Forecasting the USD/COP Exchange Rate: A Random Walk with a Variable Drift

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Abstract

This study develops three exchange rate models as well as a simple statistical model defined as a random walk with a *variable* drift. The exchange rate models all use the purchasing power parity hypothesis to account for the long-term relationships between prices and the exchange rate, together with error correction models to represent any short-term dynamics. The models are estimated for the USD/COP rate of exchange, and their forecast performance is compared to that of a simple random walk as well as to that of the random walk with a variable drift term. Two of the models are shown to outperform the simple random walk on the 12 and 24-months forecasting horizon. However, all the models are outperformed by the random walk with a variable drift, where the drift term is estimated using a Kalman filter. The results suggest that fundamental models might only be a useful tool for forecasting of the exchange rate in the very long run.

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Contents

1 Introduction

Modelling and forecasting exchange rates using fundamentals is a hazardous activity. In 1983, Richard A. Meese and Kenneth Rogoff wrote their seminal paper, *Empirical Exchange Rate Models of the Seventies: Do They Fit Out of Sample?*,[1] where they showed that the main empirical exchange rate models have inferior out-of-sample forecasting ability compared to naïve models such as a random walk. These rather gloomy results have turned out to be very difficult to overturn, and they have been confirmed by a large number of studies.

Some recent studies, using not only the long-term relationships between fundamental variables and the exchange rate but also the short-term dynamics between the variables represented by an error correction model, have, nevertheless, shown some positive results.[2]

This study develops three such models for the USD/COP exchange rate,[3] and compares their forecasting performance to that of a simple random walk as well as with that of a random walk process with a *variable* drift term. The first two exchange rate models use an Engle, Granger and Hallman (1989) framework, where a long-term cointegrating relationship based on purchasing power parity (PPP) is estimated using quarterly data from 1973 up until 2002. This is combined with an error correction model estimated with data from 1992 up until 2002. For the first model the error correction model is based on a PPP framework, and for the second on the framework of a monetary model. Both these models are shown to outperform a simple random walk on the 12 and 24-month forecasting horizon. The model using monetary data also outperforms the simple random walk on the 6-month horizon.

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[1] Meese and Rogoff (1983a).
[2] See, for example, Kim and Mo (1995), MacDonald and Taylor (1984), Rowland and Oliveros (2003), and Tawadaros (2001).
[3] According to the notation used by foreign exchange markets, USD/COP stands for the rate of exchange between the United States and Colombia expressed as Colombian pesos per U.S. dollar.

The third model uses a Johansen (1988) framework of multivariate cointegration, estimated with quarterly data from 1970 to 2002, using the USD/COP exchange rate, consumer price indices for Colombia and the United States together with an index of the Balassa-Samuelson effect. This model does, however, not manage to beat a simple random walk on any of the forecast horizons.

The paper also develops a statistical process defined as a random walk with a variable drift. The drift variable is estimated using a Kalman filter. When used to forecast the USD/COP exchange rate, this simple model turns out to outperform all the fundamental models on all forecast horizons.

The paper, consequently, confirms the Meese-Rogoff results of 1983, that monetary models do not provide a good framework for short and medium-term forecasting of exchange rates, at least not the USD/COP rate. The results of the study suggest that a random walk with a variable drift is the best way to forecast the USD/COP rate of exchange.

The paper also discusses long-term forecasts of the exchange rate. A long-term PPP relationship is shown to exist, even if this is not a parity relationship in the strong sense. This, together with the support of long-term PPP found in other studies, suggests that purchasing power parity might provide a good framework for long-term forecasts.

Chapter 2 starts by defining the random walk process with a variable drift. Thereafter, the purchasing power hypothesis and the monetary models of the exchange rate are introduced. The chapter also discusses the structural models of inflation and the impact they have on PPP. A brief literature review is also included in the chapter. Chapter 3 contains the empirical analysis and the results. The data set is discussed, the empirical frameworks used are defined, and the estimation results are presented. In chapter 4, the forecasting performance of the models is tested against a naïve random walk as well as against the random walk with a variable drift. Long-term forecasting is also discussed. Chapter 5 concludes the paper.

2 Exchange Rate Models

This chapter discusses the purchasing power parity hypothesis and the monetary models of the exchange rate. Nevertheless, the chapter starts by defining a simple statistical process referred to as a random walk with a variable drift. Section 2.1 defines this process and discusses how it can be used to represent the exchange rate. In section 2.2 the PPP hypothesis is introduced. This is a classical concept in international economics, and if is used as a base for the monetary models used in this paper. The main critique against PPP comes from the structural models of inflation, of which the Scandinavian model is discussed in section 2.3. Section 2.4 discusses how the classical PPP hypothesis can be adjusted for structural inflation. In section 2.5 the main monetary models of the exchange rate are introduced, and section 2.6 contains a short review of the relevant literature.

2.1 A Random Walk with a Variable Drift

The forecasts are compared with a random walk as well as with, what we will call, a random walk with a *variable* drift. The latter is here defined as

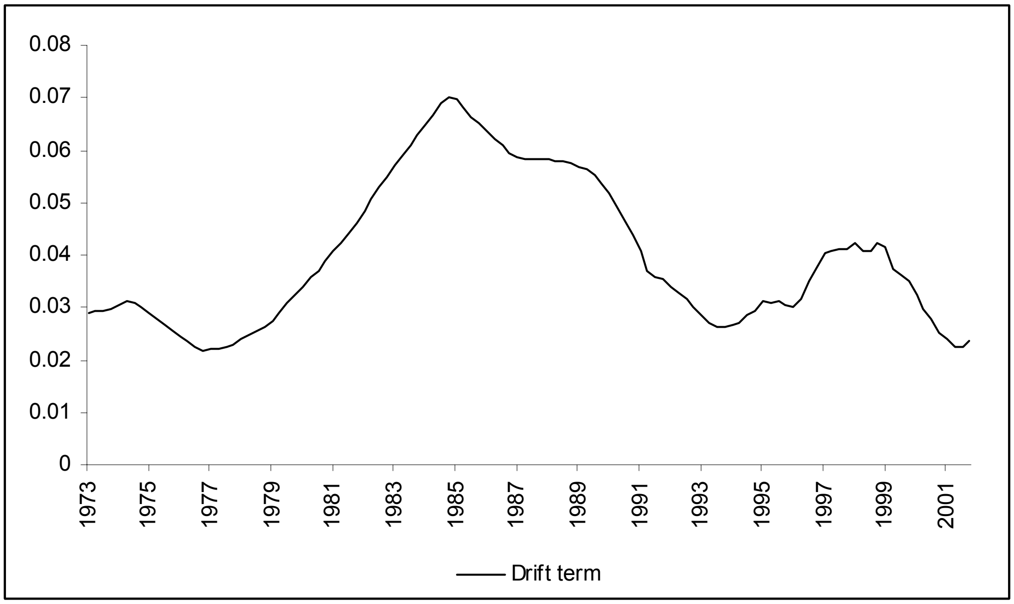
$$s_t = u_t + s_{t-1} + \varepsilon_t \qquad (2.1)$$

where

$$u_t = u_{t-1} + v_t \qquad (2.2)$$

The error terms, ε_t and v_t, are white noise processes and should be independent and normally distributed. The term u_t defines the drift, and s_t is the exchange rate as earlier. If u_t would be a time-independent constant, we would have a normal random walk with a drift. The residual time series, ε_t and v_t, are estimated using a Kalman filter on the time-series data of the exchange rate s, and the drift variable is computed from equation (2.2). All the data is in logarithms as before.

Figure 2.1. The variable drift term



Source: Banco de la República, and own computations

The reason why this statistical representation of the exchange rate was chosen, rather than a normal random walk with a drift, is that the depreciation rate of the USD/COP rate of exchange has changed significantly over time. This is illustrated by figure 2.1, which graphs the drift term. It is apparent that this varies from around 2 percent to around 7 percent, which represents the expected quarterly depreciation of the exchange rate.

2.2 The Purchasing Power Parity Hypothesis

Two alternative forms of purchasing power parity have evolved over time, *absolute* PPP and *relative* PPP.[4] The absolute PPP hypothesis states that the exchange rate between the currencies of two countries should equal the ratio of the price levels of the two countries. In logarithmic form this is written as

$$s = p - p^{*} \tag{2.3}$$

where s is the nominal exchange rate measured in units of domestic currency per unit of foreign currency, p is the domestic price level, and p^{*} is the foreign price level, all variables in logarithms. The relative PPP hypothesis, on the other hand, states that the exchange rate should be proportionate to the ratio of the price level, which, again in logarithmic form, is stated as

$$s = k + p - p^{*} \tag{2.4}$$

where k is a constant parameter. Since information on national price levels normally is available in the form of price indices rather than absolute price levels, absolute PPP may be difficult to test empirically. We will of this reason use relative PPP for the study in this paper, in line with earlier empirical studies.

The PPP hypothesis does not make any general assertion about the direction of causality between the variables. It only states the relationship. Causality between prices and the exchange rate might very well run in both directions. The exchange rate may respond to a change in the ratio of the national price levels, while an exchange rate depreciation might feed inflation.

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[4] The theory, which by now is a classical concept in international economics, dates back to the sixteenth century and the Salamanca School of Spain. However, it was not until 1918 that the Swedish economist Gustav Cassel coined the term *purchasing power parity*. See Cassel (1918). For an historical overview, See Officer (1982), and Griece-Huchinson (1952). For a definition and discussion of purchasing power parity in general, see Dornbush (1982), Isard (1995), and Rogoff (1996).

2.3 The Scandinavian Model of Inflation

The Scandinavian model of inflation is a structural model that was developed and tested by a number of Scandinavian economists in the 1970s.[5] It provides an explanation to why two economies with different productivity growth rate will enjoy a different rate of inflation even if the exchange rate remains constant. In such a case, the classical PPP hypothesis would not hold, and needs to be modified. This should be highly relevant when studying Colombia, since it is a developing economy and should enjoy a considerably higher labour productivity growth rate than that of the United States.

