocks have real effects. The following proposition shows why this is important:

Proposition 2. *Suppose that all agents in the economy except firms have perfect and complete information. Moreover, assume that intermediate goods producers cannot hold inventories, so their problem becomes:*

$$V(q_0, d_0)_0 = \max_{\{p_t, s_t, y_t\}} E \sum_{t=0}^{\infty} Q_{0,t} \left(p_t s_t - q_t y_t^{\frac{1}{\gamma}} \right) \tag{53}$$

$$s.t.$$

$$s_t = d_t p_t^{-\epsilon} \tag{54}$$

$$y_t = s_t \tag{55}$$

If prices are flexible, and if there is perfect communication within firms such that pricing managers perfectly observe their input prices and demand, then nominal shocks do not have real effects on the economy regardless of the information friction on aggregate variables.

Proof. See appendix B.5. □

Hellwig and Venkateswaran (2012) prove a result similar to Proposition 2 for a simpler model.[8] If firms do not accumulate inventories or capital, then as long as firms observe their current demand and input prices, information frictions are irrelevant. The intuition is simple: in such a model firms only need to know their current demand and input prices in order to infer their best response. A firm does not need to know the actual value of C, X, P or even its own demand shock χ, because d and q contain all the information that is relevant. This proposition implies, for example, that the models of Mankiw and Reis (2002), Paciello

[8] This is formalized in their Proposition 1.

and Wiederhold (2011), and Klenow and Willis (2007) would not display real responses to monetary shocks if perfect communication within firms was allowed. However, Proposition 2 does not hold when intermediate goods firms can accumulate inventories or capital. This is summarized in the following proposition.

Proposition 3. *Suppose that all agents in the economy except intermediate firms have perfect and complete information. If intermediate goods firms can accumulate inventories or capital and their input prices and demand do not reveal the aggregate state of the economy, the economy exhibits money non-neutrality.*

Proof. See appendix B.6. □

These results are related to Angeletos and La'O (2012), who distinguish between nominal and real information frictions. Notice that one key difference between the problem faced by firms in Propositions 2 and 3 is the existence of *real* information frictions in the latter setting. Nominal shocks have real effects in the environment specified in Proposition 3 because investment decisions are based on noisy information about the state of nature, which makes the information friction *real*. In the environment of Angeletos and La'O (2012), however, nominal shocks would not have real effects if input prices and demand were perfectly observed. This is because firms could perfectly infer the aggregate state of the economy based on that information.[9]

Intuitively, when firms makes investment decision, future aggregate conditions play an important role in firms' problem. This is because the stock of inventories or the stock of capital affect future profits. Hence, when current input prices and demand do not perfectly reveal current aggregate conditions, firms make forecast mistakes because their inference about the state of the nature is wrong.

For instance, assume that firms accumulate inventories and that the aggregate input prices go down keeping everything else constant. If firms observe the aggregate state, they will react by adjusting output prices down, and real variables will be unchanged. But, if firms only observe aggregate variables with a lag, they will initially only observe their own input prices going down. Firms do not know the source of that movement. They only know that it could be because *(i)* the aggregate economy has experienced a positive productivity shock,

[9] Input prices and demand do not reveal the aggregate state of the economy as long as the number of variables observed by firms is lower than the number of aggregate and idiosyncratic shocks in the economy (see proof of proposition 3 in appendix B.6). In Angeletos and La'O (2012), firms would observe five variables: their productivity, their demand, the wage rate of their sector, tax rates, and the aggregate price index (price of investment); and firms will face the same number of shocks: productivity shocks, consumption preference shocks, labor preferences shocks, tax shocks, and nominal shocks.

(ii) the aggregate economy has experienced a contractionary nominal shock, *(iii)* the firm has experienced a positive idiosyncratic shock, or *(iv)* a combination of these. Therefore, firms' responses will be a combination of the optimal responses for each case. Given that firms want to accumulate inventories when they are shocked by a positive idiosyncratic shock, they will respond to lower input prices by accumulating inventories, which has a positive effect on the firm's current price. How strong their responses are will depend on their expectations and the probability for each case. This points out why inventories help to explain the non-neutrality of money when perfect communication within firms is assumed.

In light of proposition 3, it is worth explaining why this paper introduces money non-neutrality by allowing firm to accumulate inventories and not capital as the previous proposition also suggests. As this paper shows, inventories impose an endogenous upper bound on firms' expected price increases and make firms' prices more persistent, and these features may have important implications for the transition mechanism of monetary policy that have not been discussed in the previous literature. However, this does imply that inventories are more relevant than capital accumulation for the monetary authority. That question could be addressed by future work. The main message of this paper is that investment decisions are key for money non-neutrality under noisy information, flexible prices and perfect communications within firms. Similarly, in the spirit of Lucas (1972), this work aims to point out that firms' input prices and demand contain noisy but important information about aggregate conditions, and how firms process that information is also key for understanding real responses to monetary shocks. The relevant literature, including Angeletos and La'O (2012), abstracts from this signal extraction problem faced by firms.

4.1 Recursive Competitive Equilibrium

Given the information friction that was introduced above, it is convenient to define the competitive equilibrium in recursive form. Denote ξ as the vector of aggregate state variables, which will be defined below. The household's recursive optimization problem is:

$$U(K, M, \xi) = \max_{M', K', C, H, X} \frac{C^{1-\sigma}}{1-\sigma} - \Psi \frac{H^{1+\eta}}{1+\eta} + \beta E\left[U(K, M, \xi')\right] \tag{56}$$

s.t.

$$M' + PC + PX \leq WH + RK + \Pi^F + iM \tag{57}$$

$$K' = (1-\delta_K)K + X \tag{58}$$

$$\xi' = \omega^h(\xi) \tag{59}$$

Where equation (60) is the household's perceived law of motion of ξ. The solution to this problem is given by decision rules $M'(K, M, \xi)$, $K'(K, M, \xi)$, $C(K, M, \xi)$, $H(K, M, \xi)$, $X(K, M, \xi)$. Similarly, the intermediate goods firms' recursive optimization problem is:

$$V(I, q, d, \xi_{-1}) = \max_{p,s,y,I'} \pi + E_{\{q',d',\xi,Q|q,d,\xi_{-1}\}} [QV(I', q', d', \xi)] \tag{60}$$

s.t.

$$\pi = ps - qy^{\frac{1}{\gamma}} \tag{61}$$

$$s = dp^{-\epsilon} \tag{62}$$

$$y = s + I' - I \tag{63}$$

$$I' \geq 0 \tag{64}$$

$$\xi' = \omega^F(\xi) \tag{65}$$

Where equation (66) is the firms' perceived law of motion of ξ. Since firms observe aggregate variables with a one period lag, the firm's problem depends on ξ_{-1} and not on ξ as in the household's problem. Hence, the solution in this case is given by decision rules $p(I, q, d, \xi_{-1})$, $s(I, q, d, \xi_{-1})$, $y(I, q, d, \xi_{-1})$, $I(I, q, d, \xi_{-1})$.

Given the assumed information friction, the vector of aggregate state variables will be given by:

$$\xi = [\mu, A, \Lambda, K, \mu_{-1}, A_{-1}]' \tag{66}$$

Given that the only two sources of aggregate uncertainty are the productivity and nominal shocks, agents in this economy can perfectly infer the current distribution of firms (Λ) and stock of capital (K) by observing ξ_{-1}. This is why Λ_{-1} and K_{-1} are not relevant for the law of motion of the economy.

The household's decision rule for capital accumulation along with the firms' decision rules for inventories induce a law of motion for the aggregate variables $\omega(\xi)$. In the recursive rational expectations equilibrium the actual and the perceived law of motions are equal. To economize on notation, I henceforth let $x(\cdot)$ denote the decision rule for x.

Definition: A recursive competitive equilibrium is defined by pricing functions $\{P(\xi), W(\xi), R(\xi), i(\xi), q(\xi)\}$, a law of motion for the aggregate variables $\omega(\xi)$, and a set of decision rules $\{C(\cdot), K'(\cdot), M'(\cdot), H(\cdot), X(\cdot), s(\cdot), y(\cdot), I(\cdot), p(\cdot)\}$ with associated value functions $\{U(K, M, \xi), V(I, q, d, \xi_{-1})\}$ such that:

1. $K(\cdot)$, $M(\cdot)$, $C(\cdot)$, $H(\cdot)$, $X(\cdot)$ and $U(K, \xi)$ solve the household's recursive optimization problem, taking as given $P(\xi)$, $W(\xi)$, $R(\xi)$, $i(\xi)$, and $\omega(\xi)$.

2. $p(\cdot)$, $s(\cdot)$, $y(\cdot)$, $I(\cdot)$ and $V(I, q, d, \xi_{-1})$ solve the intermediate goods firms' problem, taking as given $q(\xi)$, $P(\xi)$, $W(\xi)$, $i(\xi)$, and $\omega(\xi)$.

3. The final good producer optimizes taking as given $P(\xi)$, $W(\xi)$, $R(\xi)$, $i(\xi)$, and $\omega(\xi)$:

$$P(\xi) = \frac{1}{A_t} \left(\int_0^1 \chi_{jt} p(\cdot)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}} \tag{67}$$

$$C(\cdot) + X(\cdot) = A_t \left(\int_0^1 \chi_{jt}^{\frac{1}{\epsilon}} s_{jt}^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}} \tag{68}$$

4. Markets clear:

$$H(\cdot) = \int \phi_{w,j} h_j dj \tag{69}$$

$$K(\cdot) = \int \phi_{r,j} k_j dj \tag{70}$$

$$Y_t = C(\cdot) + X(\cdot) + I(\cdot) - I \tag{71}$$

5. The perceived law of motion for the aggregate variables is consistent with the actual law of motion:

$$\omega(\xi) = \omega^h(\xi) = \omega^F(\xi) \tag{72}$$

6. The distribution of firms evolves according to

$$\lambda(I', q', d', \xi') = \int \mathbf{1}_{\{I(I,q,d,\xi_{-1})=I'\}} \cdot pr(q' \wedge d' | q, d) \cdot d\lambda(I, q, d, \xi) \tag{73}$$

Where $\mathbf{1}_{\{I(I,q,d,\xi_{-1})=I'\}}$ is an indicator function that is equal to 1 if a firm with initial stock of inventories I, input price q, and demand d, chooses a stock of inventories for the next period equal to I'.

