

# Capital Destruction, Optimal Defense and Economic Growth

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## Abstract

The effects of capital destruction are endogenized in a neoclassical growth model where the economy can optimally allocate part of its labor force to defend capital from being destroyed. The purpose is to explain the optimal allocation of the labor force between productive and deterrence activities along the optimal growth path in response to exogenous terrorist attacks against the material wealth of the economy. The article makes special case to the recent Colombian case and sheds some light on the dramatic increase in the defense activities in Colombia as a result of the increase of terrorist attacks by rebel groups.

**Key words:** capital accumulation, deterrence, capital destruction and economic growth.

**JEL classification:** D23, D74, E21, O41

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*Once the Europeans found themselves reasonably secure  
from outside aggression (eleventh century on),  
they were able, as never before  
and as nowhere else, to pursue their own advantage.  
(Landes, 1998, p. 40)*

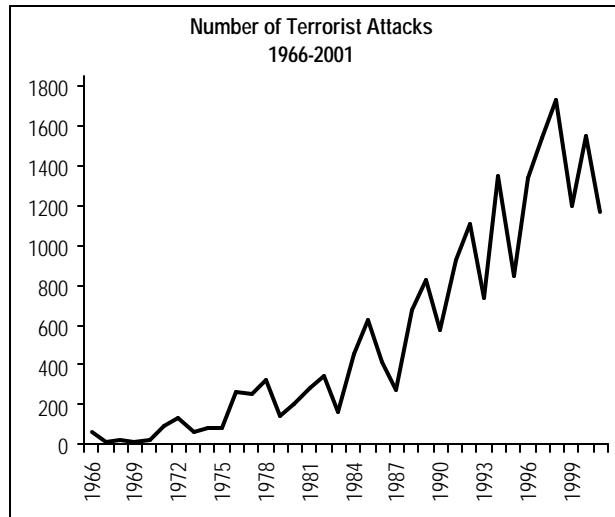
## **1. Introduction**

One of the main problems that Colombia has confronted over the last two decades is the increase of criminality and terrorist attacks by rebel groups. As Barro (2002) wrote recently “... it was clear yet and even clearer now that standard economic issues and the caliber of the country’s economic advisers are sideshows in Colombia. The country’s future is wrapped in issues about guerrilla warfare, drug trafficking, political will to fight terrorism, and the efficiency of the military”. Figure 1 shows the growth in the number of actions classified by the National Police as terrorist attacks<sup>1</sup>.

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<sup>1</sup> Explanations of the increase in criminality in Colombia can be found in: Posada (1994), Montenegro y Posada (1995), Rubio (1999), Montenegro et al. (2000), Sánchez y Nuñez (2001) y Gaviria (2001) and Montenegro y Posada (2001).

**Figure 1**



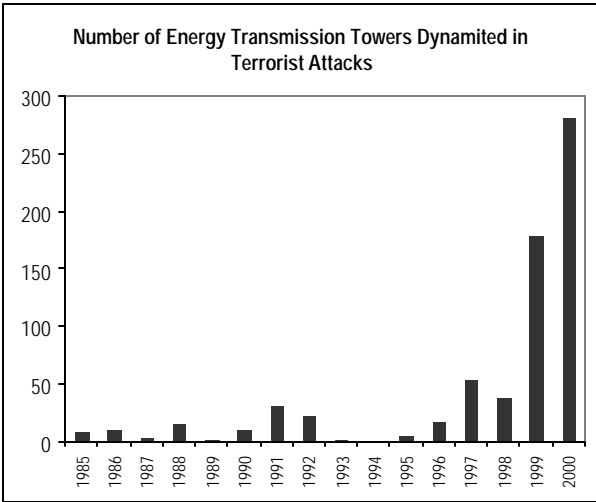
Source: National Police Magazin

In addition to the murders, kidnappings, massacres and extortions perpetrated by the rebel groups, their attacks against the oil infrastructure, the energy transmission towers and the communications nets destroy important components of the material wealth of the country. When the attacks are perpetrated against roads and bridges, they leave vast isolated zones when interrupting the transport by highway of the provisions of foods and necessary goods for its on-speed operation.

One of the main costs of criminality and violence arises from turning aside resources of productive activities towards the defense of the physical capital and the repair of material damages. This deflection does not generate value in itself; it only avoids more capital destruction and the loss of greater production. Collier (1999) identifies three main channels through which civil wars affect the economy: destruction of resources, disruption caused by the suppression of civil liberties and diversion of expenditure from productive activities.

In one of the pioneering studies on the costs of the violence in Colombia, Trujillo and Badel (1998) affirmed that the terrorist actions by the guerrilla groups have been concentrated against the electrical infrastructure. In effect, ISA (the generating and distributing company of energy in Colombia) spent US\$ 18 million during 1999 and 2000 in repairing the energy transmission towers dynamited by insurgent groups; additionally ISA estimated (in March of 2001) that the costs of the repairs still needed were approximately US\$ 4 million. These replacement costs are small compared to the associated opportunity costs of the suspension of the services of transmission and, possibly, generation. Figure 2 shows the number of dynamited towers between 1985 and 2000. As it is seen there, its increase has been remarkable in the last three years (to half of March of 2001 already 73 towers had been dynamited).