The Scandinavian model of inflation divides the economy into two sectors, of which one is producing tradables, exposed to foreign competition, and the other is producing non-tradables, sheltered from foreign competition.[6] Input-output relations between the two sectors are, furthermore, disregarded, and export and import prices are assumed to move in parallel directions.

The assumption of a small open economy is made, which implies that the economy is a price taker in the world market and, therefore, faces an infinitely elastic excess demand or supply for tradables. The exchange rate is assumed to be fixed and, accordingly, the domestic rate of price increase for tradables π_T has to be the same as the rate of price increase on the world market π_W. This can be written as

$$\pi_T = \pi_W \,. \qquad (2.5)$$

[5] See Aukrust (1977), Branson and Myhrman (1976), Calmfors (1977), Lindbeck (1979), and Maynard and van Rijckeghem (1976). For empirical tests of some of the assumptions and inferences of the model, see Aukrust (1977), Branson and Myhrman (1976), Calmfors (1977), Maynard and van Rijckeghem (1976), and Ringstad (1974).

[6] The economists originally developing the Scandinavian model used the expressions exposed sector for the sector producing tradables and sheltered sector for the sector producing non-tradables. I will, however, here use the distinctions tradables and non-tradables in accordance with, for example, Calmfors (1977) and Lindbeck (1979). In the original Swedish study made by Edgren, Faxén and Odhner (1970) the traded goods sector was defined to include competitive production of raw materials, intermediate products for exports, import competing production and finished goods industry. Maynard and van Rijckeghem (1976) used another division where the sector producing tradables is defined to include agriculture, forestry, fishing, mining and quarrying, and manufacturing. The sector producing non-tradables is then defined to include construction, public utilities, transport and communication, commerce, services, and miscellaneous.

The rate of wage increase in the sector producing tradables $\dot{W}_T$ is equal to the rate of growth of the value of output per worker, which equals the productivity growth rate in the sector producing tradables $\dot{Q}_T$ added to the rate of price increases in this sector.[7] The profit margin in this sector is, consequently, assumed to be constant. This is expressed as

$$\dot{W}_T = \pi_T + \dot{Q}_T \,. \tag{2.6}$$

The productivity growth rate is assumed to be given exogenously.

A fundamental assumption in the structural models is that the growth rate of wages is uniform throughout the economy. This implies that the rate of wage increase in the sector producing non-tradables $\dot{W}_N$ is given by the rate of wage increase in the sector producing tradables $\dot{W}_T$ and, therefore,

$$\dot{W}_N = \dot{W}_T \,. \tag{2.7}$$

In the sector producing non-tradables, prices are determined by a constant mark-up on unit labour costs. In this way the profit margin will be constant. The rate of price increase π_N in this sector is obtained as the difference between the rate of wage increase $\dot{W}_N$ and the rate of productivity increase $\dot{Q}_N$ which is assumed to be given exogenously. This can be written as

$$\pi_N = \dot{W}_N - \dot{Q}_N \,. \tag{2.8}$$

Since the rate of productivity increase is assumed to be lower in the sector producing non-tradables than in the sector producing tradables, the rate of price increase will be

[7] In this section, the percentage rate of change over time of a variable is indicated by a dot over the variable. The different sectors are indicated in subscript.

higher for non-tradables than for tradables. The total rate of increase in consumer prices is finally determined by the weighted average of the rate of price increase in the two sectors,

$$\pi = a\ \pi_N + (1 - a)\ \pi_T \tag{2.9}$$

where a is the weight of non-tradables in the consumer price index. This weight is assumed to be constant over time.

The system of these five equations (2.5) to (2.9) describes the Scandinavian model. By obvious substitutions this can also be written on the reduced forms

$$\pi_N - \pi_T = \dot{Q}_T - \dot{Q}_N \tag{2.10}$$

or

$$\pi = a(\dot{Q}_T - \dot{Q}_N) + \pi_W \tag{2.11}$$

where the latter is referred to as the Aukrust equation.[8]

This implies that the rate of domestic inflation equals the rate of international price increase for tradables added to the weighted difference in the rate of productivity increase between the two sectors. Consequently, an increase in the rate of world inflation as well as a greater difference between the rates of productivity growth in the two sectors will lead to a higher rate of domestic inflation. Furthermore, a higher rate of productivity growth in the sector producing tradables will lead to a rise in the overall inflation rate, while a higher rate of productivity growth for non-tradables will lead to a fall in the inflation rate. Since the difference between the rates of productivity growth does not have to be the same in all countries, the model also permits different inflation rates in different

[8] See Lindbeck (1979), and also Aukrust (1977).

countries even if exchange rates are fixed. More specifically, countries with a large difference between the rates of productivity growth in the two sectors will experience a high rate of inflation relative to the rest of the world, and vice versa.

The first term in equation (2.11), the weighted difference in the rates of productivity increase, reflects what may be called *structural inflation*, while the second term, the rate of international price increase for tradables, is said to express *imported inflation*.[9] In this way the long-term inflation rate in a small open economy can be divided into structural and imported inflation.

Critical to the validity of the Scandinavian model are the controlling mechanisms postulated by equations (2.6), (2.7) and (2.8). The question is whether such mechanisms exist and how exact the relationships dictated by them are. Concerning the controlling mechanisms described by equation (2.7) and (2.8), there should not be too much doubt, at least not for the Scandinavian countries. Data from Norway and Sweden has shown remarkable stability over time of the relation between wages in the two sectors. It has also shown that profit shares in the sector producing non-traded goods have remained relatively stable over time. Profits of industries producing tradables, on the other hand, have fluctuated considerably. The relationship postulated by equation (2.6) does, therefore, not hold in the short run. It might, however, hold as a long-term tendency, even if some doubt still remains.[10] The Scandinavian model is, consequently, not a short-term model, but might be helpful in explaining the long-term trend of inflation.

The model can easily be extended to an economy with a flexible exchange rate. Equation (2.5) should, in a two-country model with a flexible exchange rate, be altered to

$$\pi_T = \pi_T^* + \dot{S} \tag{2.5'}$$

[9] See Lindbeck (1979), p. 44.
[10] See Calmfors (1977).

where π_T^* is the price increase of tradables in the foreign economy denoted in the foreign currency, and $\dot{S}$ is the nominal exchange rate appreciation or depreciation between the foreign and the domestic currency. The Aukrust equation can now be rewritten as

$$\pi = a(\dot{Q}_T - \dot{Q}_N) + \pi_W + \dot{S} \tag{2.11'}$$

2.4 Purchasing Power Parity and the Balassa-Samuelson Effect

As explained by the Scandinavian model of inflation, two economies with different rates of labour productivity growth will normally enjoy different inflation rates even when the exchange rate does not change. In such a case, the classical PPP hypothesis does not hold, but has to be adjusted for the different rates of labour productivity growth.

If output in the tradable and non-tradable sectors is defined by Cobb-Douglas production functions,[11]

$$Y_T = A_T\, L_T^{\theta}\, K_T^{1-\theta} \tag{2.12}$$

$$Y_N = A_N\, L_N^{\delta}\, K_N^{1-\delta} \tag{2.13}$$

where Y is output, L is labour, K is capital, A represents the effectiveness of labour,[12] and θ and δ represent the labour intensity of production in each of the sectors. As in the previous section, we assume labour to be perfectly mobile between the sectors, which implies nominal wage equalisation,

$$W_T = W_N \tag{2.14}$$

[11] As in the previous section, the tradable and non-tradable sectors are indicated by subscripts.
[12] This is sometimes also said to represent knowledge. See Romer (1996) for a further discussion on the Cobb-Douglas production function.

As earlier discussed, the profit margin in the two sectors is assumed to be constant, and workers are paid the value of their marginal product, which is expressed as

$$\frac{\partial Y_j}{\partial L_j} = \frac{W_j}{P_j}; \qquad\qquad j = T, N \tag{2.15}$$

It can easily be shown that the ratio of marginal productivities is proportional to the ratio of average productivities under Cobb-Douglas production technology, i.e.

$$\frac{\partial Y_T / \partial L_T}{\partial Y_N / \partial L_N} = \frac{\theta Y_T / L_T}{\delta Y_N / L_N} \tag{2.16}$$

Inserting (2.14) and (2.15) into (2.16) yields

$$\frac{P_N}{P_T} = \frac{\theta Y_T / L_T}{\delta Y_N / L_N} = \frac{\theta}{\delta}\frac{Q_T}{Q_N} \tag{2.17}$$

where labour productivity Q is defined as output Y divided by labour L. Assuming that labour intensity is equal in the two sectors, i.e. that θ and δ are equal, and expressing equation (2.17) in logs, we have

$$p_N - p_T = q_T - q_N \tag{2.18}$$

In line with the Scandinavian model we assume the price level in the economy to be equal to the weighted average of the price levels in the two sectors, that is

$$p = a\, p_N + (1 - a)\, p_T \tag{2.19}$$

For the foreign economy this equation will be

$$p^* = a\, p_N^* + (1 - a)\, p_T^* \tag{2.20}$$

if we assume that the weight of non-tradables a is the same as in the domestic economy. In line with the Scandinavian model, we assume PPP between prices in the tradable sectors of the two economies, which is stated as

$$s = k + p_T - p_T^{*} \tag{2.21}$$

Equation (2.21) can, together with equation (2.19) and (2.20), be written as

$$s = k + p - p^{*} - a\, p_{NT} \tag{2.22}$$

where

$$p_{NT} = (p_N - p_T) - (p_N^{*} - p_T^{*}) \tag{2.23}$$

is called the Balassa-Samuelson effect after two seminal papers by Balassa (1964) and Samuelson (1964), which laid the ground for the structural models of inflation. If equation (2.18) is inserted in equation (2.23) the Balassa-Samuelson effect can also be expressed in terms of labour productivity differentials, i.e.

$$p_{NT} = (q_T - q_N) - (q_T^{*} - q_N^{*}) \tag{2.24}$$

The advantage of expressing PPP as equation (2.22) rather than (2.21) is that, if it is used to forecast the exchange rate, it is normally easier to forecast consumer prices and the Balassa-Samuelson effect then to forecast prices of tradable goods. Forecasting the latter generally boils down to forecasting the exchange rate itself.