4.2 Computation with Information Frictions

I solve this problem for small deviations around the steady state by following the methodology of Reiter (2009). This has an important implication: the law of motion for the aggregate variables is linear. Denoting $\mathbf{Y}$ as the vector of jump variables, this economy can be described by the following two equations:

$$\widehat{\xi}' = F\widehat{\xi} + \mathbf{V} \tag{74}$$
$$\widehat{\mathbf{Y}} = G\widehat{\xi} \tag{75}$$

Where $\widehat{x}$ denotes the deviation in levels of x around the steady state, F and G are coefficient matrices, and $\mathbf{V} \equiv \left[\varepsilon_\mu, \varepsilon_A, \mathbf{O}_{\mathbf{1}\times(\mathbf{2}\times\mathbf{ni}\times\mathbf{nz}+\mathbf{4})}\right]'$ is the vector of i.i.d. shocks. ni is the number of grid points for the stock of inventories and nz is the number of grid points for the idiosyncratic shocks.

To find the equilibrium of this economy, I start with a guess for matrices F and G. Given this guess, the household's and firms' decision rules induce a law of motion and two new matrices $F^{(new)}$ and $G^{(new)}$. In equilibrium, these matrices have to be equal. If they are not, I update these matrices until a fixed point is reached.

One should note that the intermediate goods firms face a signal extraction problem. They observe their current input price (q) and demand (d) but do not have information about the current aggregate variables. These firms need to form expectations about the evolution of their input prices and demand in order to make their pricing and inventory decisions. To see this notice that:

$$d = \chi D \tag{76}$$
$$q = \varphi \bar{q} \tag{77}$$

Where $D \equiv A^{\epsilon-1}(C+X)P^{\epsilon}$ is the aggregate nominal demand, and $\bar{q} \equiv \left(\frac{R}{\alpha}\right)^{\alpha}\left(\frac{W}{1-\alpha}\right)^{1-\alpha}$ is the aggregate nominal input price. Since the law of motion for the aggregate variables is linear, I use the Kalman Filter to compute the expectations of the intermediate goods firms. Taking logs in equations (77) and (78) we get:

$$log(d) = log(D^{ss}) + D^{ss}\widehat{D} + log(\chi) \tag{78}$$
$$log(q) = log(\bar{q}^{ss}) + \bar{q}^{ss}\widehat{\bar{q}} + log(\varphi) \tag{79}$$

Where x^{ss} denotes the value of x in steady state. Notice that firms observe $log(d)$ and $log(q)$, but they do not observe $\widehat{D}$, $\widehat{\bar{q}}$, χ, φ. Therefore, this signal extraction problem can be expressed as:

$$\begin{bmatrix} log(d) \\ log(q) \end{bmatrix} = \begin{bmatrix} log(D^{ss}) \\ log(\bar{q}^{ss}) \end{bmatrix} + \begin{bmatrix} G_D \\ G_{\bar{q}} \end{bmatrix} \widehat{\xi} + \begin{bmatrix} \chi \\ \varphi \end{bmatrix} \tag{80}$$

$$\widehat{\xi}' = F\widehat{\xi} + \mathbf{V} \tag{81}$$

Where G_D and $G_{\bar{q}}$ are the rows of matrix G associated with the jump variables D and $\bar{q}$. Hence, this system can be solved using the Kalman Filter.

4.3 Impulse responses with Information Frictions

Assuming the same parameter values as for the perfect information model, I report the steady state for this economy in Table 2 and the ergodic distribution of inventories in the second panel of Figure 2. The only significant difference between the steady state with perfect information and the steady state with information frictions is that the stock of inventories now represents 15% of total output. Given that aggregate uncertainty is greater with information frictions and given that the final goods firms value function ($V(I, q, d, \xi_{-1})$) is strictly concave, intermediate firms have more incentive to invest in inventories, which provide insurance against negative shocks from the point of view of the firm.[10]

4.3.1 Productivity shock

Figure 4 plots the impulse response functions to a 1% increase in productivity, and Figure 5 compares these function with those generated by the model with perfect information. One of the most striking results is that inventories increase after the productivity shock in the model with information frictions. To explain this, suppose for simplicity that the idiosyncratic cost φ has a uniform distribution.[11] This implies that the nominal price of frims' inputs q is also distributed uniform between $\left[q^l, q^u\right]$ with mean $\bar{q}$ as shown in Figure 6. Firms located between $\left[q^l, \bar{q}\right]$ have more incentive to accumulate inventories than those located between $[\bar{q}, q^u]$. After an aggregate productivity shock, the average input price $\bar{q}$ decreases to $\bar{q} - \phi$, where $\phi > 0$. Figure 6 also shows how the distribution shifts. Given that firms do not know that the economy has been experienced a positive productivity innovation, all the firms have an incentive to accumulate more inventories. Firms located between $[\bar{q}, q^u - \phi]$ (part A in Figure 6) are in the right tail of the new distribution, but they are not sure that

[10] Using language from consumer theory, firms have a precautionary motive for holding inventories.

[11] I solve the model assuming that this schock is log-normal, but assuming a uniform distribution is helpful for discussing the intuition behind the results.

the distribution has changed. As a consequence, those firms do not sell as many inventories as they would under full information. In the model with perfect information, those same firms *know* that the economy has been shocked, they *know* that their input price is relatively high, and they *know* about the big jump in total demand. Therefore, these firms sell a high volume of inventories when the economy experiences a positive productivity innovation in a model with perfect information. Similarly, firms facing an input price between $\left[q^l, \bar{q}\right]$ (part B of Figure 6) attach some probability under imperfect information that they are facing low real input prices with respect to the whole distribution. Therefore, they accumulate more inventories than they would absent information frictions. Finally, firms between $\left[b^l - \phi, b^l\right]$ (part C in Figure 6) know that the input price distribution has changed, since their input price has probability zero under the old distribution. Hence, those firms accumulate inventories not only because they know that their input price is relatively low, but also because they have better expectations about the evolution of the economy, and they know that aggregate demand will keep increasing for another couple of periods.

The aggregate price index falls in the model with information frictions as the economy is able to produce more goods at a lower price. However, in comparison with the model with perfect information, the magnitude of the price decline is smaller. This is because the firms in the right tail of the idiosyncratic input price distribution do not sell as many inventories. Hence, these firms set a higher price. Since firms accumulate more inventories under imperfect information, current profits decline. This explain why the increase in the aggregate demand and output is smaller under imperfect information, since household's income is expanding at a slower rate.

4.3.2 Nominal Shock

Figure 7 plots the impulse response functions of this economy to a 1% decrease in the money growth rate. After the shock, intermediate goods firms observe a decrease in their nominal input price and nominal demand. They do not know the source of these changes. They only know that they could be facing a positive productivity shock (aggregate or idiosyncratic), a contractionary nominal shock, or a combination of both. Given that there is some probability that they are facing a positive productivity shock, firms accumulate inventories in the first period. As explained above, this response is amplified by the fact that firms located in the right tail of the input price distribution do not sell their stocks of inventories as much as they would under perfect information.

The large increase in inventories reduces current profits (Π^F), and as consequence household income. Since households want to smooth their consumption, they consume a part of their capital and work more. In the second quarter, when firms see that the economy was shocked by a lower money growth, they realize that they made a mistake by accumulating inventories. So they reduce their production and sell a large fraction of their inventories.

The dynamics of total investment (capital plus inventory) follow the output dynamics. However, the magnitude of fluctuations is larger for investment than for output. Notice that output decreases 0.18% in the first quarter while investment goes down by 0.67%. The output and investment troughs are in quarter two, when output decreases 0.38% and investment falls 2.26%.

4.3.3 Business Cycle Moments

Following Cooley and Hansen (1995), Tables 3 and 4 show variables' standard deviations, cross-correlations with output, and correlations with the money growth rate from simulating the model with perfect information (Table 3), and the model with information frictions (Table 4). For each table, the economy was simulated for 2,100 quarters, and the first 100 observations were dropped. The artificial series were logged and then detrended using the Hodrick-Prescott filter. To assess these models, I compare these tables with the numbers reported in Table 7.1 in Cooley and Hansen (1995), which presents business cycle statistics for the U.S. economy for the period 1954:1-1991:2.

It is not surprising that the standard deviations increase in the model with information frictions, since this model adds more uncertainty to the intermediate firms' problem, and generates real responses to nominal shocks. Also, total investment and change in inventories become the most volatile variables in the model with information frictions, which is consistent with the empirical evidence.[12] Similarly, prices become more stable in the model with imperfect information. The standard deviations of the price level and inflation are smaller, and they are even smaller in relative terms when compared to output. This is because firms carry more inventories on average to smooth shocks. The correlations with output in the model with information frictions are also closer to the data. In particular, inventory investment is pro-cyclical in the model with information frictions, and total investment is strongly correlated with output.

Finally, Figure 8 shows the optimal price series (left panel) for a firm facing a particular series of demand and input price shocks (right panel). The black line in the left panel shows the optimal price series for a firm that cannot accumulate inventories; the red line shows

[12] In the data, the standard deviation for investment is approximately 8%.

the price set by a final goods firm that can accumulate inventories and that has perfect information; and the blue line shows the price set by a firm that can accumulate inventories but that faces the information friction. Table 5 presents some statistics for this simulation.

Notice that the correlation between firms' output prices and input prices is very strong (0.998) when firms cannot accumulate inventories. In contrast, when firms can accumulate inventories, this correlation decreases by almost 40%. Hence, inventories break the strong relationship between current input prices and current output prices. Also, introducing inventories adds persistence to prices. The first autocorrelation of the output price increases from almost zero to 0.55. As discussed above, inventories are used to smooth the marginal cost of production, which also implies price smoothing in the context of monopolistic competition.

5 Conclusions

In the past decade, much progress has been made on models studying the impact of information frictions on aggregate supply. However, an assumption in the existing literature is that pricing managers do not interact with production managers within firms. If this assumption is relaxed, nominal shocks would not have real effects on the economy in existing models. Hence, it is not clear why nominal shocks have real effects when prices are flexible and there is perfect communication within firms (input prices and demand are perfectly observed by pricing managers).