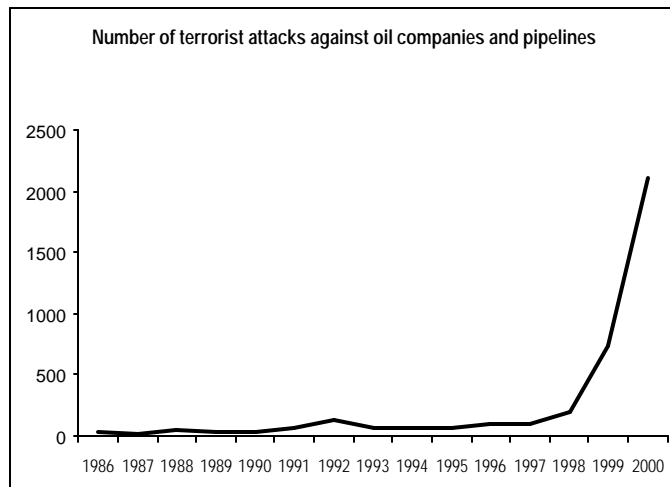
**Figure 2**



Source: ISA

Another sector that has to support high costs due to the terrorist attacks is oil. The number of attacks against oil facilities and pipelines increased tremendously as of 1998 (Figure 3). ECOPETROL (The Colombian National Oil Company) considers the total valuation of the damages by attacks to the pipe line Caño Limon – Coveñas during year 2000 at US\$ 12,3 million, of which US\$ 4 million correspond to repair costs, US\$ 0,5 million environmental decontamination and US\$ 7,8 million to the spilled value of the crude.

**Figure 3**



Source: Division of Corporate Security - Ecopetrol

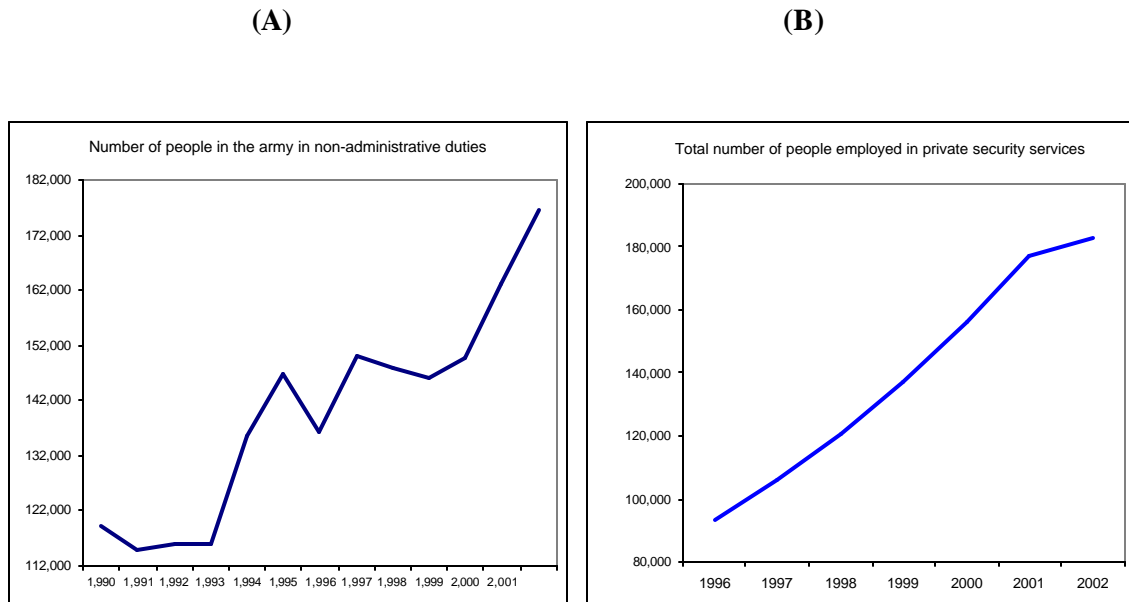
In a recent document, Levitt and Rubio (2000), present a summary of the literature concerning the costs of violence in Colombia. In this document the costs are divided, as in Collier (1999), in three different groups. First it includes those associated to repression and deterrence of criminal activities; the second alludes to the costs of destruction and damages on the physical and human capital, and the third refers to the negative impact that violence has on investment decisions.

In the first group it is possible to emphasize the substantial increase of defense expenses, as much private as public <sup>2</sup>. According to the same study, the estimated public costs of security and justice reached at the present time levels near 5% of the GDP, which implies an increase of nearly 2 percentage points during the 90's. It is possible that the increases in expenditure on public security (and private) and justice have been an answer to the increase of criminal actions by the rebel groups and its operational capacity (Mejía, 2000 and Posada and González, 2001). What has been observed in the last decade is a reallocation of productive resources towards the defense sector, which, as said before, does not generate any value itself. In Figure 4A and 4B are shown the total number of people employed in the National Army and in private security services respectively. While the number of people employed in operational (non-administrative duties) in the National Army increased 48.3% between 1990 and 2002, the increase in the number of people employed in private security companies between 1996 and 2002 is 95.3%.

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<sup>2</sup> Although the information available on this heading is little, there seems to exist evidence that the increase in the number of people employed in private activities of protection has been greater, in the last years, than the observed one in the number of policemen. The same study mentions one of Ospina (1996) according to which the relation policemen/private guards increased from 2,5 in 1980 to 1 in 1995.

**Figure 4**



The response (public as well as private) to the increase in terrorist attacks, kidnappings and criminality has clearly been a substantial increase in resources allocated to defense activities.

The second group of costs, those corresponding to the destruction of capital, not only considers the loss of physical capital derived from the attacks against the oil infrastructure, the energy transmission towers and the road infrastructure, but also the costs of human life lost as a result of the murders, kidnappings and forced displacement of people who decide to leave the country as a result of threats to their life. Guerrero and Londoño (1999), in a study for some countries of Latin America, calculated that the costs of the criminality caused by direct material losses in Colombia are near 6,4% of the GDP, whereas in the other countries, except in Venezuela, whose cost rises to 6,6% of the GDP, the numbers by this concept are

significantly smaller<sup>3</sup>. On the other hand, these authors show that the costs of violence determined by the losses of productivity and investment could be near 2% of GDP. Trujillo and Badel (1998) considered that the net cost of terrorism and loss of lives was around 46% of the total net cost (consolidated) of the violence in Colombia between 1990 and 1996. This was on average, 1,2% of the annual GDP, a much smaller cost compared to that calculated by Guerrero and Londoño but, even so, quite significant. On the other hand, Echeverry et al. (2001) considered that the loss of annual growth of GDP with respect to its long run trajectory could be near 0,5%, a number that seems small, but is significant when it is compounded throughout several years of extraordinary criminality and violence.