2.5 The Monetary Models of the Exchange Rate

Three main types of monetary models have emerged over time. These are the flexible-price monetary (Frenkel-Bilson) model, the sticky-price monetary (Dornbusch-Frankel) model, and the sticky-price asset (Hooper-Morton) model.[13] A reduced form specification of all three models, that has been widely employed when testing these models is stated as

$$s = a_1 + a_2\,(m - m^*) + a_3\,(y - y^*) + a_4\,(r - r^*) + \\ + a_5\,(\pi - \pi^*) + a_6\,CCA + a_7\,CCA^* + \varepsilon_t \qquad (2.25)$$

where s, as before, is the exchange rate, m is the money supply, y is output, r represents interest rates, π is the rate of inflation and CCA is the cumulative current account. Many studies have used the cumulative trade balance instead of the cumulative current account, and others have used different measures of net foreign assets.

All the models assume that the exchange rate exhibits first-degree homogeneity in the relative money supplies, i.e. a_2 is assumed to be equal to one. The Frenkel-Bilson model, which assumes PPP, constrains a_5, a_6 and a_7 to zero. The Dornbusch-Frankel model allows for slow domestic price adjustment and, therefore, departure from PPP. In this model a_6 and a_7 are constrained to zero. In the Hooper-Morton model, on the other hand, none of the coefficients are constrained to be zero. This model extends the Dornbusch-Frankel model in that it allows for changes in the long-term real exchange rate.[14]

[13] See Bilson (1978, 1979), Frenkel (1976), Dornbusch (1976), Frankel (1979, 1981), and Hooper and Morton (1982).
[14] Meese and Rogoff (1983a).

There is an extensive theoretical literature discussing these models, and a derivation or further discussion of them is outside the scope of this paper.[15] The model that will be used as a base for one of the studies in this paper, is the Frenkel-Bilson model, which can be stated as

$$s = a_1 + a_2\,(m - m^*) + a_3\,(y - y^*) + a_4\,(r - r^*) + \varepsilon_t \tag{2.26}$$

2.6 Brief Review of the Literature

In their seminal papers, Meese and Rogoff (1983a, 1983b) reported the results of an extensive set of thoughtfully formulated tests, in which existing empirical models failed to significantly outperform a random walk model in out-of-sample forecasts of exchange rates. Their study focused on exchange rates of the U.S. dollar against the Deutsche mark, the British pound and the Japanese yen, as well as the nominal effective exchange rate of the United States. The procedure that Meese and Rogoff used to construct out-of-sample forecasts has since then become the accepted methodology. Their results have been confirmed by an extensive number of studies.[16]

A number of recent studies have employed the Johansen framework of multivariate cointegration to estimate the monetary models and to forecast out of sample. Some of these studies have yielded positive results. MacDonald and Taylor (1984) use an unrestricted monetary model to forecast the exchange rate of the U.S. dollar to the British pound. The model is shown to outperform the random walk. Tawadaros (2001) presents similar results from studying the Australian dollar to the U.S. dollar exchange rate. Kim and Mo (1995) studies the U.S. dollar to the Deutsche mark exchange rate, and their results are also positive.

[15] See Isard (1995) for a derivation and a discussion of these models.
[16] See, for instance, Isard (1995) for a discussion and a review.

A study of the Colombia nominal effective exchange rate by Rowland and Oliveros (2003) uses a PPP based model in combination with a framework of multivariate cointegration. Also in this case the forecasts of a random walk are outperformed.

When it comes to purchasing power parity, this has been tested in a large number of studies, and empirical evidence strongly confirms that PPP is not a valid hypothesis about the relationship between nominal exchange rates and national price levels in the short term.[17] The main theoretical argument against the validity of long-term PPP comes from the structural models of inflation, which have been discussed in the previous sections. In the long term, PPP has, nevertheless, received considerable empirical support. Flood and Taylor (1996) shows that cross-sectional data yields very high correlations between changes in nominal exchange rates and relative national price levels over 10- or 20-year horizons. A number of studies from the mid 1980s and onwards have also tested if divergence from PPP between national price levels can be explained in terms of the Balassa-Samuelson effect.[18] The literature does, however, not provide a unanimous agreement on how to interpret the evidence.[19]

[17] See, for example, Isard (1995) for a discussion.

[18] See, for example, Edison and Klovland (1987), and Marston (1987). See also Froot and Rogoff (1995) for an extensive discussion.

[19] Froot and Rogoff (1995) argue that the Balassa-Samuelson effect may be relevant in the medium run, but that the spreading of knowledge, together with the mobility of physical as well as human capital, generates a tendency toward absolute PPP over the very long run.

3 Estimation of the Models

In this chapter, three empirical exchange rate models are defined and estimated using the USD/COP rate of exchange together with fundamental time-series data from Colombia and the United States. The data used is discussed in section 3.1. One obstacle to the analysis is the structural breaks created by the changes in exchange rate regimes, which are discussed in section 3.2. In section 3.3, a long-term purchasing power relationship is estimated, and in section 3.4 the estimation of two different error correction models is discussed. The analysis so far is based on the Engle, Granger and Hallman (1989) framework. Section 3.5 continues by introducing the Johansen (1988) framework of multivariate cointegration, which is used for the estimation of the third model, presented in section 3.6.

3.1 The Data Set

For the empirical analysis we use quarterly data up until the second quarter 2002. Most of the time-series used are available from 1970 onwards. Table 3.1 lists the span of the time-series together with the data sources. For the exchange rate, end-of-period data is used. Industrial production has been used as a proxy for output, and as a measure of the money supply, the M3 aggregate is used. We are choosing a broad monetary aggregate, to avoid the instability and the influence of financial innovation associated with narrower aggregates. All the data is, furthermore, seasonally adjusted, apart from exchange rate and interest rate data.[20]

[20] For seasonal adjustment an additive X11 process is used, apart from the multivariate cointegration analysis in section 3.6, where seasonal dummy variables are used.

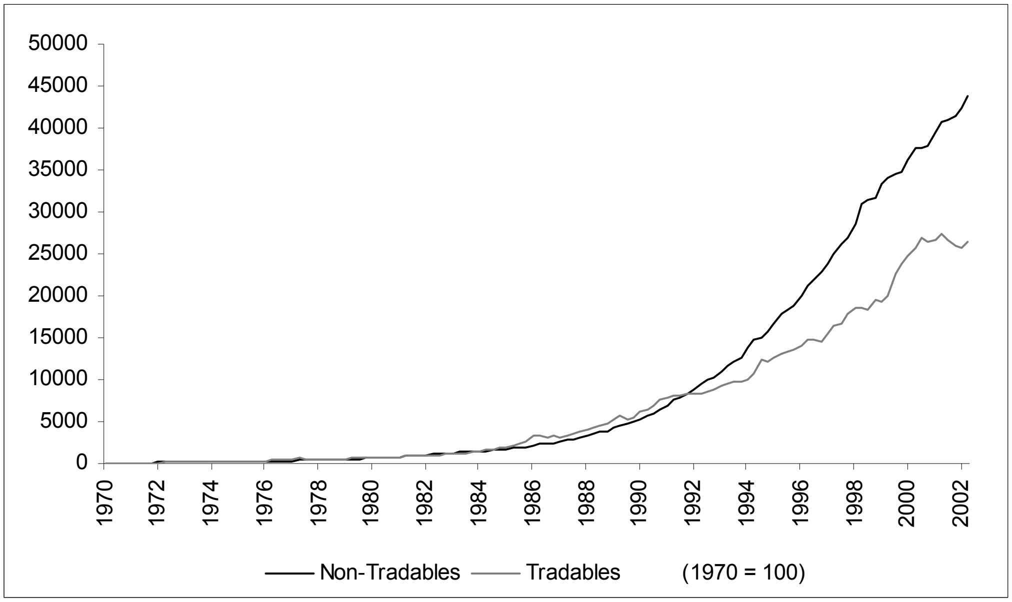
Table 3.1. Data and data sources

Data Series	Availability of Quarterly Data from Source	Source
USD/COP exchange rate	Before 1970	Banco de la República
CPI (CO)	Before 1970	IFS
CPI (US)	Before 1970	IFS
Export price index (CO)	From 1970	IFS
Export price index (US)	From 1970	IFS
Import price index (CO)	From 1970	IFS
Import price index (US)	From 1970	IFS
Balassa-Samuelson Effect	From 1970	Own calculations
Industrial production (CO)	From 1980	IFS
Industrial production (US)	Before 1970	IFS
Money supply M3 (CO)	From 1970	Banco de la República
Money supply M3 (US)	Before 1970	IFS
Interest rates (CO)	From 1984	Banco de la República
Interest rates (US)	Before 1970	The Federal Reserve

Note: IFS refers to *International Financial Statistics* from the International Monetary Fund. For interest rates we use the 3-month certificate-of-deposit rate (DTF) for Colombia and the 3-month Treasury bill rate for the United States. CO and US in parentheses indicate Colombian and U.S. data respectively.

The Balassa-Samuelson effect is calculated from equation (2.23) using the consumer price index to represent prices of non-tradables and an average of the export price index and the import price index to represent prices of tradables. The reason for using this definition is that consumer price, export price and import price indices are all readily available from 1970 and onwards, while sectoral price data, which might have provided a better definition, is not available that far back in time. The definition used here also guarantees that the same definition is used for Colombia and for the United States.