In this paper, I present a model with information frictions, output inventories, and perfect communication within firms in which nominal shocks have real effects on the economy. In this model, intermediate goods firms observe aggregate variables with a lag but receive information on their nominal input prices and demand in real time. In this model, inventories helps to explain the non-neutrality of nominal shocks for the following reason: given that firms only observe their nominal input prices and demand, they will accumulate inventories (by producing more) as long as they think that they are facing low real input prices. After a contractionary nominal shock, firms observe lower nominal input prices. They do not know what the source of this change is, but they know that it could be due to a positive productivity innovation or due to a nominal shock. Since positive shocks have a positive probability, firms will increase their stock of inventories. This will prevent firms' prices from decreasing, which will distort relative prices, and will make current profits and households' income go down. As a consequence, aggregate demand and real output fall.

According to my model simulations, a negative nominal shock reduces output by 0.17%

in the first quarter and by 0.38% in the second quarter, followed by a slow recovery to the steady state. Contractionary nominal shocks have also significant effects on investment, which remains 1% below the steady state for the first 6 quarters. Investment responds to an aggregate nominal perturbation by -0.67% in the initial quarter and reaches its trough in the second quarter when it falls by 2.26%. I also find that information frictions make the model more consistent with the empirical evidence on inventory behavior. In the model with information frictions, inventory investment is counter-cyclical; and its standard deviation is closer to the data.

I show that this model does not generate real effects of nominal shocks when there is perfect communication within firms if firms do not accumulate inventories or capital, even when firms have imperfect information about aggregate shocks. In contrast, I show that if firms make investment decisions (capital accumulation or inventory decisions) and if their nominal input prices and demand do not perfectly reveal the aggregate state of nature, the economy exhibits money non-neutrality even under flexible prices and perfect communication within firms (Proposition 3). In those situations, firms need to forecast future aggregate conditions in order to make optimal current decisions. Hence, when current input prices and demand do not perfectly reveal aggregate conditions, firms make forecast errors because their inference about the state of the nature is wrong, and their real decisions deviate from the decision that would have been taken under perfect information.

This paper introduces money non-neutrality by allowing firm to accumulate inventories and not capital as the Proposition 3 also suggests. However, this does imply that inventories are more relevant than capital accumulation for the monetary authority. The relative importance of inventories versus capital accumulation is left for future work. The main point of this paper is that investment decisions are key for money non-neutrality under flexible prices and perfect communication within firms. Similarly, in the spirit of Lucas (1972), this work points out that firms input prices and demand contain noisy but important information about aggregate conditions, and how firms process that information is key for understanding real responses to monetary shocks. The relevant literature, including Angeletos and La'O (2012), abstracts from this signal extraction problem.

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A Tables and Figures

Table 1: Steady State Values.
Model with Perfect and Complete Information

Variable	Value	Description
Y	1.02	Output
C	0.89	Consumption
I	0.13	Inventories
K	8.97	Capital
P	0.69	Price index
W	1.00	Nominal Wage
$\frac{I}{Y}$	0.13	Inventories-Output ratio
$\frac{K}{Y}$	8.80	Capital-Output ratio

Note: This table reports the steady state values for the endogenous model variables in the model with perfect and complete information.

Table 2: Steady State Values.
Model with Information Frictions

Variable	Value	Description
Y	1.02	Output
C	0.89	Consumption
I	0.15	Inventories
K	8.97	Capital
P	0.69	Price index
W	1.00	Nominal Wage
$\frac{I}{Y}$	0.15	Inventories-Output ratio
$\frac{K}{Y}$	8.80	Capital-Output ratio

Note: This table reports the steady state values for the endogenous model variables in the model in which final goods firms do not have information about current aggregate variables.

Table 3: Business Cycles Statistics: Model with Perfect Information

Variable	SD(%)	Relative	Cross Correlation of Output with									Corr
			x(-4)	x(-3)	x(-2)	x(-1)	x	x(+1)	x(+2)	x(+3)	x(+4)	with μ
Output	1.232	1.000	0.009	0.156	0.381	0.665	1.000	0.665	0.381	0.156	0.009	-0.729
Consumption	0.209	0.170	0.420	0.521	0.632	0.722	0.772	0.361	0.045	-0.179	-0.313	-0.129
Capital	0.412	0.334	0.607	0.644	0.636	0.553	0.368	0.022	-0.224	-0.382	-0.467	0.367
Hours	0.670	0.544	-0.175	-0.028	0.214	0.544	0.959	0.705	0.475	0.282	0.149	-0.892
Price level	2.060	1.670	-0.006	-0.088	-0.229	-0.416	-0.622	-0.443	-0.284	-0.166	-0.076	0.495
Inflation	1.630	1.320	0.104	0.179	0.238	0.261	-0.228	-0.201	-0.150	-0.115	-0.090	0.344
Investment	9.116	7.402	-0.058	0.089	0.322	0.624	0.990	0.686	0.423	0.208	0.066	-0.795
Change Inv	0.124	0.101	0.150	0.318	0.502	0.691	-0.078	-0.115	-0.128	-0.101	-0.102	0.298
Real Int rate	0.042	0.034	-0.197	-0.055	0.186	0.515	0.948	0.706	0.484	0.297	0.167	-0.909
Money growth	0.177	0.144	0.439	0.317	0.084	-0.266	-0.729	-0.650	-0.547	-0.439	-0.353	1.000

Note: This table presents the standard deviations, cross-correlation with output, and correlations with the money growth rate after simulating the model with perfect information. The economy was simulated for 2,100 quarters, and the first 100 observations were dropped. The artificial series were logged and then detrended by using the Hodrick-Prescott filter. “Relative” is the relative standard deviation with respect to output. Fixed Inv- Fixed capital investment; Change Inv- Change in inventories; Real Int rate- Real Interest rate.

Table 4: Business Cycles Statistics: Model with Information Frictions

Variable	SD(%)	Relative	Cross Correlation of Output with									Corr
			x(-4)	x(-3)	x(-2)	x(-1)	x	x(+1)	x(+2)	x(+3)	x(+4)	with i
Output	0.979	1.000	-0.007	0.124	0.341	0.562	1.000	0.562	0.341	0.124	-0.007	0.287
Consumption	0.221	0.225	0.274	0.378	0.473	0.558	0.639	0.385	0.087	-0.098	-0.211	0.544
Capital	0.920	0.939	0.211	0.244	0.206	0.160	-0.064	0.086	-0.098	-0.129	-0.156	0.545
Hours	0.974	0.995	-0.105	-0.047	0.106	0.263	0.597	0.231	0.230	0.125	0.068	-0.309
Price level	1.460	1.488	0.017	0.022	-0.030	-0.058	-0.171	0.078	-0.027	-0.010	-0.011	0.553
Inflation	1.37	1.404	-0.005	0.055	0.031	0.119	-0.265	0.112	-0.019	0.001	-0.009	0.236
Investment	7.150	7.308	-0.069	0.055	0.280	0.510	0.983	0.549	0.368	0.163	0.041	0.203
Change Inv	7.430	7.59	0.020	-0.033	-0.015	-0.156	0.262	-0.112	0.016	-0.004	0.010	-0.189
Real Int rate	0.061	0.062	-0.133	-0.047	0.111	0.355	0.544	0.308	0.252	0.145	0.075	-0.262
Money growth	1.586	1.620	-0.047	0.011	0.094	0.179	0.287	0.452	0.411	0.182	0.066	1.000

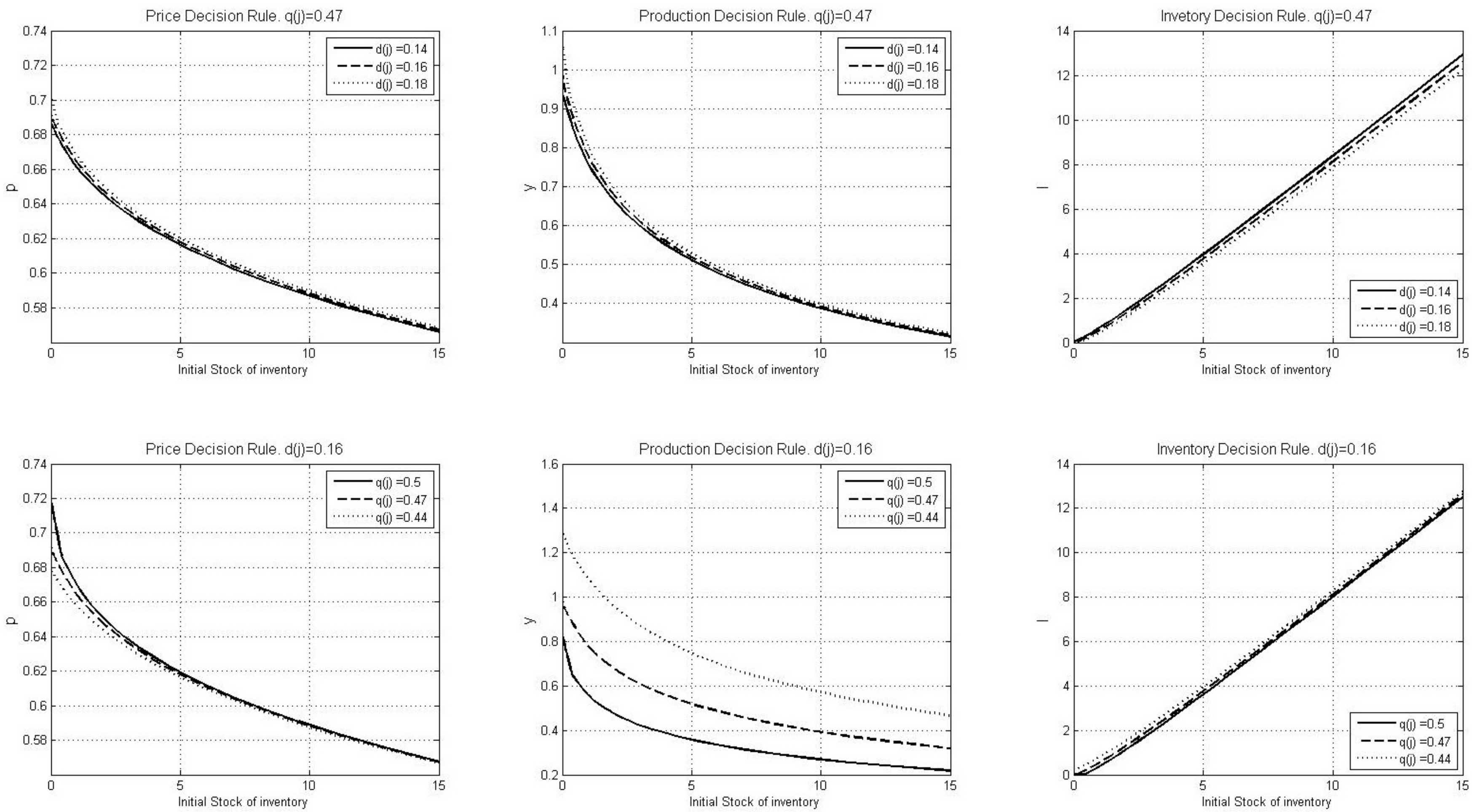
Note: This table presents the standard deviations, cross-correlation with output, and correlations with the money growth rate after simulating the model with information frictions. In this model, final goods firms observe aggregate variables with one period lag. The economy was simulated for 2,100 quarters, and the first 100 observations were dropped. The artificial series were logged and then detrended by using the Hodrick-Prescott filter. "Relative" is the relative standard deviation with respect to output. Fixed Inv- Fixed capital investment; Change Inv- Change in inventories; Real Int rate- Real Interest rate.