In the third group of costs are the ones related to investments forgone due to violence and the uncertainty that it generates. As far as the investment in human capital, different studies indicate that the groups of organized crime recruit young people who, in other cases, could be studying; in addition, they emphasize the effect that insecurity has on the rates of desertion from school. With respect to the physical capital, the empirical evidence is significant about the negative effect that violence has on investment decisions (Alesina et al., 1996). As long as there exists a greater risk the agents diminish their planning horizon and they do not carry out projects that require long periods of maturity. The other possibility is that agents become more impatient and consumption-investment decisions are modified in favor of the former.

In what follows we will refer to a specific cost of violence: the opportunity cost that is derived from the allocation of part of the labor force to an unproductive activity (instead of using it in the production of the final good) that consists of repairing damages and trying to prevent them, in a model which does not fit the possibility of establishing a “strategic game”.

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<sup>3</sup> El salvador (4.9%), México (3.6%), Brasil and Perú (1.4%).



Therefore, the model that will be presented in the next section captures the first two groups of costs that were identified above.

Before presenting the model it seems convenient to make explicit some basic ideas about the relation between criminal destruction, optimal defense and economic growth that, without being original, illustrate the construction of the model and can be defended within the proposed model. The first idea is the following one: all societies face an insecurity problem: somebody, from the inside or the outside, can threaten the life of its members or their material wealth by very diverse causes, between these, plundering, extortion, etc. Therefore, the society must make decisions on the matter: what to do? fight without truce and limit? Or, on the contrary, surrender without resistance? Run the risk of setting an ominous precedent to confront future threats? Combine strategies? The forms that such threats take and the manner in which a society reacts to them have been essential in defining the material and cultural developments of societies. John Stuart Mill, the great philosopher and English economist of the nineteenth century, was brilliant in this subject (as in many others) and he advanced in a quite precise hypothesis that is worth mentioning:

*“ The first of these (the duties of the government) is the protection of person and property... Insecurity of person and property, is as much as to say, uncertainty of the connexion between all human exertions or sacrifice, and the attainment of the ends for the sake of which they are undergone. It means, uncertainty whether they who sow shall reap, whether they who produce shall consume, and they who spare to-day shall enjoy to-morrow. It means, not only that labour and frugality are not the road to acquisition, but that violence is. When person and property are to a certain degree insecure, all the possessions of the weak are at the*

*mercy of the strong. No one can keep what he has produced, unless he is more capable of defending it, than others who give no part of their time and exertions to useful industry are of taking it from him...*

*...Nations have acquired some wealth, and made some progress in improvement, in states of social union so imperfect as to border on anarchy: but no countries in which the people were exposed without limit to arbitrary exactions from the officers of government, ever yet continued to have industry or wealth” (Mill, 1994, chapter VIII, pages 256 to 258).*

From the text it is seen that: a) no society rationally chooses extreme positions of fight or surrender before a permanent threat; b) there exists, in the margin, a trade-off between allocating resources to the defense sector and the alternative use, production of goods and services. It is rational, therefore, to establish some equilibrium in the allocation of these resources; c) that the origin of the State lies in the convenience for the “common” members of society to accept some degree of extortion exerted on the part of powerful in exchange for which, one establishes some degree of relative security in the society, that is to say, some degree of (relative) monopoly of the “rapacity”, d) that most probable, an increasing degree of security has an increasing cost in repression terms and loss of freedom, that beyond certain limits it is able to restrain or even extinguish the engines of social and economic development that, in Mill’s words are “*the energies of free men in union and cooperation*”.

The trade-off identified in part b) above is also identified by Olson (2000) when considering the example of a family that has to decide how to respond to the endemic thievery in an anarchic environment: “*This Family will best serve its interests by allocating its efforts so*

*that the last unit of efforts devoted to protecting against theft has the same return as the last unit devoted to production... ”. Therefore, he concludes, “...this family loses the production that would have come from the resources allocated to guarding the assets” (Olson, 2000, chapter 3, pgs. 63-64).*

Several of these ideas are gathered by Grossman and Kim (1996) and used in their model to describe the conditions of long-term economic dynamics in a predation scene. In this model the trade-off between securing the property (by allocating a fraction of wealth to defense activities instead of productive activities) and tolerating predation is introduced to explain why generations who choose to deter predation accumulate capital more slowly and can grow slower than generations who choose to tolerate predation: *“This result obtains because deterrence requires a large allocation of inherited wealth to defensive fortifications. ...”* (Grossman and Kim, 1996, pp. 345-6).

Our model is less ambitious; it only utilizes a basic idea: before a prolonged or permanent threat of criminal or violent destruction of material capital, the society allocates (rationally) only some fraction of its labor capacity to its defense, allocating the rest to production; such fraction has an optimal level (that is, maximum security is not usually the optimal level), and, through time, this level varies along with the stock of capital of the economy although it depends on other factors such as the degree of “autonomous” efficiency of the defense sector.