Figure 3.1. Price indices for non-tradables and tradables in Colombia



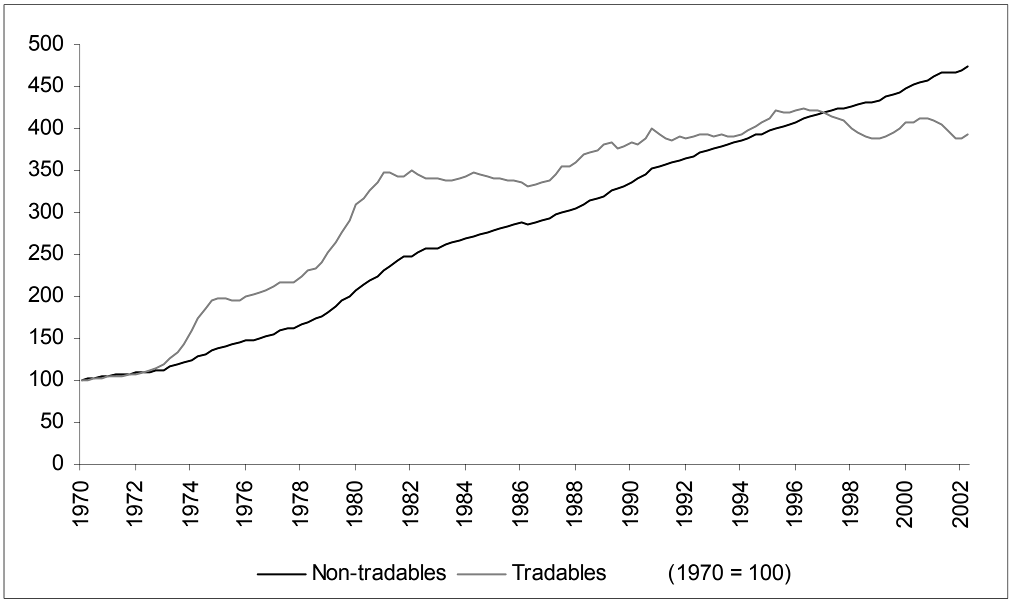
Note: Consumer prices have been used as a proxy for non-tradables and the average of export and import prices as a proxy for tradables

Source: IMF International Financial Statistics

As illustrated by figure 3.1, in Colombia, prices of non-tradables have grown considerably faster than prices of tradables, as predicted by the Scandinavian model of inflation. A significant part of inflation in Colombia during the period should, consequently, have been structural.

For the United States, the price difference between non-tradables and tradables has shown a very different pattern of development, as shown by figure 3.2. During the 1970s, prices of tradables increased much faster than those of non-tradables. This might be explained by large increases in the oil price during those years. Nevertheless, import and export prices can be shown to have followed each other closely throughout this period. However, during the 1980s and the 1990s consumer prices grew faster than import and export prices, and the situation seems to have returned to normal.

Figure 3.2. Price indices for non-tradables and tradables in the United States

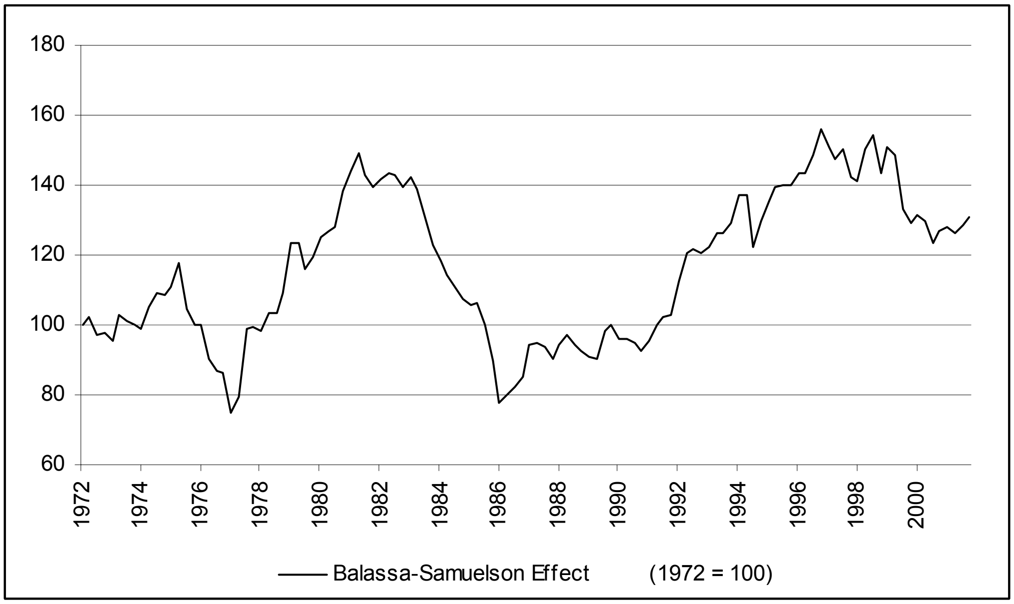


Note: Consumer prices have been used as a proxy for non-tradables and the average of export and import prices as a proxy for tradables

Source: IMF International Financial Statistics

Figure 3.3 graphs the Balassa-Samuelson Effect. It might seem strange that this has not increased more, since Colombia is a developing country and should have had a much faster growth rate during the period. Colombia has indeed grown faster than the United States, but the difference is not that large. Between 1970 and 2001, real gross domestic product increased by 205 percent in Colombia compared to 161 percent in the United States.

Figure 3.3. The Balassa-Samuelson effect between Colombia and the United States



Source: IMF International Financial Statistics, and own calculations.

A further point should be made concerning the Balassa-Samuelson effect in Colombia. The central assumptions in the structural models are that the profit margin in the tradable and non-tradable sectors of the economy is assumed to be constant, and that nominal wages are the same in the two sectors. The first assumption is stated by equation (2.15), or, in the case of the Scandinavian model, by equation (2.6) and (2.8), while the second assumption is stated by equation (2.14), and, for the Scandinavian model, by equation (2.7). The first assumption is likely to be valid in the long term in Colombia, while the second assumption is questionable. Wage equalisation between the sectors has been shown to be a valid assumption for the Scandinavian as well as many other developed countries. However, in Colombia, salaries in the countryside are developing very differently from the salaries in the cities,[21] and people are not easily moving between the two areas.

[21] This can be shown by comparing the development of salaries in agriculture with salaries in non-agricultural production.

3.2 The Different Exchange Rate Regimes in Colombia

From 1967 and up until 1991, the exchange rate regime in Colombia was defined by a crawling peg. The Colombian peso was pegged to the U.S. dollar at a pre-specified exchange rate and was not allowed to depart significantly from this rate. This exchange rate was, furthermore, devalued daily at a pre-determined and continuous devaluation rate. The exchange rate regime was combined with a system of thorough capital controls, where all foreign exchange transactions had to be made through the Banco de la República.[22]

The crawling peg regime was abolished in June 1991, following a sharp fall in international coffee prices and a deterioration in the trade balance. A market for foreign exchange was created, where the exchange rate was freely determined.[23] However, the Banco de la República continued to intervene in the market, and in practice the new exchange rate regime was a managed floating regime with many similarities to a crawling exchange rate band.

In January 1994, the central bank introduced an official crawling band regime. This was to regain control over monetary variables, after a period of very low real interest rates in combination with very large capital inflows. The exchange rate was allowed to fluctuate around a pre-determined central rate, which initially was to be continuously devalued at an annual rate of 11 percent. The actual exchange rate could depart with as much as 7 percent from the central rate. As shown by figure 3.4, the regime, in fact, very much resembled a managed float, since the limits of the band were shifted several times, and since the band was relatively wide.[24]

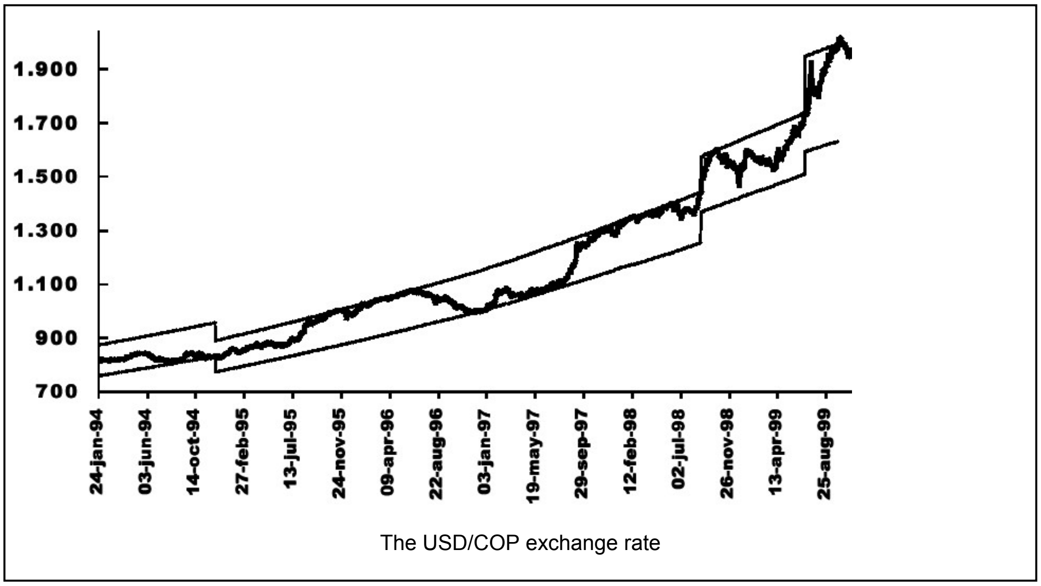
[22] For a thorough discussion on the Colombian exchange rate regimes, see Villar and Rincón (2000), as well as Cárdenas (1997). The discussion here draws heavily from Villar and Rincón (2000), as well as from Rowland (2003).
[23] The market traded Exchange Rate Certificates (Certificados de Cambio) which were US dollar denominated interest bearing papers issued by the Banco de la República. See Villar and Rincón (2000), pp 27ff.
[24] Villar and Rincón (2000), p. 30.

In September 1999, the exchange rate band was dismantled, and the exchange rate was allowed to float freely. This followed a period of economic difficulties. Colombia was in a recession, the government was running a large fiscal deficit, and the credibility of the currency band system had rapidly been deteriorating. The floating regime, which has been in place since then, is close to a free float. The central bank can only intervene to reduce short-term exchange rate volatility, and has not done so until earlier this year.[25]

Figure 3.5 shows the exchange rate development since 1970, and figure 3.6 shows the exchange rate variability. It is apparent from figure 3.5 that the exchange rate left its path of a long-term stable depreciation rate in 1991, when the crawling peg was abandoned. As expected, the short-term variability of the exchange rate also increased significantly, as shown by figure 3.6. However, there was no significant change in exchange rate variability between the crawling band regime and the floating regime, which was introduced in 1999. If we calculate the average absolute weekly change for the periods January 1994 to September 1999 and October 1999 to August 2002 we receive values of 0.72 percent and 0.68 percent respectively.