Table 5: Price Statistics for a Simulated Final Goods Firm

Model	Mean	St. Dev	First Auto-correlation	Correlation with q(j)	Correlation with d(j)
No Inventories	0.878	0.079	0.031	0.998	-0.067
No Information Frictions	0.851	0.062	0.548	0.673	-0.049
Information Frictions	0.845	0.061	0.551	0.626	0.069

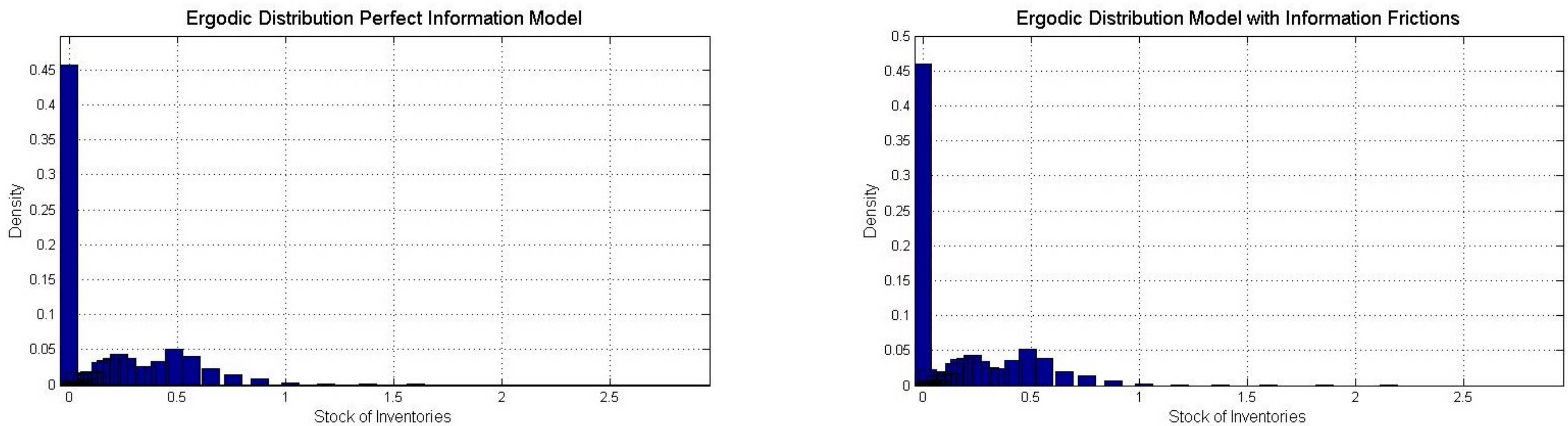
Note: This table present the mean, standard deviation, first autocorrelation, and the correlation with the input price and demand for a final goods firm. No inventories- price charged by a final goods firm in a model in which firms cannot accumulate inventories and have perfect information. No Information Frictions- price charged by a final goods firm in a model in which firms can accumulate inventories and have perfect and complete information. Information Frictions- price charged by a final goods firm in a model in which firms observe aggregate variables with a lag and can accumulate inventories.

Figure 1: Intermediate Goods Firms' Decision Rules



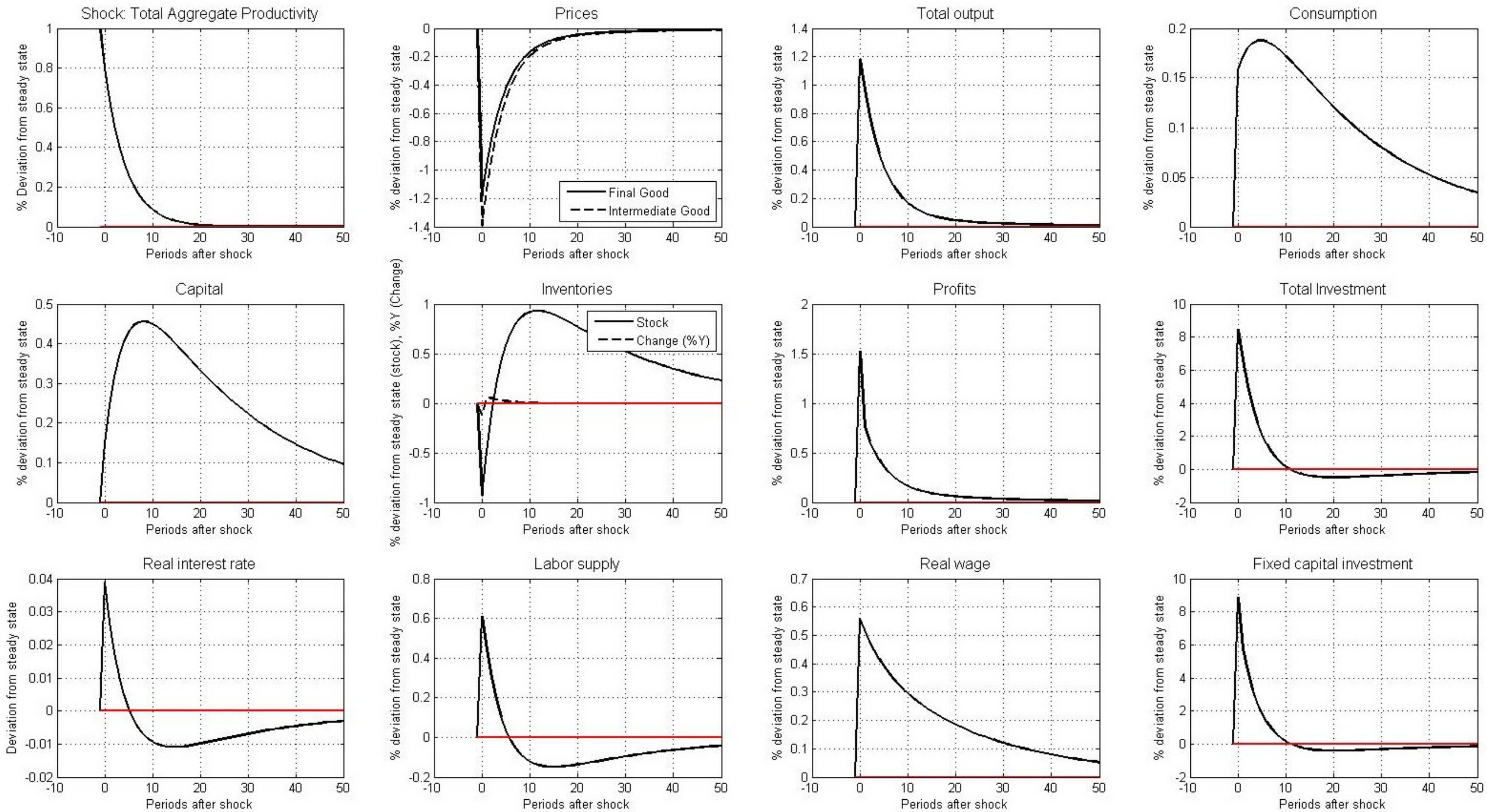
Note: This Figure shows the intermediate goods firms decision rules in steady state in a model with perfect and complete information. q(j)- Nominal input price. d(j)- Nominal demand. p- firm's output price. y- firm's production. I- end of period inventories.