The limited power of our model can also be appreciated by the fact that we do not try to make explicit the motivations nor the decisions of those who destroy the capital. On the contrary, our model implies that its actions are exogenous, which means that we do not include the establishment of a “strategic game” between authorities and a criminal or rebel group. This

exclusion could be excessively restrictive in certain cases but justifiable in others like, for example, when the violent actions are perpetrated by several groups or different people, not coordinated with each other (or competing) in such a way that for the authorities it is not feasible to engage in a strategic game with a single group. Yet, our intuition on the matter is the following one: the main conclusions of the model are compatible with those that can be extracted from a more complicated one that describes a strategic game between the authorities and a violent group. If our intuition is correct we would be drawing pertinent conclusions with an extremely simple model.

## **2. The model**

The proposed model is an extension of the neoclassical growth model (Ramsey-Cass-Koopmans) in which a part of the labor force is unproductive (it is not assigned to the production of the final good) and endogenizes the rate of depreciation appearing as the rate at which an external agent to the model destroys the physical capital of the economy. We will assume, to simplify the analysis, that the total population is equal to the labor force of the economy and it does not grow over time, normalizing it to 1. A proportion  $(1 - r_t)$  of the labor force will be assigned to the production of the final good and the remaining part will be assigned to a sector whose objective is to attenuate the destruction of capital. This sector will be called, from now on, the defense sector. The decision that agents in this economy take, by means of a dynamic optimization program, will consider the marginal productivity of the labor force in each one of sectors; this way, the proportion of the assigned labor force to each one of the sectors will be optimal. To simplify the model we do not make the functions of a

government nor the existence of public finances explicit. Somehow, members of society (or their leaders) make decisions on sectorial allocation of the labor force through compulsive mechanisms (among them we could include the taxes or the unavoidable benefit of a service in working time) or of market. The model does not make explicit the mechanism of sectorial allocation of the workers; it simply assumes that whatever is the mechanism it is compatible with the one that would be established by a competitive market economy (and without externalities in the production), as will be seen later. The previous explanation means that it is enough to establish the social objectives and the basic restrictions society confronts (the alternative source of the production and its uses, and the availability and alternative uses of the labor force), without paying attention to the public finances (nor to subjects of credit or money, etc.).

It will be assumed a Cobb-Douglas production function with diminishing marginal productivities in each one of the factors and constant returns to scale together.

$$Y_t = AK_t^a (1 - r_t)^{1-a}; \quad 0 < a < 1 \quad (1)$$

Since we don't have any reason to think that the technical change is more or less intense in the production of the final good than in the defense activity, we will consider that its rate is equal in both activities. This consideration is equivalent to accepting that an equal technical change in each sector is something that lacks importance for the decision of the percentage distribution of the labor force between sectors. The dynamic restriction of capital accumulation that the economy confronts is given by<sup>4</sup>:

$$\dot{K} = Y_t - C_t - d_t K_t \quad (2)$$

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<sup>4</sup> Assuming a closed economy.

$\dot{K}$ ,  $Y$  and  $C$  being the accumulation of capital, output and consumption, respectively. The term  $d_t K_t$  does not refer to the depreciation of capital in the usual sense, and we will make abstraction of this. Such term refers to the destruction of capital derived from criminal activities. This approach is, in some sense, the same as the one in Collier (1998) in which the destruction effect is captured by the rate of depreciation. We will assume that, in absence of deterrence, control and repair actions there would be a certain destruction of capital whose magnitude is exogenous ( $\bar{d}$ ) but that, thanks to such measures, implied by the allocation of a fraction  $r_t$  of the labor force to protection and repair, total destruction or net, by capital unit, is smaller. In other terms, we postulated the following function for the rate of destruction of capital<sup>5</sup>:

$$d_t = \bar{d} - g \frac{r_t}{K_t}; \quad \bar{d} > 0$$

Where  $g$  is the parameter that measures the degree of efficiency of the labor force dedicated to prevent and to repair the damages of the criminal attacks to capital units (given that we assumed the absence of technical change). As well, the parameter  $g$  is divided into two components, one that is independent (presumably derived from technical, organizational, moral factors, etc.), denominated  $\bar{g}$ , and other that depends itself on the rate of exogenous destruction, is to say:

$$g = \bar{g} + g_1 \bar{d}; \quad \bar{g} > 0; \quad g > 0$$

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<sup>5</sup> The modeling of the defense sector is taken from Posada and Gonzalez (2001).

In words, the previous modeling tries to capture the idea that the same criminal destruction can affect the efficiency with which the personnel dedicated to the defense sector manages to protect the capital. Thus, for example, if the criminal attacks lead to improvements in strategy and defensive organization, or elevate the morale of the personnel in this sector the parameter,  $g_1$  would be positive, whereas if the violent attacks generate inefficiency by congestion, disorder or discouragement of the morale, etc., this parameter would be negative. Therefore, the net rate of violent or criminal destruction per capital unit is:

$$d_t = \bar{d} - \left[ \bar{g} + g_1 \bar{d} \right] \frac{r_t}{K_t} \quad (3)$$

Replacing equations (1) and (3) into (2) we have the dynamic restriction of capital accumulation in its extensive form:

$$\dot{K}_t = AK_t^a (1 - r_t)^{1-a} - C_t - \left\{ \bar{d} - \bar{g} \frac{r_t}{K_t} - g_1 \bar{d} \frac{r_t}{K_t} \right\} K_t \quad (4)$$

The preferences of the agents are assumed to depend only on consumption and, for simplicity, take the CRRA form:

$$U(C_t) = \frac{C_t^{1-s} - 1}{1-s} \quad (5)$$

The equilibrium in the final good market is established, as usual, by the equality between the sum of consumption and gross investment and the production of the final good:

$$Y_t = C_t + I_t \quad (6)$$

As usual, the problem a representative agent faces consists of maximizing the present value of the flow of future utilities subject to the dynamic restriction of capital accumulation.

$$\max \int_0^{\infty} e^{-\rho t} U(C_t) dt$$

$$\text{s.t.} \quad \dot{K}_t = AK_t^a(1-r_t)^{1-a} - C_t - \left\{ \bar{d} - \bar{g} \frac{r_t}{K_t} - g_1 \bar{d} \frac{r_t}{K_t} \right\} K_t \quad (7)$$

and,  $K_0 > 0$ .