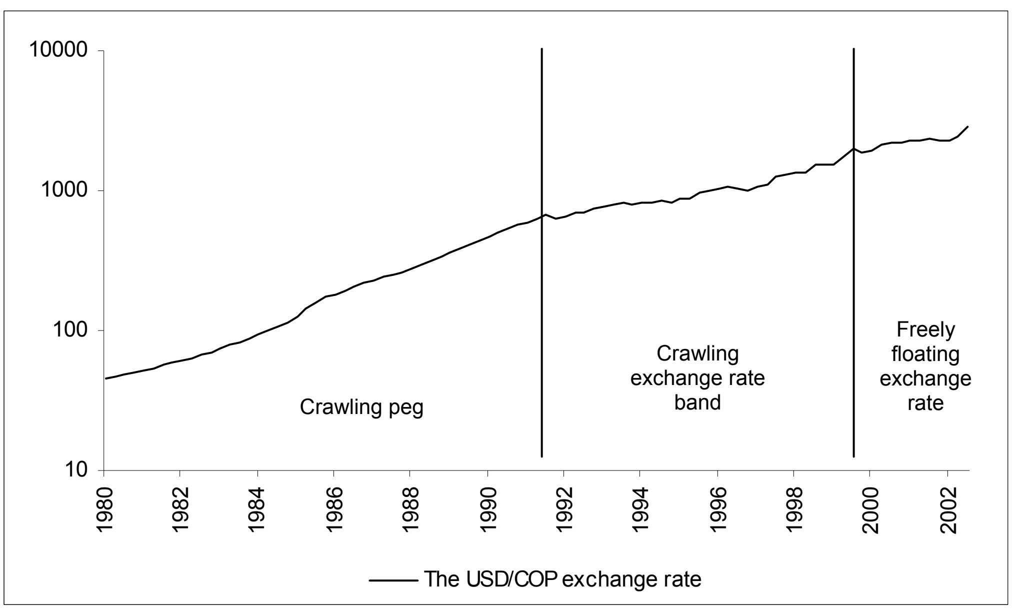
[25] The central bank can only intervene if the average exchange rate of a given day deviates more than 4 percent from its 20-day moving average.

Figure 3.4. The Colombian exchange rate band 1994 – 1999



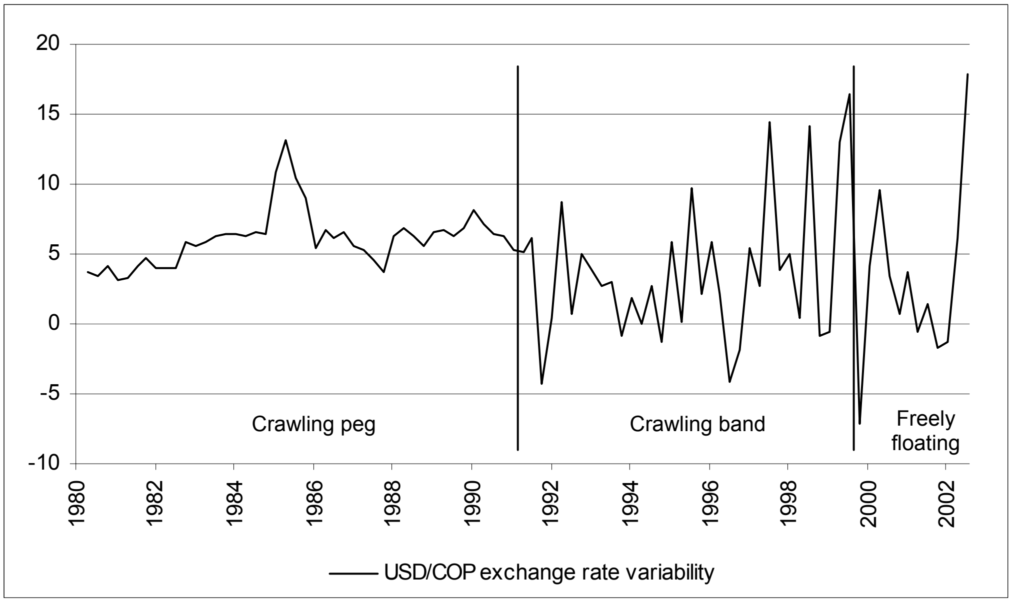
Source: Villar and Rincón (2000), p. 31.

Figure 3.5. The USD/COP exchange rate under the different regimes (logarithmic scale)



Source: Banco de la República.

Figure 3.6. Short-term variability of the USD/COP exchange rate, expressed as percentage change from previous quarter



Source: Banco de la República.

3.3 Analysing the Long-Term Cointegrating Relationships

We will in this section and in the two that follows estimate three different exchange rate models. We start by testing the PPP hypothesis using the Engle-Granger methodology.[26] The existence of a valid cointegrating vector between the exchange rate, Colombian consumer prices, U.S. consumer prices and the Balassa-Samuelson effect is investigated. We are testing for long-term equilibrium relationships. These are then used in the next section to estimate two different error correction models, which can be used to forecast the exchange rate. A third exchange rate model is estimated in section 3.5 using the Johansen methodology.

To test the PPP hypothesis, we start by testing the time series for unit roots. These tests are summarised in table 3.2. We can assume all the variables to be integrated of order

[26] See Engle and Granger (1987), or for an overview, Enders (1995).

one. The null-hypothesis that the variables are integrated of order two (i.e. that the first difference is integrated of order one) can be rejected at 5 percent significance level for all the variables except the U.S. price level. For the U.S. price level, the null-hypothesis can, nevertheless, be rejected at the 10 percent level, and we will, therefore, also consider this variable to be integrated of order one.

Table 3.2. Unit root tests for the time series
(using quarterly data from Q3 1973 to Q2 2002)

Variable	Level	First Difference	Second Difference
s	ADF(4) = -0.65	ADF(3) = -2.90	
p	ADF(2) = -2.31	ADF(1) = -3.98	
p^*	ADF(4) = -2.34	ADF(3) = -2.66	ADF(1) = -11.99
p_{NT}	ADF(4) = -1.60	ADF(3) = -5.37	

Note: The Augmented Dickey-Fuller test is used to test for unit roots. The value in parentheses is the order of the lag used, which is decided by using the Schwartz information criteria. The null hypothesis in each case is that the variable is integrated of order one and, thereby, non-stationary. The 5 percent rejection region for non-stationarity for the Dickey-Fuller statistic is ADF < -2.89, and the 10 percent rejection region is ADF < -2.58, according to Fuller (1976).

To investigate the existence of a cointegrating vector, we estimate the logarithmic form of the relative PPP hypothesis with the Balassa-Samuelson effect added as stated by equation (2.22), i.e.

$$s = b_1 + b_2\, p + b_3\, p^* + b_4\, p_{NT} + \varepsilon_t \qquad (3.1)$$

For relative PPP to be valid, b_2 should be positive and approximately equal to one, b_3 should be negative and approximately equal to one, and b_4 should be negative and between zero and one. The parameter b_1 depends on the level of the price indices, i.e. what year is used as base for calculating the indices. The results of the regressions are presented in table 3.3.

Table 3.3. Regressions for the time period Q3 1973 to Q2 2002

The unrestricted model

$s = 6.010 + 1.112\, p - 0.924\, p^{*} - 0.939\, p_{NT}$
(14.92) (43.23) (-8.28) (-15.54)

$R^2 = 0.99$; Adjusted $R^2 = 0.99$; ADF(4) = -3.31

Restriction: $b_4 = 0$

$s = 5.825 + 1.034\, p - 0.787\, p^{*}$
(8.18) (23.18) (-4.00)

$R^2 = 0.98$; Adjusted $R^2 = 0.98$; ADF(4) = -2.25

Restriction: $b_2 = 1$ and $b_3 = -1$

$s = 6.765 + 1.000\, p - 1.000\, p^{*} - 0.364\, p_{NT}$
(230.75) (-3.22)

$R^2 = 0.08$; Adjusted $R^2 = 0.07$; ADF(4) = -1.10

Note: The t-ratios are given in parentheses below the parameter estimates. The Augmented Dickey-Fuller statistic tests the null hypothesis that the residuals are integrated of order one and, thereby, non-stationary. The 5 per cent rejection region for non-stationarity for the Augmented Dickey-Fuller statistic within the Engle-Granger framework is ADF < -3.17 and the 10 percent rejection region is ADF < - 2.91. See Enders (1995), p. 383. The value in parentheses shows the order of the lag used for the augmented Dickey-Fuller test.