Figure 2: Ergodic Distributions of Inventories



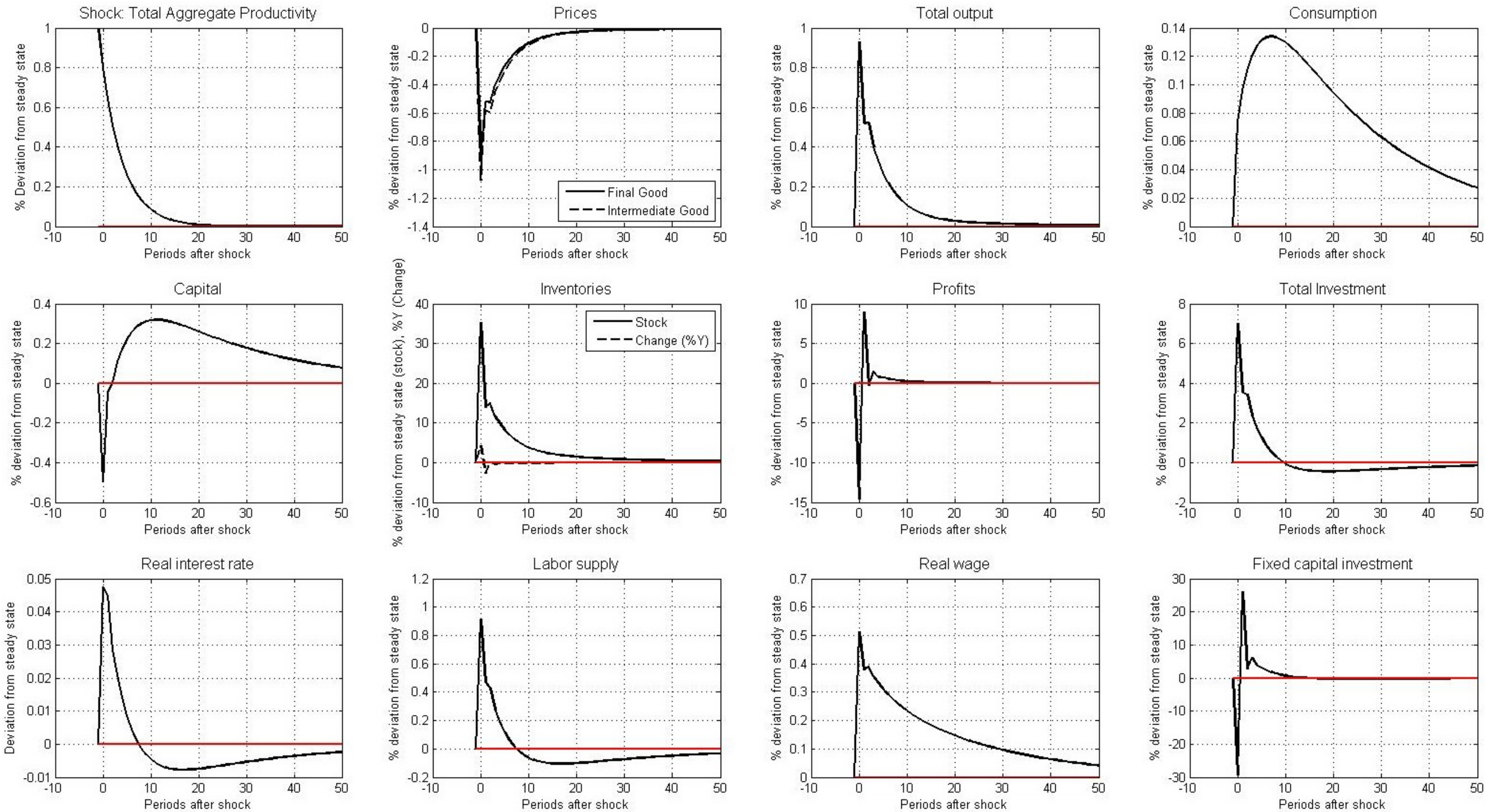
Note: This Figure plots the ergodic distribution of inventories in a model with perfect and complete information (left panel) and in a model with information frictions (right panel).

Figure 3: Impulse Response Functions to a Productivity Shock. Model with Perfect and Complete Information



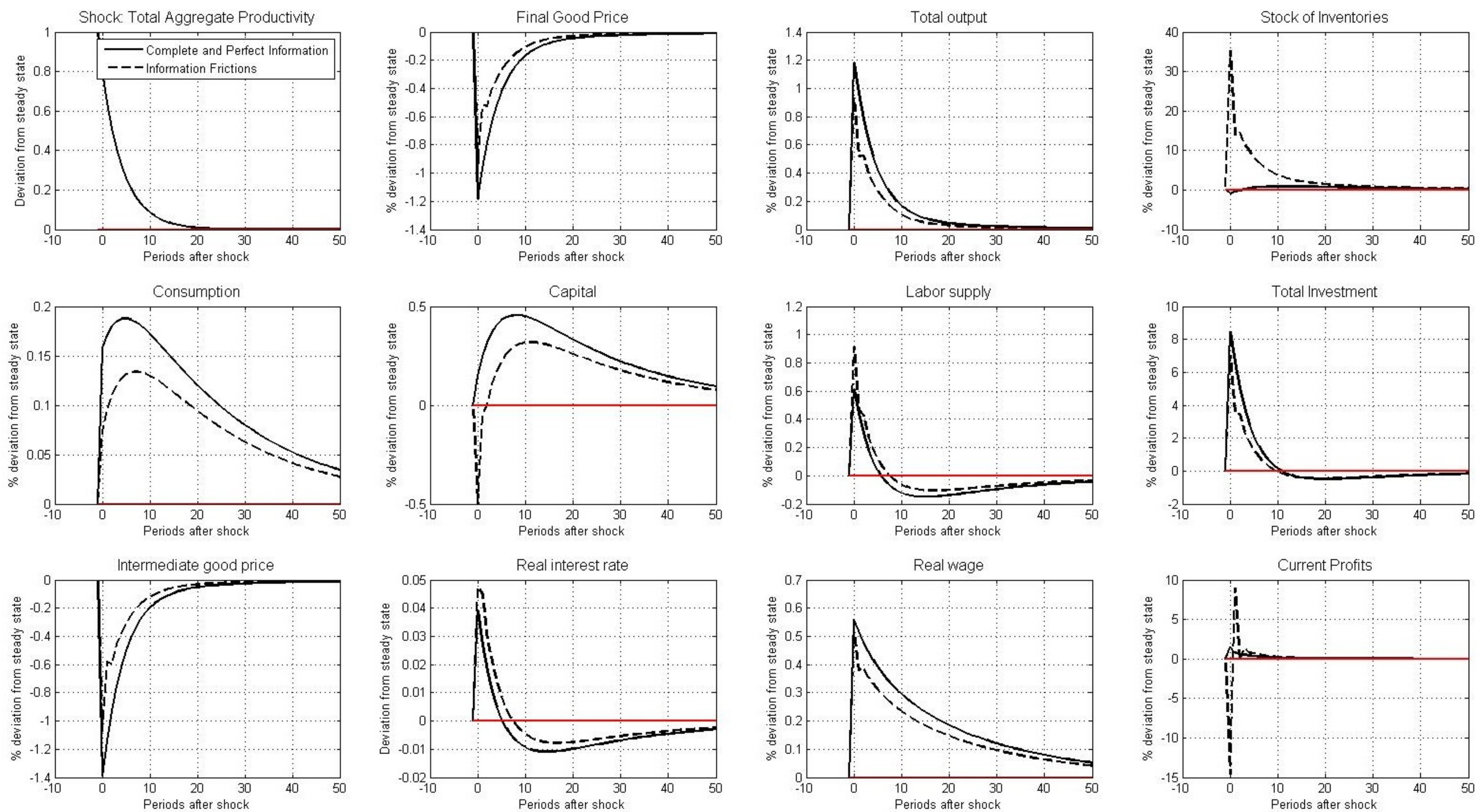
Note: This Figure plot the impulse response functions to a 1% increase in total aggregate productivity, $\bar{b}$ in a model with perfect and complete information. All figures are deviations with respect to the deterministic steady state. Change in inventories is the change in inventories as a fraction of output in steady state. Total investment is the sum of fixed capital investment and inventory investment. Intermediate good price is the average of the input prices.

Figure 4: Impulse Response Functions to a Productivity Shock. Model with Information Frictions



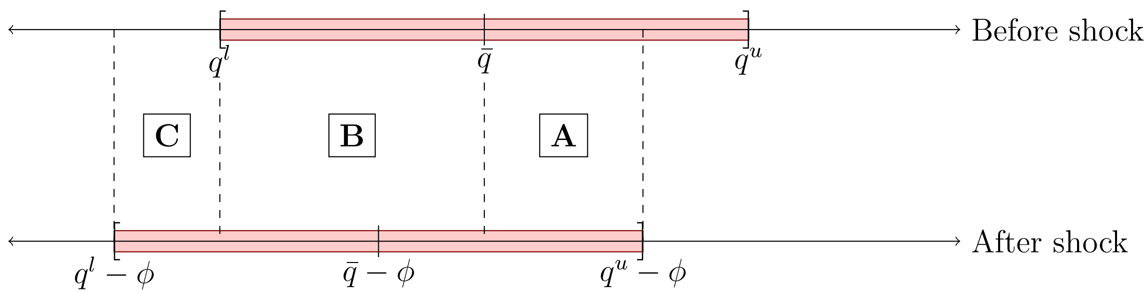
Note: This Figure plots the impulse response functions to a 1% increase in the total aggregate productivity in a model in which final goods firms observe aggregate variables with one period lag. All figures are deviations with respect to the steady state. Change in inventories is the change in inventories as a fraction of output in steady state. Total investment is the sum of fixed capital investment and inventory investment. Intermediate good price is the average of the input prices.

Figure 5: Impulse Response Functions to a Productivity Shock. Model with Perfect and Complete Information Vs Model with Information Frictions



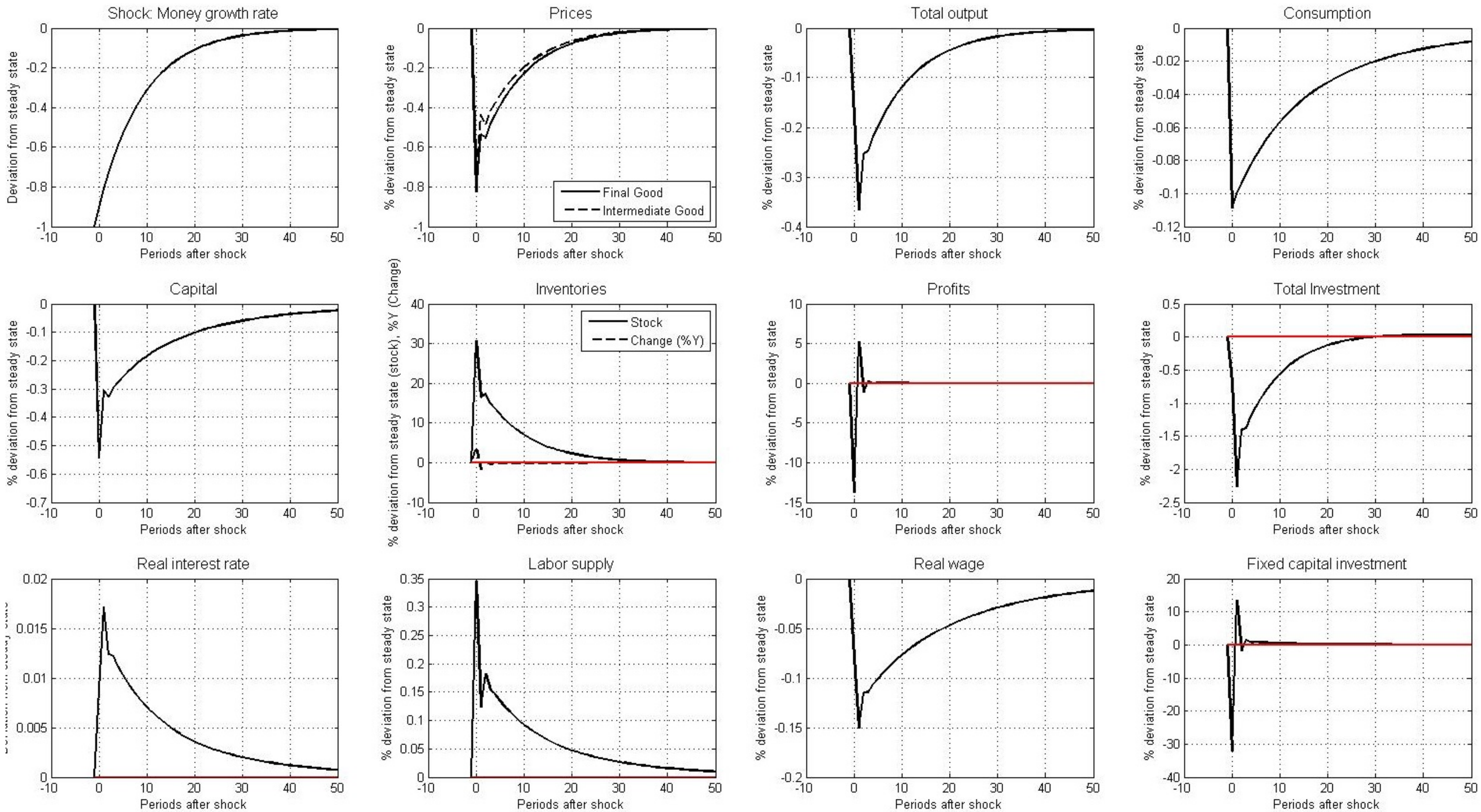
Note: This Figure compares the impulse response functions to a 1% increase in total aggregate productivity for two different models. Complete and Perfect Information- Model with heterogeneous firms and output inventories in which agents have perfect and complete information. Information Frictions- Model with heterogeneous firms and output inventories in which firms observe aggregate variables with one period lag.