The details and first order conditions of the maximization problems can be found in the appendix<sup>6</sup>.

To determine the optimal allocation of time between the two sectors one has to take into account the marginal productivity of labor in each one of them. The left side of equation (8) is the marginal productivity of labor in the sector that produces the final good and the right hand side is the marginal productivity in the defense sector. In equilibrium the two should be equal.

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<sup>6</sup> Leisure is not a control variable. In effect, not to excessively complicate the analytical processing of the problem we will assume that the labor day has an exogenous magnitude. An alternative model could incorporate in the utility function leisure and a good denominated "security". The inclusion of leisure and security would imply solving a problem that society faces: to distribute the total time between work, defense and leisure. The inclusion of a denominated good " security " demands to establish the division of the working time in two productive sectors: the one of the final good and the one of security. These characteristics would make, without a doubt, the model more complicated. The important subject would be to know if the additional benefit, measured in terms of the new or different conclusions that are allowed to extract, can justify the additional complications.



$$A(1-\mathbf{a})K_t^a(1-r_t)^{-a} = \bar{\mathbf{g}} + \mathbf{g}_1 \bar{\mathbf{d}} \quad (8)$$

This equation determines, therefore, the allocation of the labor force between both sectors. It will depend on the stock of capital at each moment of time and the efficiency parameters values in each sector. The reason why individuals do not allocate all their time to the production of or the final good is that the marginal productivity of allocating an additional unit of labor to defend the capital stock when this is low (that is to say, when its marginal productivity is high) is greater than the one they would obtain if they allocate it to the production of or the final good. Rearranging terms in equation (8) and log-linearizing one obtains:

$$\frac{\dot{K}_t}{K_t} = \frac{(1-r_t)}{(1-r_t)}$$

In words, in the optimal path, the growth rate of the proportion of the labor force allocated in the final good sector should be equal to the growth rate of the capital stock.

From equation (8) also, one can obtain the optimal allocation of time in the defense sector as a function of the capital stock and the productivity parameters<sup>7</sup>:

$$r_t = 1 - \left\{ \frac{AK_t^a(1-\mathbf{a})}{\bar{\mathbf{g}} + \mathbf{g}_1 \bar{\mathbf{d}}} \right\}^{1/a} \quad (9)$$

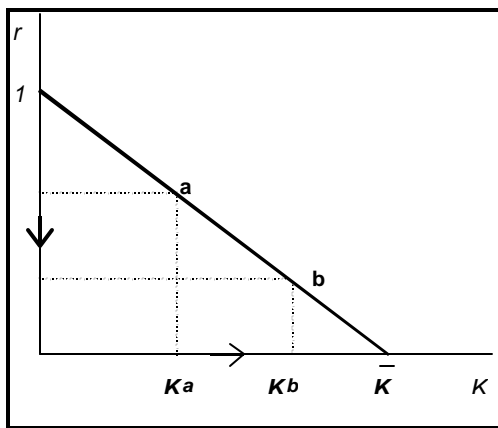
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<sup>7</sup> The linearity of the relation between the optimal defense allocation and the stock of capital comes from the assumption of a linear defense technology.

From where one obtains that:  $\frac{\partial r_t}{\partial K_t} < 0$ , or, the optimal proportion of time allocated to the

defense of capital diminishes as the capital stocks increases (Figure 4A).

**Figure 4A**



The previous analysis relies on the assumption that the defense activity does not use capital whereas the production of the final good does. However, as the neoclassical production function demands, capital accumulation increases the marginal productivity of labor whereas it reduces the marginal productivity of capital in the final good sector, without increasing the productivity of labor in the defense sector. Is because of this that each additional unit of capital is less productive than the previous one and it is less profitable to defend it<sup>8</sup>.

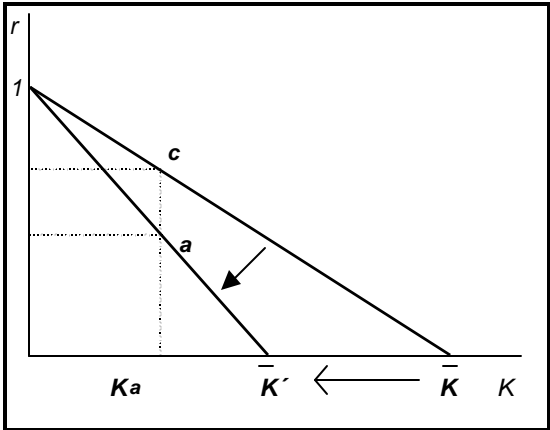
<sup>8</sup> This conclusion heavily relies on the decreasing returns to capital accumulation assumption ( $\mathbf{a} < 1$ ).

Additionally, from equation (9):  $\frac{\partial r_t}{\partial \bar{g}} > 0$ , and if  $g_1 > 0$ , then  $\frac{\partial r_t}{\partial \bar{d}} > 0$ . But in the other

case, if  $g_1 < 0$  then  $\frac{\partial r_t}{\partial \bar{d}} < 0$

Therefore, the optimal degree of defense increases with the efficiency of labor in the defense sector; but, if due to a greater level of criminal destruction efficiency is reduced ( $g_1 < 0$ ), it will turn out to be optimal to increase the personnel dedicated to defense in a smaller proportion than that that would indicate only the effect of capital reduction. If we assume that the first case is true, that is to say that the efficiency of labor in the defense sector increases with increases in the criminal destruction, a permanent decrease of capital destruction makes the proportion of the labor force dedicated to the defense sector decrease faster. Following the same analysis, the capital stock level at which the optimal defense rate becomes zero is smaller when the rate of exogenous destruction diminishes (Figure 4B).