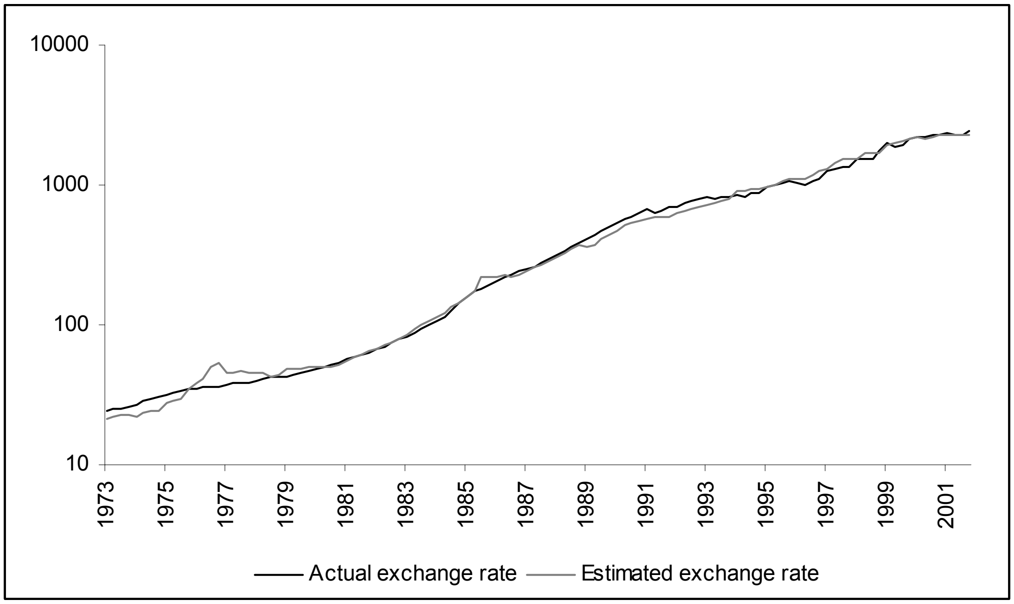
The parameter estimates of b_1 are positive and significantly different from zero in all the regressions. The parameter estimates of b_2 and b_3 are all significant and relatively close to one and minus one respectively, and the parameter estimates of b_4 are all significant and between zero and one. In the first two regressions, where the Balassa-Samuelson effect is included, we have a valid cointegrating relationship, since the residuals are stationary, as shown by the augmented Dickey-Fuller test. When the Balassa-Samuelson effect is excluded, the regressions do, however, not yield any valid cointegrating vector, as shown by the second regression, where the residuals are clearly non-stationary. And we do not have cointegration when b_2 and b_3 are restricted to one and minus one respectively, which should be the case for PPP to hold in the strong sense.

We can therefore conclude that the PPP hypothesis seems to hold in the long term if it the Balassa-Samuelson effect is included, even if it does not hold in the strong sense. The deviation of the parameter estimates of b_2 and b_3 from their theoretical values of one and minus one, and the fact that restricting these variables to their theoretical values, does not yield a cointegrating relationship, can possibly be explained by the price indices used.[27] The Balassa-Samuelson effect is derived using the consumer price index as a proxy for a price index of non-tradables. The theory, furthermore, assumes price equalisation between the price indices of tradables of the two countries. However, the baskets of tradables look very different between Colombia and the United States, and even if there is price equalisation between the individual tradable goods, the composite price indices of tradables might not follow each other due to their different make-up.

Figure 3.7 shows the actual exchange rate and the exchange rate estimated by the unrestricted model. The latter is a long-term equilibrium exchange rate. Figure 3.8 graphs the error between the actual and the estimated exchange rate. It is apparent from figure 3.8 that the departure of the actual exchange rate from the equilibrium rate seems to have decreased after the 1970s, and that the departures from the equilibrium rate seem less persistent.

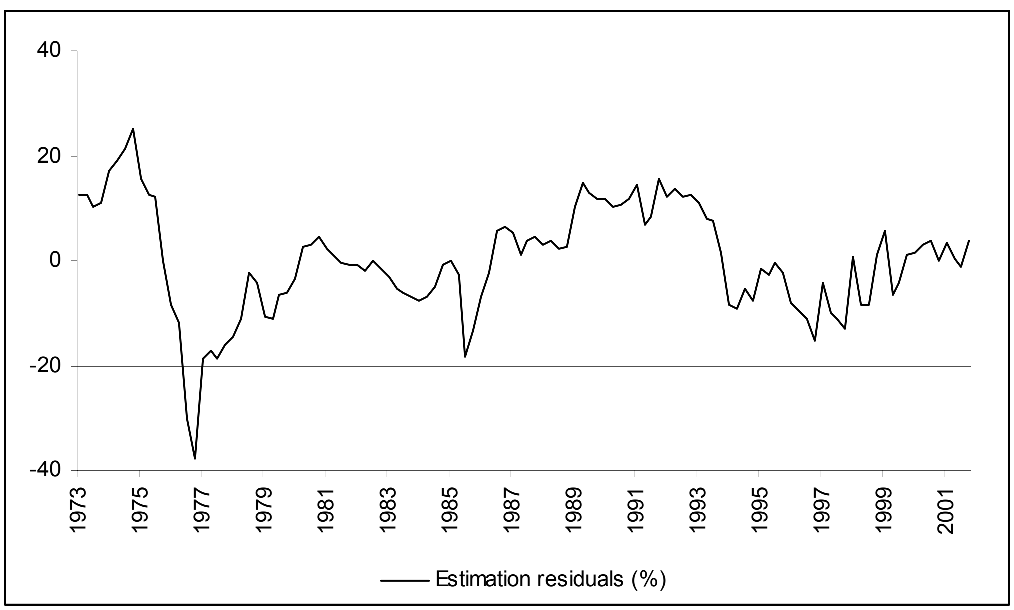
[27] We are using the consumer price indices. Their composition do, however, differ significantly between Colombia and the United States.

Figure 3.7. Estimated and actual exchange rate (long-term Engle-Granger model)



Source: Banco de la República, and estimations.

Figure 3.8. Estimation residuals in approximate percentage terms



Note: The residuals are calculated as the difference between actual and estimated time series in logarithmic terms. The difference between two logarithmic values is approximately the same as the relative difference between the non-logarithmic values.

We are continuing our analysis by testing the Frenkel-Bilson monetary model.[28] In this case we are using annual data from 1970 up until 2001, since Colombian interest rates are not available quarterly before 1984.[29] Again we are using the Engle-Granger methodology, testing for long-run equilibrium relationships.

The unit root tests of the time series are summarised in table 3.4. It is obvious that the exchange rate and the ratio of Colombian to U.S. money supply are integrated of order two, while the output ratio and the interest rate differential are integrated of order one. A condition for cointegration within the Engle-Granger framework is that all the variables are integrated of order one. We can, therefore, conclude that a valid cointegrating relationship, as suggested by the Frenkel-Bilson model, does not exist in the data set analysed.

Table 3.4. Unit root tests for the time series
(using annual data from 1970 to 2001)

Variable	Level	First Difference	Second Difference
s	ADF(1) = -0.29	ADF(0) = -2.28	ADF(0) = -6.95
$(m - m^*)$	ADF(1) = -2.17	ADF(0) = -1.50	ADF(1) = -4.17
$(y - y^*)$	ADF(0) = -0.34	ADF(0) = -5.45	
$(r - r^*)$	ADF(1) = -1.47	ADF(0) = -4.74	

Note: The Augmented Dickey-Fuller test is used to test for unit roots. The value in parentheses is the order of the lag used, which is decided by using the Schwartz information criteria. The null hypothesis in each case is that the variable is integrated of order one and, thereby, non-stationary. The 5 percent rejection region for non-stationarity for the Dickey-Fuller statistic is ADF < -2.89, and the 10 percent rejection region is ADF < -2.58, according to Fuller (1976).

[28] This was defined in section 2.5.
[29] The data sources are the same as stated in table 3.1. As a measure of output we are using the gross domestic product rather than industrial production. The source is *IMF International Financial Statistics*. The source for Colombian interest rates before 1984 is Posada (1999).

We could add the inflation rate together with the cumulated current account to the analysis of the monetary model, as suggested by equation (2.25). This would add little to the results, since we already have a problem with different orders of integration of the variables.

3.4 Estimating the Error Correction Models

We now proceed with the third step in the Engle-Granger methodology, the estimation of the error correction model. We are, however, facing one problem. The time period that we are studying, from 1973 up until 2002 include at least one considerable structural break when the crawling peg was abolished in June 1991. This was discussed in section 3.2, and it is clearly shown by figure 3.6. To avoid this problem we will only use data from the first quarter 1992 for the estimation of the error correction model,[30] which is in line with the methodology used in Engle, Granger and Hallman (1989).

We will, furthermore, estimate two different error correction models. For the first we use data from the PPP analysis, i.e. the exchange rate, Colombian and U.S. consumer prices and the Balassa-Samuelson effect. For the second one we use data suggested by the monetary model, i.e. the exchange rate, the money supply ratio, the output ratio and the interest rate differential. Both error correction models are estimated using data from the first quarter 1992 to the second quarter 2002. In both cases we are using the cointegrating relationship estimated for the unrestricted PPP model, i.e. the first result of table 3.3. We are, consequently, estimating the VARs using the residuals from this regression. The two error correction models that we are estimating can be stated as

$$\Delta s_t = \sum_{i=1}^{k} \gamma_1^i \Delta s_{t-i} + \sum_{i=1}^{k} \gamma_2^i \Delta p_{t-i} + \sum_{i=1}^{k} \gamma_3^i \Delta p_{t-i}^* + \sum_{i=1}^{k} \gamma_4^i \Delta p_{NT,t-i} + \alpha z_{t-1} + \varepsilon_t \qquad (3.2)$$

[30] We chose to begin in the first quarter 2002 to avoid including the large adjustment of the exchange rate that took place following the abolishment of the crawling peg in June 1991.

and

$$\Delta s_t = \sum_{i=1}^{k} \delta_1^i \Delta s_{t-i} + \sum_{i=1}^{k} \delta_2^i \Delta (m - m^*)_{t-i} + \sum_{i=1}^{k} \delta_3^i \Delta (y - y^*)_{t-i} +$$

$$+ \sum_{i=1}^{k} \delta_4^i \Delta (r - r^*)_{t-i} + \beta z_{t-1} + u_t \qquad (3.3)$$

where γ_n^i, α, δ_n^i and β are parameters to be estimated, k is the maximum distributed lag length, Δ is the difference operator and ε_t and u_t are independent and identically distributed error terms. The parameter estimates of α and β, which are presented in table 3.6, should both be negative and between zero and minus one. The maximum lag length k was chosen to be long enough for the error terms to be normally distributed and not serially correlated. Table 3.7 and 3.8 present the residual tests of the two models.