Figure 6: Distribution For The Input Price q



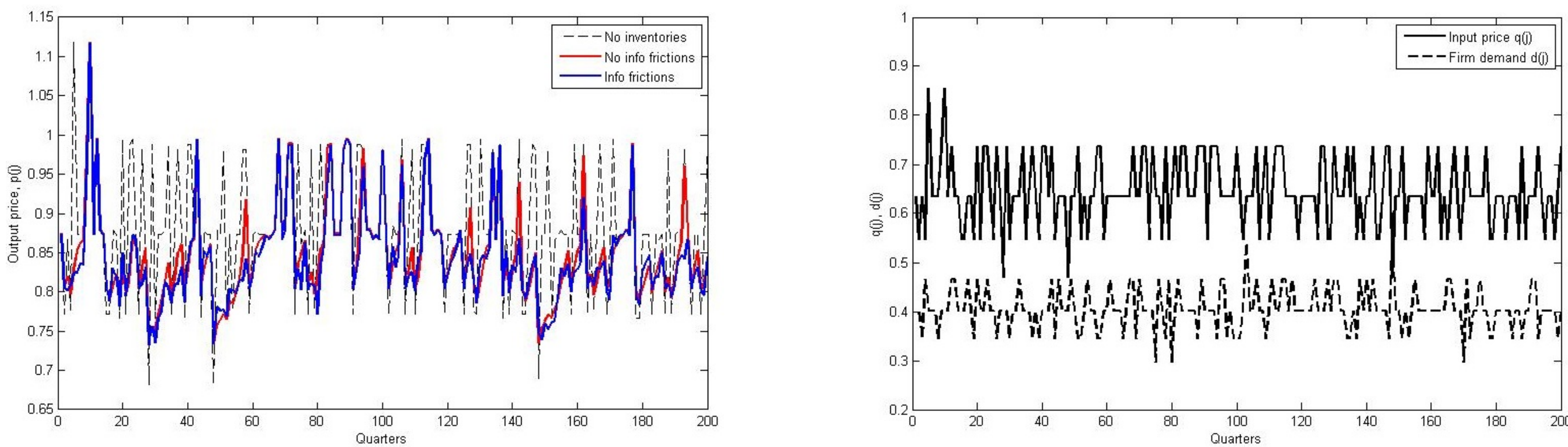
Note: This figure illustrates how the distribution of final goods firms changes after a productivity shock. Assuming that the idiosyncratic cost (φ) distributes uniform, the distribution of the input price is also uniform between $\left[q^l, q^u\right]$ (Before shock). After a productivity shock, the distribution shifts to the left (After shock). Firm in part A and B, do not have enough evidence to conclude that the economy was shocked, and they think that they have more incentives to accumulate inventories. Only firms in part C conclude that the distribution changed.

Figure 7: Impulse Response Functions to a Nominal Shock. Model with Information Frictions



Note: This Figure plots the impulse response functions to a 1% increase in the nominal interest rate in a model in which final goods firms observe aggregate variables with one period lag. All figures are deviations with respect to the steady state. Change in inventories is the change in inventories as a fraction of output in steady state. Total investment is the sum of fixed capital investment and inventory investment. Intermediate good price is the average of the input prices.

Figure 8: Simulated Price for Three Different Models



Note: This Figure plots the simulated output price charged by a particular final goods firms in three different models. The right panel plot the simulated input price and demand. The left panel shows the price charged by the final goods firm. No inventories- The final goods firm cannot accumulate inventories. No info frictions- firm can accumulate inventories and has perfect and complete information. Info frictions- firm can accumulate inventories but observes aggregate variables with one period lag.

B Proofs

B.1 Lemma 1

p_{jt} is strictly decreasing in I_{jt}

Proof. First, notice that $V(I,q,d)^i_t$ is a strictly increasing and concave function in I. Notice that the firm's problem could be written as follows:

$$V(I_0, d_0, q_0)_0 = \max_{\{y_{jt}, I_{jt+1}\}} E_0 \sum_{t=0}^{\infty} Q_{0,t} \left[d_{jt} \left(y_{jt} + I_{jt} - I_{jt+1}\right)^{\frac{\epsilon-1}{\epsilon}} - q_{jt} y_{jt}^{\frac{1}{\gamma}} \right] \tag{82}$$

s.t.

$$I_{jt+1} \geq 0 \tag{83}$$

Since $\epsilon > 1$ and $\gamma \leq 1$, the first term in (83) is strictly concave, and the second term is convex. Hence, this problem is strictly concave. Then, using the envelope theorem, we get that $V(I,q,d)_0$ is strictly increasing in I. Thus, $V(I,q,d)$ is strictly increasing and concave in I. Then, using the envelope theorem:

$$\frac{\partial V(I,q,d)_t}{\partial I_t} = \left(\frac{\epsilon - 1}{\epsilon}\right) p_{jt} > 0 \tag{84}$$

$$\frac{\partial^2 V(I,q,d)_t}{\partial I_t^2} = \left(\frac{\epsilon - 1}{\epsilon}\right) \frac{\partial p_{jt}}{\partial I_{jt}} < 0 \tag{85}$$

□

B.2 Lemma 2

Assuming that $\epsilon > 1$ and that $\gamma \leq 1$, the optimal decision rules for p_{jt} and I_{jt+1} have the following properties:

- The current optimal price (p^*_{jt}) is strictly increasing in the firm's current demand (d_{jt}) and input prices (q_{jt}).

  Proof. By the envelope theorem and the symmetry of the second derivatives, we get that:

$$\frac{\partial p}{\partial d} \propto \frac{\partial^2 V}{\partial d \partial I} = \frac{\partial^2 V}{\partial I \partial d} = p^{-\epsilon} \frac{(1-\epsilon)}{\epsilon} \frac{\partial p}{\partial I} > 0 \tag{86}$$

$$\frac{\partial p}{\partial q} \propto \frac{\partial^2 V}{\partial q \partial I} = \frac{\partial^2 V}{\partial I \partial q} = -\left(\frac{1}{1-\gamma}\right)\left(\frac{y}{p}\right) \frac{\partial p}{\partial I} > 0 \tag{87}$$

□

- The current optimal price (p^*_{jt}) is weakly increasing in the firm's future demand (d_{jt+1}) and input prices q_{jt+1}.

  Proof. Using these results and the optimality condition for inventories (39), we obtain:

$$p_{jt} \geq E\left[Q_{t,t+1} p_{jt+1}\right] \tag{88}$$

  Hence, for $X = \{d_{jt+1}, q_{jt+1}\}$:

$$\frac{\partial p_{jt}}{\partial X} = E\left[Q_{t,t+1} \frac{\partial p_{jt+1}}{\partial X}\right] > 0 \qquad \text{if } I^*_{jt+1} > 0 \tag{89}$$

$$\frac{\partial p_{jt}}{\partial X} = 0 \qquad \text{if } I^*_{jt+1} = 0 \tag{90}$$

□

- The optimal next period's stock of inventories (I^*_{jt+1}) is weakly decreasing in the firm's current demand (d_{jt}) and input prices q_{jt}. Moreover, if the initial stock of inventories is positive ($I_{jt} > 0$), I^*_{jt+1} is strictly decreasing in d_{jt} and q_{jt}.

  Proof. For $X = \{d_{jt}, q_{jt}\}$ and using the optimality condition for inventories (39), we obtain:

$$\frac{\partial p_{jt}}{\partial X} = E\left[Q_{t,t+1} \frac{\partial p_{jt+1}}{\partial X}\right] > 0 \qquad \text{if } I^*_{jt+1} > 0 \tag{91}$$

$$\frac{\partial p_{jt+1}}{\partial X} = 0 \qquad \text{if } I^*_{jt+1} = 0 \tag{92}$$

Hence

$$\frac{\partial p_{jt+1}}{\partial X} \propto -\frac{I_{jt+1}}{X} < 0 \qquad \text{if } I^*_{jt+1} > 0 \tag{93}$$

$$\frac{\partial p_{jt+1}}{\partial X} \propto -\frac{I_{jt+1}}{X} = 0 \qquad \text{if } I^*_{jt+1} = 0 \tag{94}$$

$\square$

- The optimal next period's stock of inventories (I^*_{jt+1}) is weakly increasing in the firm's future demand (d_{jt+1}) and input prices (q_{jt+1}).