Figure 4B



Finally, the effect of variations in the elasticity of output to the stock of capital is ambiguous: in certain rank, an increase of this parameter reduces the optimal degree of defense; but further on it increases it (Table 1).

**Table 1**

<b>Optimal defense depending on alpha</b>	
<b>a</b>	<b>r</b>
0.3	0.402
0.4	0.178
0.5	0.059
0.6	0.038
0.7	0.109
0.8	0.274
Based on equation 9 with parameter values: A = 1; K = 10 ; $\bar{g} + g_1 \bar{d} = 1.63$	

The optimal consumption path is obtained after some algebraic manipulation of the FOC. As usual in this kind of optimization problems, in order for the individuals to be indifferent between consumption and investment in the equilibrium path, the marginal loss from scarifying one unit of consumption should be equal to the net marginal returns from investment (Equation 10).

$$r + s \frac{\dot{C}_t}{C_t} = AaK_t^{a-1}(1-r_t)^{1-a} - \bar{d} \quad (10)$$

Or, rearranging, the growth rate of consumption on the equilibrium path is (Equation 11)

$$\frac{\dot{C}_t}{C_t} = \frac{1}{s} \left\{ A \mathbf{a} K_t^{a-1} (1-r)_t^{1-a} - \bar{\mathbf{d}} - r \right\} \quad (11)$$

If we replace the optimal fraction of the labor force allocated to the defense sector (equation 9) in equation (11) we get:

$$\frac{\dot{C}_t}{C_t} = \frac{1}{s} \left\{ \frac{A^{1/a} \mathbf{a} (1-\mathbf{a})^{(1-a)/a}}{(\bar{\mathbf{g}} + \mathbf{g}_1 \bar{\mathbf{d}})^{(1-a)/a}} - \bar{\mathbf{d}} - r \right\} \quad (12)$$

The dynamics of the endogenous variables (output, consumption, capital stock and the proportion of the labor force used in the defense sector) can be divided in three stages: a) Stage 1, in which the rate of growth of these variables is constant if the parameters remain constant and if  $r > 0$ , that is to say, while some part of the labor force can be reallocated from the defense sector to the final goods sector; during this phase the dynamics of the model are similar to those ones of the endogenous growth models b) Stage 2 that starts when all the labor force has all been “transferred” to the defense sector; during this stage the growth rate of the endogenous variables is positive but decreasing; this stage is similar to the transition of the model of exogenous growth; and c) steady state.

In fact, if the exogenous rate of destruction does not change ( $\bar{\mathbf{d}}$ ), as the capital stock of the economy increases, the proportion of the labor force allocated to the defense sector ( $r_t$ ) diminishes (equation 9) until it reaches zero (because the marginal productivity of labor in the final good sector increases as the capital stock does). From equation 9, the capital stock level associated with this point is given by equation (13):

$$\bar{K} = \left[ \frac{\bar{g} + g_1 \bar{d}}{A(1-a)} \right]^{1/a} \quad (13)$$

However, this is not the steady state level of capital. Before the economy has reached this level of capital, the economy grows at a constant rate because labor force is being transferred from the defense to the final good sector. When the economy reaches this level, its rate of growth (which we can measure by that of consumption) begins to diminish as the capital stock increases, just as in the standard model of exogenous growth. From equation (11) the rate of growth of consumption from this level of capital on will be given by equation (14):

$$\frac{\dot{c}_t}{c_t} = \frac{1}{s} \left\{ A a K_t^{a-1} - \bar{d} - r \right\} \quad (14)$$

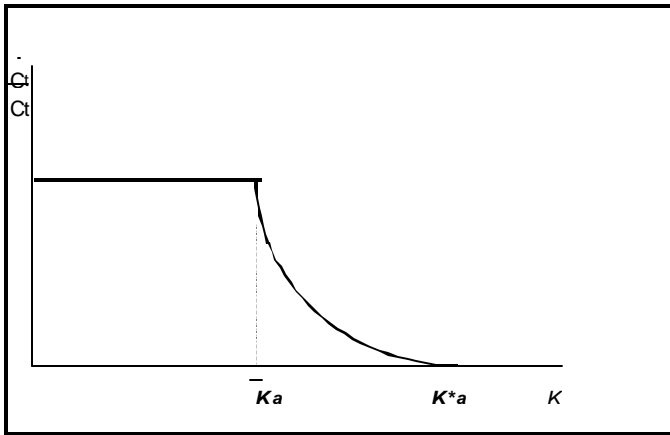
In the absence of technological progress, once the economy reaches the steady state there will be no growth of the endogenous variables. In order to find the steady state level of capital we can make the rate of growth of the consumption equal to zero in the previous equation and, rearranging, we obtain equation (15):

$$K^* = \left[ \frac{Aa}{\bar{d} + r} \right]^{1/(1-a)} \quad (15)$$

This level of capital is equal to the one in the traditional model of exogenous growth (Cass-Koopmans-Ramsey) in the absence of technical progress. As it was already explained, the

rate of growth of consumption will be constant until the level of capital given by  $\bar{K}$  and will diminish thereafter (Figure 5A).