Table 3.6. Parameter estimates of α and β
(using quarterly data from Q3 1973 to Q2 2002, and a maximum lag length $k = 1$)

	Equation for variable			
Equation (3.2)	Δs_t	Δp_t	Δp_t^*	$\Delta p_{NT,t}$
α	-0.237	0.022	0.003	0.043
t-statistic	(-2.41)	(0.92)	(0.67)	(0.57)
Equation (3.3)	Δs_t	$\Delta(m - m^*)_t$	$\Delta(y - y^*)_t$	$\Delta(r - r^*)_t$
β	-0.216	0.077	0.068	-7.887
t-statistic	(-2.09)	(1.34)	(1.47)	(-1.11)

Note: Only the equations for Δs_t are stated by equation (3.2) and (3.3). The other equations, i.e. the equations for Δp_t and so on, can easily be derived from these equations. For the error correction models stated by equation (3.2) and (3.3) to be valid within the Engle-Granger framework, the exchange rate s has to be exogenous while all the other variables should be endogenous. That this is the case is shown by the fact that the parameter estimates of α and β are significant only in the equations for Δs_t, as indicated by the t-statistics.

Table 3.7. Residual tests of the error correction model based on equation (3.2) (using quarterly data from Q3 1973 to Q2 2002, and a maximum lag length $k = 1$)

Test	Test Statistic	P-value
Multivariate Normality		
Lütkepohl test	$\chi^2(8) = 13.20$	0.105
Autocorrelation		
Portmanteau test	Portmanteau(12) = 178.60	0.431
LM test	LM(12) = 20.46	0.200

Table 3.8. Residual tests of the error correction model based on equation (3.3) (using quarterly data from Q3 1973 to Q2 2002, and a maximum lag length $k = 1$)

Test	Test Statistic	P-value
Multivariate Normality		
Lütkepohl test	$\chi^2(8) = 7.02$	0.534
Autocorrelation		
Portmanteau test	Portmanteau(12) = 190.36	0.218
LM test	LM(12) = 9.75	0.879

Estimation of the error correction models adding a term for the lagged interest rate differential, $(r_{t-1} - r^*_{t-1})$, to equations (3.2) and (3.3), was also tried. The parameter estimates for this term was, however, not significantly different from zero in any of the cases.

We now have two models that build on the long-term cointegrating relationship that was estimated in section 3.3. The short-term dynamics are, however, represented by different error correction models, one based on variables derived from the PPP hypothesis, and the other based on variables derived from the monetary Frenkel-Bilson model.

3.5 The Johansen Framework

We will now continue by estimating a third model using a framework of multivariate cointegration that was originally developed by Johansen (1988).[31] In this section we briefly present the econometric framework, and in the next we present the estimation results.

The Johansen estimation test procedure, which by now is well known, is a method for estimating the cointegrating relationships that exist between a set of variables as well as testing these relationships. The application of this framework on the PPP relationship with the Balassa-Samuelson effect, as stated by equation (2.22), can very briefly be introduced as follows. First, a vector autoregressive model with a maximum distributed lag length of k is defined,

$$\boldsymbol{X}_t = \boldsymbol{r}_1 \boldsymbol{X}_{t-1} + \ldots + \boldsymbol{r}_k \boldsymbol{X}_{t-k} + \boldsymbol{e}_t \,, \qquad t = 1, \ldots, T \tag{3.4}$$

where $\boldsymbol{X}_t = (s_t, p_t, p_t^*, p_{NT,t})^{\mathrm{T}}$, $\boldsymbol{r}_i$ are 4x4 coefficient matrices and $\boldsymbol{e}_t$ is a 4x1 vector of independent and identically distributed error terms.[32] The distributed lag length k should be specified long enough for the residuals not to be serially correlated. The cointegrating matrix $\boldsymbol{r}$, which defines the long-term solution of the system, is defined as

$$\boldsymbol{r} = -\boldsymbol{I} + \boldsymbol{r}_1 + \ldots + \boldsymbol{r}_k \tag{3.5}$$

where $\boldsymbol{I}$ is the 4x4 identity matrix. The Johansen procedure now continues with decomposing the matrix $\boldsymbol{r}$ into two $N \times r$ matrices $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$,

$$\boldsymbol{r} = \boldsymbol{\alpha} \boldsymbol{\beta}^{\mathrm{T}} \tag{3.6}$$

[31] See also Johansen (1990, 1991, 1995).

[32] A 'T' in superscript behind a vector or matrix indicates that this vector or matrix is to be transposed.

The rows of the matrix β now define the cointegrating relationships among the four variables in the vector $\boldsymbol{X}$, and the rows of the matrix α show how these cointegrating vectors are loaded into each equation in the system. Johansen, furthermore, suggests a maximum likelihood estimation procedure to estimate the two matrices α and β together with test procedures to test the number of distinct cointegrating vectors. Linear parameter restrictions on the data can, furthermore, be tested by testing the matrix β, and the direction of causality within the system can be tested by testing the matrix α.

3.6 Likelihood Estimation and Results

As discussed earlier, we are using a set of variables in logarithms, (s, p, p^*, $p_{NT,}$), where s stands for the nominal effective exchange rate, p and p^* are the Colombian and U.S. consumer price indices, and p_{NT} is the Balassa-Samuelson effect. The model includes dummy variables to account for any seasonal patterns. The data set used spans from the first quarter 1970 to the second quarter 2002.

We are, furthermore, using dummy variables to represent the different exchange rate regimes. We are using one dummy variable to represent the exchange rate band regime, running from the third quarter 1991 to the third quarter 1999, and another dummy variable to represent the floating rate regime from the fourth quarter 1999 and onwards.[33]

The results of the estimation procedure are presented in table 3.9 and 3.10, and table 3.11 shows the residual tests. The maximum distributed lag length is chosen long enough for the residuals to be independent and to fulfil the normality assumption. The results of the likelihood ratio test in table 3.9 leads us to assume one cointegrating vector.

[33] Since the change from the crawling band regime to the floating regime did not seem to represent a significant structural break, we could, alternatively, have chosen to use one dummy variable to represent the whole period from the third quarter 1991 and onwards.

Table 3.9. Likelihood ratio test of the number of distinct cointegrating vectors (using quarterly data from Q1 1970 to Q2 2002)

Hypothesised Number of Cointegrating Equations	Eigenvalue	Test Statistic	5 Percent Critical Value	1 Percent Critical Value
Trace Statistic				
None	0.3283	80.80	47.21	54.46
At most 1	0.1518	31.46	29.68	35.65
At most 2	0.0852	11.05	15.41	20.04
At most 3	0.0000	0.00	3.76	6.65
Max-Eigen Statistic				
None	0.3283	49.34	27.07	32.24
At most 1	0.1518	20.41	20.97	25.52
At most 2	0.0852	11.04	14.07	18.63
At most 3	0.0000	0.00	3.76	6.65

We impose the restriction that the U.S. price level is exogenous,[34] as suggested by economic theory. This restriction is easily passed by the validity test. None of the other variables pass this test, so we assume them to be endogenous. The estimated cointegrating vector is, however, both in the unrestricted and the restricted case, relatively far away from its theoretical value of (1, -1, 1, *a*), where *a* represents the weight of non-tradables in the consumer price index and should, therefore, be positive and between zero and one. The parameter estimate of the U.S. consumer price index is far away from one, and the parameter estimate of the Balassa-Samuelson index is not only far away from its theoretical value of between zero and one, but also of the wrong sign. Trying to restrict any of the parameters to their theoretical value is, furthermore, rejected by the tests.

[34] Weakly exogenous in this case.

Table 3.10. Estimation of the model
(using quarterly data from Q1 1970 to Q2 2002)

Model	VAR(6): Drift
Variables	$(s, p, p^*, p_{NT,})$
Unrestricted cointegrating vector $\beta^T = (\beta_{11}\ \ \beta_{12}\ \ \beta_{13}\ \ \beta_{14})$	$\beta^T = (1.000\ \ -1.299\ \ 2.833\ \ -0.485)$
Restriction test $(\alpha_{13} = 0)$	$\chi^2(1) = 0.527$ P-value: 0.468
Restricted cointegrating vector $\beta^T = (\beta_{11}\ \ \beta_{12}\ \ \beta_{13}\ \ \beta_{14})$	$\beta^T = (1.000\ \ -1.317\ \ 2.996\ \ -0.703)$
Speed of adjustment $\alpha^T = (\alpha_{11}\ \ \alpha_{12}\ \ \alpha_{13}\ \ \alpha_{14})$	$\alpha^T = (-0.024\ \ 0.028\ \ 0.000\ \ -0.034)$

Note: In EViews, which is used for these estimations, the variable α_{13} is called α_{31}, so the restriction is denoted $\alpha_{31} = 0$.

Table 3.11. Residual tests of the restricted model
(using quarterly data from Q1 1970 to Q2 2002)

Test	Test Statistic	P-value
Multivariate Normality		
Lütkepohl test	$\chi^2(8) = 15.52$	0.050
Autocorrelation		
Portmanteau test	Portmanteau(30) = 447.61	0.050
LM test	LM(30) = 14.80	0.539

4 Forecasting the USD/COP Exchange Rate

In the previous chapter we developed and estimated three different exchange rate models. In this chapter the forecasting performance of these models is evaluated. The forecasting power of the models is compared to a simple random walk process as well as to a random walk process with a variable drift term. Section 4.1 presents the methodology for evaluation of the forecasts, which is in line with that used by Meese and Rogoff (1983a). The comparison of the forecasting performance of the different models takes place in section 4.2. Section 4.3 ends the chapter with a discussion of long-term exchange rate forecasting.

4.1 The Methodology for Evaluating the Forecasts

The data set used in our study comprises of quarterly data to the second quarter 2002, as discussed earlier. The estimation of the long-term cointegrating relationship within the Engle-Granger framework used data from the third quarter 1973, while the estimation of the error correction models uses data from the first quarter 1992. The estimation of the Johansen model uses the full dataset. All the three models are initially estimated using data up until the last quarter 1996.[35] Forecasts are then generated at horizons of 3, 6, 12 and 24 months.[36] The data for the first quarter 1997 is then added to the sample, and the model is re-estimated, and new forecasts generated. These rolling re-estimations are continued until the second quarter 2002 is included in the sample.

We have chosen to begin our forecast period in the first quarter 1997 to give the models enough time to adjust to the crawling band that was introduced in June 1991. The error

[35] For the estimation of the error correction models, we are using the residuals from the initial estimation of the cointegrating relationship, which used data from the third quarter 1973 up until the second quarter 2002. The long-term cointegrating relationship is, consequently, not re-estimated for the different forecast iterations.