  Proof. From the second part of this lemma and for $X = \{d_{jt+1}, q_{jt+1}\}$

$$\frac{\partial p_{jt}}{\partial X} \propto \frac{I_{jt+1}}{X} > 0 \qquad \text{if } I^*_{jt+1} > 0 \tag{95}$$

$$\frac{\partial p_{jt}}{\partial X} \propto \frac{I_{jt+1}}{X} = 0 \qquad \text{if } I^*_{jt+1} = 0 \tag{96}$$

$\square$

B.3 Lemma 3

At the firm level, inventories impose an upper bound for the increase in the firm's price. In particular,

$$1 \geq \mathbb{E}\left[Q_{t,t+1}\frac{p_{t+1}}{p_t}\right] \tag{97}$$

Proof. This comes directly from multiplying both sides of equation (39) by $\epsilon/(\epsilon-1)$ $\square$

B.4 Proposition 1

The set of real allocations $\{C_t,\ K_t,\ I_t,\ Y_t,\ X_t,\ H_t,\ y_{jt},\ h_{jt},\ k_{jt}\}$ and distribution of final goods firms $\{\lambda(I,q,d)_t\}$ that are consistent with the existence of a competitive equilibrium is independent of the path for money.

Proof. Notice that we can re-write the set of equations that describe the competitive equilibrium in a form that does not involve the nominal interest rate. To see this, we need to define the real rental rate of capital $r_t = R_t/P_t$, the real wage rate $w_t = W_t/P_t$, and relative prices $\tilde{p}_{jt} = p_{jt}/P_t$ and $\tilde{q}_{jt} = q_{jt}/P_t$. Also, the stochastic discount factor becomes:

$\tilde{Q}_{0,t} = \beta u'(C_t)/u'(C_0)$. By defining and replacing these variables in the set of equations that describe the competitive equilibrium, we get a system of equations that are independent of the nominal interest rate. $\square$

B.5 Proposition 2

Suppose that all agents in the economy except firms have perfect and complete information. Moreover, assume that intermediate goods producers cannot hold inventories, so their problem becomes:

$$V(q_0, d_0)_0^i = \max_{\{p_t, s_t, y_t\}} E \sum_{t=0}^{\infty} Q_{0,t} \left(p_t s_t - q_t y_t^{\frac{1}{\gamma}} \right) \tag{98}$$

s.t.

$$s_t = d_t p_t^{-\epsilon} \tag{99}$$

$$y_t = s_t \tag{100}$$

If prices are flexible, and if there is perfect communication within firms such that pricing managers perfectly observe their input prices and demand, then nominal shocks do not have real effects on the economy regardless of the information friction on aggregate variables.

Proof. In Proposition 1, I showed that the set of equations that describe the competitive equilibrium under perfect information could be written in a form that does not involve the nominal variables. Since the only equations that change under information frictions are those involving intermediate goods firms, I only need to show that those equations can be written in a form that does not involve nominal variables. First, notice that the intermediate goods firms problem can be re-stated as:

$$V(q_0, d_0)_0 = \max_{\{p_t\}} E_0 \sum_{t=0}^{\infty} Q_{0,t} \left(p_t^{1-\epsilon} d_t - q_t \left(d_t p_t^{-\epsilon} \right)^{\frac{1}{\gamma}} \right) \tag{101}$$

Hence, from the first order condition, we find that:

$$p_t^* = \left[\left(\frac{\epsilon}{\epsilon - 1} \right) \frac{q_t}{\gamma} d_t^{\frac{1-\gamma}{\gamma}} \right]^{\frac{\gamma}{\gamma + \epsilon(1-\gamma)}} \tag{102}$$

And, using the definition of d_t, we have:

$$p_t^* = P_t \cdot \left[\left(\frac{\epsilon}{\epsilon - 1} \right) \left(\frac{q_t}{P_t} \right) \frac{(\chi_t A_t^{\epsilon-1}(C_t + X_t))^{\frac{1-\gamma}{\gamma}}}{\gamma} \right]^{\frac{\gamma}{\gamma+\epsilon(1-\gamma)}} \tag{103}$$

Therefore, the firm's relative price, (p_t^*/P_t), is independent of the nominal variables, and therefore so is the set of allocations that are consistent with the existence of a competitive equilibrium. □

B.6 Proposition 3

Suppose that all agents in the economy except firms have perfect and complete information. If intermediate goods firms can accumulate inventories or capital and their input prices and demand do not reveal the aggregate state of the economy, the economy exhibits money non-neutrality.

Proof. If firms accumulate inventories their problem becomes:

$$V(q_0, d_0)_0 = \max_{\{p_t, s_t, y_t, I_{t+1}\}} E_0 \sum_{t=0}^{\infty} Q_{0,t} \left(p_t s_t - q_t y_t^{\frac{1}{\gamma}} \right) \tag{104}$$

s.t.

$$s_t = d_t p_t^{-\epsilon} \tag{105}$$

$$y_t = s_t + I_{t+1} - I_t \tag{106}$$

$$I_{t+1} \geq 0 \tag{107}$$

And the optimality conditions are given by:

$$p_t^* = \left(\frac{\epsilon}{\epsilon - 1} \right) \left(\frac{q_t}{\gamma} \right) \left[d_t p_t^{*-\epsilon} + I_{t+1}^* - I_t \right]^{\frac{1-\gamma}{\gamma}} \tag{108}$$

$$p_t^* \geq E_t \left[Q_{t,t+1} p_{t+1}^* \right] \tag{109}$$

Notice that the optimal current price depends not only on the firm's current demand and input prices but also on current inventory investment, which according to (110) depends on firm's expectations. Similarly, if a intermediate goods firm can accumulate capital, its problem becomes:

$$V(q_0, d_0, k_0)_0 = \max_{\{p_t, s_t, y_t, x_t, k_{t+1}\}} E_0 \sum_{t=0}^{\infty} Q_{0,t} \left(p_t s_t - w_t y_t^{\frac{1}{(1-\alpha)\gamma}} k_t^{\frac{-\alpha}{(1-\alpha)}} - p_{xt} x_t \right) \tag{110}$$

$$\begin{aligned}
&\text{s.t.} \\
s_t &= d_t p_t^{-\epsilon} && (111)\\
y_t &= s_t && (112)\\
k_{t+1} &= (1-\delta_K) k_t + x_t && (113)
\end{aligned}$$

Where p_{xt} is the price of investment. The optimality conditions are given by:

$$p_t^* = \left[\left(\frac{\epsilon}{\epsilon - 1} \right) \left(\frac{w_t}{\gamma(1-\alpha)} \right) d_t^{\frac{1}{\gamma(1-\alpha)} - 1} k_t^{\frac{-\alpha}{(1-\alpha)}} \right]^{\frac{\gamma(1-\alpha)}{(1-\epsilon)\gamma(1-\alpha)+\epsilon}} \tag{114}$$

$$p_{xt} = E_t \left\{ Q_{t,t+1} \left[\left(\frac{\alpha}{1-\alpha} \right) w_{t+1} \left(d_{t+1} p_{t+1}^{*-\epsilon} \right)^{\frac{1}{\gamma(1-\alpha)}} k_{t+1}^{*\frac{-\alpha}{1-\alpha}} + p_{xt+1}(1-\delta_K) \right] \right\} \tag{115}$$

Notice that in both cases investment decisions depend on firms' expectations. Hence, if firms' expectations under informational frictions are not equal to those under perfect and complete information, firms' decision rules would not be equal.

Using the same notation as in Hamilton (1994) and under these assumption, we can summarize firms' expectations by the following signal extraction problem. Denoting ξ as the vector of aggregate state variables of the economy and y as the vector of contemporaneous variables that a firm perfectly observes (input prices and demand), we get that:

$$\xi_{t+1} = \phi(\xi_t) + \upsilon_{t+1} \tag{116}$$

$$y_t = a(x_t) + h(\xi_t) + w_t \tag{117}$$

Where ϕ, a, and h are non-linear functions, x_t is a vector of observed and exogenous variables, and υ_t and w_t are vector of unobserved i.i.d. shocks. $\upsilon \sim N(0, Q)$, and $w_t \sim N(0, R)$. Hence, this system can be linearized as follows:

$$y_t = a(x_t) + h_t + H_t \left(\xi_t - \hat{\xi}_{t|t-1} \right) + w_t \tag{118}$$

$$\xi_{t+1} = \phi_t + \Phi_t \left(\xi_t - \hat{\xi}_{t|t} \right) + \upsilon_{t+1} \tag{119}$$

Where $\hat{\xi}_{t|j}$ is the expected value of ξ_t given information until period j. Hence, the contemporaneous inference about the aggregate state of the economy $\tilde{\xi}_{t|t}$ is given by:

$$\tilde{\xi}_{t|t} = \hat{\xi}_{t|t-1} + P_{t|t-1}H_t \left(H_t' P_{t|t-1} H_t + R\right)^{-1} H_t' \left[y_t - a(x_t) - h\left(\hat{\xi}_{t|t-1}\right)\right] \tag{120}$$

Notice that under perfect and complete information $\tilde{\xi}_{t|t} = \hat{\xi}_{t|t}$ for all t. Therefore, if $\tilde{\xi}_{t|t} \neq \hat{\xi}_{t|t}$ firms cannot perfectly infer the aggregate state of the nature and, as a consequence, firms' decision rules will deviate from those under perfect and complete information. This occurs when $r + n < r + k + z$ where r is the number of state variables, n the number of perfectly observed variables by a firm, k is the number of non-zero elements in the main diagonal of Q, and z is the number of non-zero elements in the main diagonal of R. In that case, the number of equations $(r + n)$ is greater than the number of unknown variables $(r + k + z)$. In other word, the number of variables observed by firms has to be lower than the total number of aggregate and idiosyncratic shocks that producers face. One should note that $z = 0$ does not guarantee that $\tilde{\xi}_{t|t} = \hat{\xi}_{t|t}$, it only implies that firms can perfectly infer the value of $h\left(\xi_t\right)$