**Figure 5A**

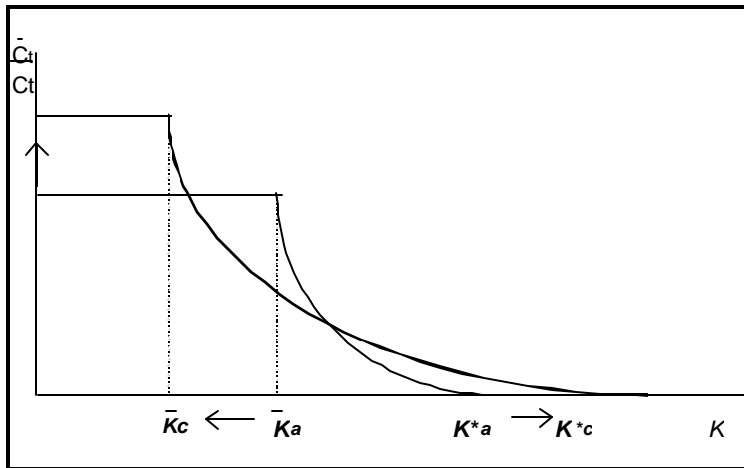


The transition of the economy to the steady state is modified if there is a permanent change in the rate of exogenous destruction (see Figure 2B for the case of a permanent reduction in this rate). On the one hand, the rate of growth of the consumption increases (with respect to a situation in which the rate of destruction is greater) while there exists labor force available to transfer from the defense sector to the final good sector (i.e. before the economy has reached the capital level  $\bar{K}$ ); but, as it was explained before, since the optimal labor force assigned to the defense sector diminishes more quickly when the exogenous destruction rate is smaller, shorter will be the time interval during which the rate of growth of consumption is constant.



This is because the level of capital from which the optimal level of defense is zero will be smaller. On the other hand, the level of capital in the steady state will be greater (Figure 5B).

**Figure 5B**



The model presented is consistent with the empirical findings of Murdoch and Sandler (2001) in the sense that civil wars ( a temporary increase in  $\bar{d}$  in our model) have a consistent and strong negative effect on short term growth rates, because physical capital is destroyed and some (higher) fraction of the population is re-allocated from the productive to the defense sector, but not on longer-term growth rates or steady state values of per-capita GDP, as long as the civil war has a limited duration.

### 3. Simulation of the main results of the model

In order to see the response of the economy to a temporary shocks on the exogenous destruction rate two simplifying assumptions are made. First, we concentrate on the Solow-version of the model (i.e. the version with a constant saving rate) and second, we will work with the discrete time version of the model, specifically the discrete time version of the dynamic restriction of capital accumulation takes the form:

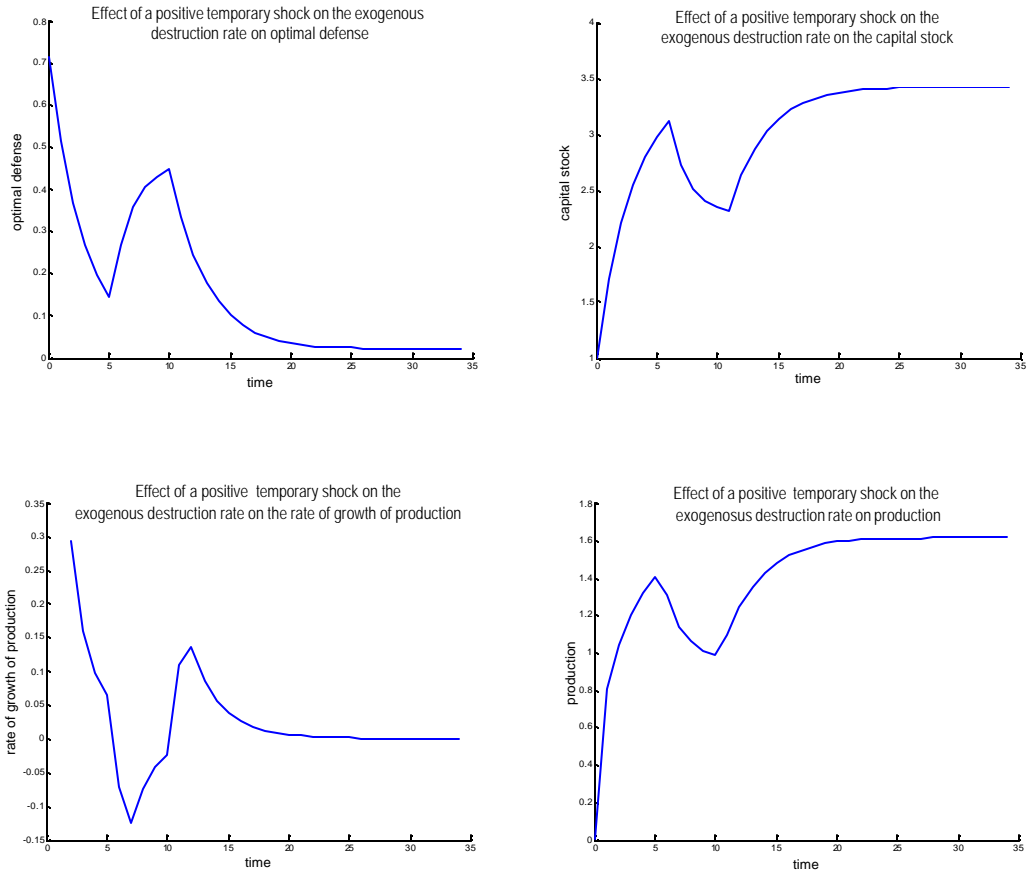
$$k_{t+1} = k_t + s \cdot Ak_t^a (1 - r_t)^{1-a} - \mathbf{d}_t k_t \quad (16)$$

Given that the optimal defense depends itself on the level of the capital stock, for a given level of  $k_0$  and parameter values one can derive the time path of the capital stock. Also, we can infer the response of the optimal defense, the capital stock, output and output growth when a temporary shock to the exogenous destruction rate takes place.

Assuming the following values for the parameters of the model:

$A=1, s=0.2, \mathbf{a}=0.4, \bar{\mathbf{d}}=0.1, \bar{\mathbf{g}}=0.95, \mathbf{g}_1=0.4$ , and an initial value for  $k_0$  we simulate equation (16) for 35 periods applying a positive shock to the exogenous destruction rate between period 6 and period 12. More specifically we want to see the response of some of the endogenous variables of the model to a temporary increase in the exogenous destruction rate. The path for optimal defense, the capital stock, the rate of growth of output and the output level are shown in the panels in Figure 6.

Figure 6



After a positive temporary shock in the exogenous destruction rate, optimal defense jumps up and only starts decreasing after the shock to the destruction rate ends. The capital stock decreases temporarily as a result of the higher rate of exogenous destruction but retakes the path to its steady state consistent with the initial parameters of the model. Perhaps, the most interesting result is with respect to the growth rate of output. Initially, and as predicted by the Solow model, the rate of growth of output is decreasing. When the shock to the destruction rate occurs, two effects settle in: first, higher capital destruction and second, and a rapid

reallocation of the labor force towards defense activities. This two effects work in the same direction and make the rate of output growth decrease very rapidly (in this particular example the rate of growth of the economy becomes negative for a short period of time). After the initial reallocation of labor towards defense activities, the economy starts again transferring labor towards productive activities and the rate of growth of output increases but is still negative. Right after the shock ends, again, two effects settle in: a rapid reallocation of labor towards productive activities (which is consistent with a rapid decrease in optimal defense as shown in the upper-left panel) and a rapid accumulation of capital (upper right panel). This two effects cause a temporary jump in the growth rate of output during a short period of time. The results of the lower right panel describe the time-path of output: during the shock, output decreases but, just after it ends output exhibits a rapid temporary increase.

Even though this is a simple characterization of the response of some macroeconomic variables to a shock on the destruction rate of the capital stock of the economy, the results replicate some of the main observations of the German and Japanese economies after the end of the World War II: a rapid reallocation of the labor force towards productive activities, rapid growth of output per worker and a temporary jump in the growth rate of the economy.

## 4. Conclusions: the implications

Since the basic ideas relative to the model have already been repeated it seems suitable to us to conclude mentioning what, in our opinion, are the main implications of the model. These are the following:

- A) The rich countries (with a high level of capital per worker) would have, *ceteris paribus*, a smaller percentage of their labor force in the defense sector and repair activities of their material wealth. For example, in the poor countries, according to the model, there will be a greater fraction of the population dedicated to defense activities<sup>9</sup>.
- B) When the efficiency of the labor force in the defense sector is greater, the proportion of the labor force (given the level of the capital stock) allocated in this sector will also be greater.
- C) Given the criminal destruction of material wealth, an increase of the labor force allocated to the defense sector must be expected, due to the fall in the capital reducing the marginal productivity of labor in the final good sector. The case of attacks to the physical infrastructure can illustrate the situation in which the labor productivity in the final good sector falls significantly and, therefore, it is justified to increase the personnel dedicated to defense activities.
- D) If the efficiency of the labor force reduces with increases in the destruction of capital ( $g_1 < 0$ ), it may not be optimal to increase the labor force allocated to defense

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<sup>9</sup> As an example, in Latin America, it is common to encounter in some blocks where cars can be parked a young person offering his/her services to guard the car, or, private guards (many times armed) taking care of residential buildings.

activities in the same proportion as the decrease in the capital stock caused by the attacks.

- E) A process of accumulation of capital can, during a long enough period of time, keep the economy growing at a constant rate (somehow replicating a process of endogenous growth, even though the true process is not an endogenous growth process) if the starting point is one in which the fraction of the labor force allocated to the defense sector is high and the stock of capital is relatively low. For example, if the departure point is the end of a war, an internal conflict or a great criminal wave, that had previously forced the society to have a relatively high fraction of the population dedicated to defense activities. Here a conjecture fits: the observed German and Japanese economic “miracles” in the period after II World War can be partially explained by the fact that these societies were able to reallocate to productive activities a much greater fraction of their population in labor age than other societies that, like the United States or the Soviet Union, could and believed it prudent to maintain relatively important defense apparatuses.

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## Appendix

The present value Hamiltonian is given by:

$$(A1) \quad H = e^{-rt} \left[ \frac{C_t^{1-s} - 1}{1-s} \right] + I_t \left\{ AK_t^a (1-r_t)^{1-a} - C_t - \left[ \bar{d} - \bar{g} \frac{r_t}{K_t} - g_1 \bar{d} \frac{r_t}{K_t} \right] K_t \right\}$$

The first order conditions of the intertemporal maximization problem are:

$$(A2) \quad \frac{\partial H}{\partial C_t} = 0 \Rightarrow e^{-rt} C_t^{-s} = I_t$$

$$(A3) \quad \frac{\partial H}{\partial r_t} = 0 \Rightarrow I_t \left\{ -A(1-a)K_t^a (1-r_t)^{-a} + \bar{g} + g_1 \bar{d} \right\} = 0$$

$$(A4) \quad \frac{\partial H}{\partial K_t} = -\dot{I}_t \Rightarrow I_t \left\{ AaK_t^{a-1} (1-r_t)^{1-a} - \bar{d} \right\} = -\dot{I}_t$$

Plus, the transversality condition:

$$(A5) \quad \lim_{t \rightarrow \infty} K_t I_t = 0$$

Rearranging terms in equation (A3) one can obtain equation (8) in the text. From equations

(A2) and (A4) one can obtain equation (10) in the text.