Hence,. $\square$

C Computation of the Model With Perfect and Complete Information

I approximate the model by assuming that the idiosyncratic shocks, φ and χ, and the inventories holdings, I, can only take values on the grids $\Gamma^{\varphi} = \left\{\varphi^1...\varphi^{nb}\right\}$, $\Gamma^{\chi} = \{\chi^1...\chi^{n\chi}\}$, and $\Gamma^I = \{0, I^2...I^{ni}\}$. I find the transition probability matrices Π^{φ} and Π^{χ} for φ and χ using the Tauchen's method. Defining the variable $z \in \Gamma^z = \left\{z^1...z^{nz\equiv nb\times n\chi}\right\}$ such that: $z = z^r$ if $\varphi = \varphi^{ceil(r/n\chi)}$ and $\chi = \chi^{mod(r,n\chi)}$, I specify the time varying distribution matrix $\mathbf{\Lambda}_t$ of size $(ni \times nz)$ such that the row l, column r element represents the fraction of firms in state (I^l, z^r).[13] Following Costain and Nakov (2011), given the decision rule $I(I,z) = arg\max_{\{I'\in R^+\}} V(I',I,z)^i$, inventories holdings are kept on the grid Γ^I by rounding $I(I,z)$ up or down stochastically without changing the mean. Specifically, for each $w \in \{1,2,...nz\}$, define matrix $\mathbf{R^w}$ of size $(ni \times ni)$ as:

$$\mathbf{R^w} = \begin{cases} \frac{I^{l_t(r,w)} - I_t^{*r,w}}{I^{l_t(r,w)} - I^{l_t(r,w)-1}} & \text{in column r, row } l_t(r,w)-1 \\ \frac{I_t^{*r,w} - I^{l_t(r,w)-1}}{I^{l_t(r,w)} - I^{l_t(r,w)-1}} & \text{in column r, row } l_t(r,w) \end{cases} \tag{121}$$

Where

$$I_t^{*r,w} = arg\max_{\{I'\in R^+\}} V(I', I = I^r, z = z^w)^i \tag{122}$$

$$I^{l_t(r,w)} = min\left\{I \in \Gamma^I : I \geq I_t^{*r,w}\right\} \tag{123}$$

Hence, the evolution of $\mathbf{\Lambda}_t$ can be computed as:

$$vec(\mathbf{\Lambda_{t+1}}) = (\Pi^{z\prime} \otimes \mathbf{I_{ni}}) \times \mathbf{R} \times vec(\mathbf{\Lambda_t}) \tag{124}$$

$$\mathbf{R} = \begin{bmatrix} \mathbf{R^1} & \mathbf{0_{ni}} & \cdots & \mathbf{0_{ni}} \\ \mathbf{0_{ni}} & \mathbf{R^2} & \cdots & \mathbf{0_{ni}} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0_{ni}} & \mathbf{0_{ni}} & \cdots & \mathbf{R^{nz}} \end{bmatrix} \tag{125}$$

[13] Being more precise, $\Gamma^z = \left\{(\varphi^1,\chi^1),(\varphi^1,\chi^2)...(\varphi^2,\chi^1),(\varphi^2,\chi^2)...(\varphi^{nb},\chi,1)...(\varphi^{nb},\chi^{n\chi}\right\}$, and its transition probability matrix is given by $\Pi^z = \Pi^{\varphi} \otimes \Pi^{\chi}$

Where $\mathbf{I_{ni}}$ is the identity matrix of size ni.[14] Similarly, the row l, column r element of the pricing, inventory and profit functions ($p(I,z)$, $I(I,z)$, and $\pi(I,z)^i$) are given by:

$$p(I^l,z^r)_t \times I(I^l,z^r)_t = E\left[Q \times p(I(I^l,z^r),z)_{t+1} \times \Pi^z(:,z^r)'\right] I(I^l,z^r)_t \tag{132}$$

$$p(I^l,z^r)_t = \left(\frac{\epsilon}{\epsilon-1}\right)\frac{q(z^r)_t}{\gamma}\left(d(z^r)_t p(I^l,z^r)_t^{1-\epsilon} + I(I^l,z^r)_t - I^l\right) \tag{133}$$

$$\pi(I^l,z^r)_t^i = d(z^r)_t p(I^l,z^r)_t^{1-\epsilon} - q(z^r)_t\left(d(z^r)p(I^l,z^r)_t^{1-\epsilon} + I(I^l,z^r)_t - I^l\right) \tag{134}$$

Where $d(z^r)_t$ and $q(z^r)_t$ are the values of d and q consistent with $z = z^r$. It is worth pointing out that the expectation in equation (133) is over the aggregate shocks of the economy. The expectation over the evolution of z is written explicitly by multiplying by Π^z. Hence, the vector of aggregate variables is given by:

$$\overrightarrow{X}_t \equiv \{vec(\mathbf{\Lambda_t}), K_t, \Pi_t, P_t, D_t, \bar{q}_t, Q_t, Y_t, C_t H_t, r_t, w_t, vec(I(I,z)_t)\} \tag{135}$$

Vector $\overrightarrow{X}_t$ along with the vector of shocks $\overrightarrow{Z}_t = (log(A_t), \mu_t)$ consist of $2(ni \times nz) + 11$

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[14] define $\tilde{\mathbf{\Lambda}}_t$ such that:

$$vec(\tilde{\mathbf{\Lambda}}_t) = \mathbf{R} \times vec(\mathbf{\Lambda}_t) \tag{126}$$

$$\tilde{\mathbf{\Lambda}}_t^{(w)} = \mathbf{R^w} \times \mathbf{\Lambda}_t^{(w)} \tag{127}$$

Where $\mathbf{X}_t^w$ is the column w of matrix $\mathbf{X}_t$, and $\mathbf{0_x}$ is the zeros matrix of size nz. Hence, the row k, column w element of matrix $\tilde{\mathbf{\Lambda}}_t$ represents the fraction of firms in state $z = z^w$ that, regardless of their initial inventories holdings, have an stock of inventories equal to I^k at the end of period t. Therefore, $\mathbf{\Lambda}_{t+1}$ can also be written as:

$$\mathbf{\Lambda}_{t+1} = \tilde{\mathbf{\Lambda}}_t \times \Pi^z \tag{128}$$

$$vec(\mathbf{\Lambda}_{t+1}) = vec(\tilde{\mathbf{\Lambda}}_t \times \Pi^z) \tag{129}$$

$$vec(\mathbf{\Lambda}_{t+1}) = (\Pi^{z\prime} \otimes \mathbf{I_{ni}}) \times vec(\tilde{\mathbf{\Lambda}}_t) \tag{130}$$

$$vec(\mathbf{\Lambda}_{t+1}) = (\Pi^{z\prime} \otimes \mathbf{I_{ni}}) \times \mathbf{R} \times vec(\mathbf{\Lambda}_t) \tag{131}$$

Where $\mathbf{I_{ni}}$ is the identity matrix of size ni.

endogenous variables that are determined by the following system of equations

$$C_t^{-\sigma} = \beta \mathbb{E}\left[\left(r_{t+1} + (1-\delta_K)\right) C_{t+1}^{-\sigma}\right] \tag{136}$$

$$\Psi H_t^{\eta} = w_t C_t^{-\sigma} \tag{137}$$

$$\frac{H_t}{K_t} = \left(\frac{r_t}{w_t}\right)\left(\frac{1-\alpha}{\alpha}\right) \tag{138}$$

$$C_t + K_t = w_t H_t + r_t K_t + \Pi_t / P_t + (1-\delta_K) K_t \tag{139}$$

$$P_t^{1-\epsilon} = \mathbf{e_{ni}}' \left[\chi(z) p(I,z)_t^{1-\epsilon} . * \mathbf{\Lambda_t}\right] \mathbf{e_{nz}} \tag{140}$$

$$\left(\frac{Y_t}{A_t}\right)^{\frac{\epsilon-1}{\epsilon}} = \mathbf{e_{ni}}' \left[\chi(z)^{\frac{1}{\epsilon}} . * y(I,z)_t^{\frac{\epsilon-1}{\epsilon}} . * \mathbf{\Lambda_t}\right] \mathbf{e_{nz}} \tag{141}$$

$$\Pi_t = \mathbf{e_{ni}}' \left[\pi(I,z)_t^i . * \mathbf{\Lambda_t}\right] \mathbf{e_{nz}} \tag{142}$$

$$p(I^l, z^r)_t \times I(I^l, z^r)_t = E\left[Q \times p(I(I^l, z^r), z)_{t+1} \times \Pi^z(:, z^r)'\right] I(I^l, z^r)_t \qquad \forall l, z \tag{143}$$

$$vec(\mathbf{\Lambda_{t+1}}) = (\Pi^{z\prime} \otimes \mathbf{I_{ni}}) \times \mathbf{R} \times vec(\mathbf{\Lambda_t}) \tag{144}$$

$$D_t = A_t^{\epsilon-1} \left(C_t + K_{t+1} - (1-\delta_K) K_t\right) P_t^{\epsilon} \tag{145}$$

$$Q_t = \beta \frac{P_t}{P_{t+1}} \left(\frac{C_t}{C_{t+1}}\right)^{\sigma} \tag{146}$$

$$\bar{q}_t = P_t \left(\frac{r_t}{\alpha}\right)^{\alpha} \left(\frac{w_t}{(1-\alpha)}\right)^{1-\alpha} \tag{147}$$

$$log(P_t) + log(Y_t) = \mu_t \tag{148}$$

Notice that given a inventory decision rule, the price decision rule and current profit are given by equations (134) and (135). Following the notation of Costain and Nakov (2011), this set of equations form a first-order system of the form:

$$\mathbb{E}_t \mathcal{F}\left(\overrightarrow{X}_{t+1}, \overrightarrow{X}_t, \overrightarrow{Z}_{t+1}, \overrightarrow{Z}_t\right) = 0 \tag{149}$$

This system can linearized by computing numerically the jacobian matrices at the deterministic steady state, in order to express this system as a first-order linear expectational difference equation system:

$$\mathbb{E}_t \mathcal{A} \Delta \overrightarrow{X}_{t+1} + \mathcal{B} \Delta \overrightarrow{X}_t + \mathbb{E}_t \mathcal{C} \overrightarrow{Z}_{t+1} + \mathcal{D} \overrightarrow{Z}_t = 0 \tag{150}$$

Where $\mathcal{A} \equiv D_{\overrightarrow{X}_{t+1}} \mathcal{F}^*$, $\mathcal{B} \equiv D_{\overrightarrow{X}_t} \mathcal{F}^*$, $\mathcal{C} \equiv D_{\mathbf{E}_{t+1}} \mathcal{F}^*$, $\mathcal{D} \equiv D_{\mathbf{E}_t} \mathcal{F}^*$. Then this system of equations can be solve using the QZ decomposition describe in Klein(2000).