[36] This corresponds to 1, 2, 4 and 8 quarters using the quarterly data.

correction models, in particular, needs this time span, since they are estimated using data only from the first quarter 1992.

In the Johansen model, only the U.S. price level p^* is exogenous, and the model does, therefore, not allow us to use our own forecasts or, as in Meese and Rogoff (1983a), actual data for the other variables when forecasting the exchange rate. We are, consequently, letting the model forecast all the variables. Since the U.S. price level is, indeed, exogenous, we could have chosen to feed the model with actual data of this variable for the forecasts. However, we will not to give the model this advantage, but instead to let it forecast the whole set of variables.

The error correction models, which do not face this limitation, are fed with actual data. This is in line with Meese and Rogoff (1983a) and many other studies.

The out-of-sample accuracy of the forecasts is measured by two statistics, root mean square error (RMSE) and mean absolute error (MAE). These are defined as follows:

$$RMSE = \left(\sum_{s=0}^{N_k - 1} \frac{\left(F(t+s+k) - A(t+s+k)\right)^2}{N_k} \right)^{1/2} \tag{4.1}$$

$$MAE = \sum_{s=0}^{N_k - 1} \frac{\left|F(t+s+k) - A(t+s+k)\right|}{N_k} \tag{4.2}$$

where k = 1, 2, 4, 8 denotes the forecast steps in quarters, N_k the total number of forecast iterations in the projection period for which the actual value of the exchange rate $A(t)$ is known, and $F(t)$ the forecasted value of the exchange rate. The forecast starts in period t.

4.2 Comparing the Models

Table 4.1 shows the statistics of the root mean square error and the mean absolute error for the forecasts of the different models as well as for the spot rate (the forecast of a random walk model) for the different forecast horizons.

The table presents some very interesting findings. It is apparent that the two models built on the PPP relationship together with an error correction model outperform the random walk on both the 12 and 24-month horizon. The model using monetary variables for the error correction model also outperforms the random walk on the 6-month horizon. The multivariate cointegration model produces the worst forecasts. What is striking is, furthermore, the performance of the random walk model with a drift. This outperforms all other models on all the forecast horizons.

An interesting observation is that none of the models based on fundamentals outperforms a random walk (without drift) on the 3-month horizon. To forecast exchange rates in the short term using fundamentals should, indeed, be more difficult than to forecast in the medium and long term. A number of studies have shown that, due to incomplete information in the short term, the behaviour of foreign exchange market participants is to a large extent based on *technical analysis* of short-term trends or other patterns in the observed behaviour of the exchange rate.[37] In support of such behaviour, simulations have shown short-term trading strategies based on technical analysis to generate significant profits.[38] The long-term behaviour of exchange rates is, on the other hand, much more governed by fundamentals. This also implies that short-term exchange rates will vary much more widely than is justified by changes in fundamentals.[39]

[37] See, for example, Taylor and Allen (1992).
[38] See Cumby and Modest (1987), Dooley and Shafer (1983), and Sweeney (1986).
[39] Models have been developed where *feedback traders* coexist with *fundamentalists* as market participants. The former base their trading strategies on the recent history of exchange rates, while the latter base their strategies on analysis of economic fundamentals. In these types of models, the fundamentalists have the predominant influence of exchange rates in the long term. However, risk aversion together with substantial uncertainties regarding news and new information, leads to feedback traders dominating the market in the short term. See Lyons (1993), Kyle (1985), Frankel and Froot (1990), and Cutler, Poterba and Summers (1990).

Table 4.1. Statistics for the forecast errors of the models

	Forecast Horizon			
	3 months	6 months	12 months	24 months
Random Walk (without drift)				
RMSE	5.46	9.16	16.09	32.37
MAE	4.40	7.88	14.80	31.19
Random Walk with a Variable Drift				
RMSE	3.66	4.94	5.50	7.79
MAE	2.39	3.17	4.36	6.88
PPP with an ECM				
RMSE	6.59	9.31	13.03	20.77
MAE	5.25	7.87	11.05	18.14
PPP with an ECM based on Monetary Variables				
RMSE	6.56	7.85	8.64	13.33
MAE	5.01	6.24	7.45	10.74
PPP in a Multivariate Cointegration Model				
RMSE	10.30	14.82	25.19	43.64
MAE	7.99	11.58	18.16	30.16

Note: All values are in approximate percentage terms (the difference between two logarithmic values is approximately the same as the relative difference between the variables).

Another interesting fact is that the model based on multivariate cointegration performs so badly. Rowland and Oliveros (2003) used a similar model to forecast the nominal effective exchange rate with relatively good results. The data used in this study runs from the first quarter 1980 to the third quarter 2002, and their forecast iterations run from the first quarter 1997 to the third quarter 2002, i.e. more or less the same period as in the study undertaken here. The results from the Rowland-Oliveros study are presented in table 4.2. In this case the multivariate cointegration model clearly outperforms the random walk on the 12 and 24-months horizon. The very different results of the studies can be explained by the fact that the nominal effective exchange rate should on average be closer to its equilibrium value than the USD/COP rate.[40] The structural break in 1991 was furthermore larger for the USD/COP rate than for the nominal effective exchange rate, since the latter includes many exchange rates that were floating against the Colombian peso also before 1991.

It is, furthermore, possible that USD/COP exchange rate models based on the Johansen framework will perform better in the future, when longer time-series data from the floating exchange rate period (from September 1999 onwards) are available.

However, none of the models here are even close to outperforming the random walk with a variable drift. This simple, but statistically brilliant, model seems to be the best tool to forecast the USD/COP exchange rate.

Table 4.2. Statistics for the forecast errors when forecasting the nominal effective exchange rate using data from Q1 1980 – Q3 2002

	Forecast Horizon			
	3 months	6 months	12 months	24 months
Random Walk (without drift)				
RMSE	5.24	7.84	12.54	23.08
MAE	3.91	6.17	11.09	20.44
PPP in a Multivariate Cointegration Model				
RMSE	5.15	8.23	12.16	22.43
MAE	4.46	7.39	9.88	17.57

Note: All values are in approximate percentage terms (the difference between two logarithmic values is approximately the same as the relative difference between the variables).

Source: Rowland and Oliveros (2003)

[40] The nominal effective exchange rate is a weighted average of several exchange rates. If these are not correlated, any deviations from their equilibrium rate should, at least to some extent, offset each other.

4.3 A Comment on Long-Term Forecasting of the Exchange Rate

To forecast the exchange rate in the long term is not much easier than forecasting the medium-term exchange rate correctly. However, in the long term the purchasing power parity relationship should be much stronger than in the short to medium term, and PPP might, therefore, be a useful tool. We have shown in section 3.3 that PPP did not hold in the strong sense between Colombia and the United States over the past 30 years. However, a clear purchasing power relationship did exist, and this relationship could be used to forecast the exchange rate in the long term, let us say 10 years ahead. In that case we would use the following equation

$$s = 6.010 + 1.112\, p - 0.924\, p^{*} - 0.939\, p_{NT} \tag{4.3}$$

as estimated in section 3.3. We would feed the model with forecasts for Colombian and U.S. consumer price as well as for the Balassa-Samuelson effect. The later might, however, present a problem. As discussed earlier, we would have problems estimating export and import prices, since this would be about the same as estimating the exchange rate. It might furthermore be difficult to estimate the productivity increases in the different sectors of the economy, which could be used to estimate the Balassa-Samuelson effect by using equation (2.24). This method may run into further problems, since equation (2.24) might actually not hold. It is based on the assumption of wage equalisation between the two sectors of the economy, and this assumption might not hold for Colombia. The best way to forecast the Balassa-Samuelson effect might simply be to assume that it will grow at the same average rate over the next 10 years as it has over the past 30 years.

There is a further problem when using equation (4.3) to forecast the exchange rate. This equation assumes all variables apart from the exchange rate to be exogenous. The Johansen analysis in section 3.6, however, indicated that only the U.S. price level is exogenous. Nevertheless, assuming only the exchange rate to be endogenous might not be an unrealistic assumption after its floatation in 1999. Economic theory stipulates a perfectly flexible exchange rate to be endogenous indeed.

5 Conclusion

> As the story goes, a general once told his weather-forecasting team, “I appreciate being informed that your forecasts are no better than random, but please keep sending them on, as the army needs your predictions for planning purposes.”[41]

As with weather forecasting (not covered in this paper), exchange rate forecasting has turned out to be notoriously difficult. The results by Meese and Rogoff (1983a), that the main monetary models perform no better in exchange rate forecasting than a simple random walk, has been difficult to overturn.

We have in this paper developed three models for the USD/COP exchange rate, and in line with Meese and Rogoff (1983a), the forecasting performance of these models was tested against a simple random walk. All the models use a cointegrating vector based on the PPP hypothesis to represent the long-term relationship, and different error correction models to represent short-term dynamics. Two of the models, using an Engle, Granger and Hallman (1989) framework for the econometric analysis, are shown to outperform the simple random walk on the 12 and 24-month forecast horizon (one of the models also outperforms on the 6-month horizon), which are good results. However, none of the models do well on the 3-month horizon, which is supported by theory suggesting that exchange rates are not determined by fundamentals in the short term.

The third model, developed using a Johansen (1988) framework of multivariate cointegration, turns out to perform badly, and does not beat the simple random walk on any of the forecast horizons. This is related to problems of modelling the considerable structural break, generated by the abandonment of the crawling exchange rate peg in 1991. The exchange rate variability increased considerably at this point in time, and the causality between the exchange rate and prices might very well have changed. The model also has problems forecasting the exchange rate around the change in exchange rate regime in 1999, when the exchange rate was floated. Other studies have, nevertheless,

[41] Rogoff (2002).

shown that this type of model might perform relatively well when fed with data without any significant structural breaks.[42] The model might, thus, be a useful tool in the future, when the floating exchange rate regime in Colombia has been in place for a longer period of time.

We also developed a simple statistical process where we added a variable drift term to the random walk process. This was estimated using a Kalman filter. This random walk with a variable drift was shown to perform better than all the three exchange rate model in out-of-sample forecasts. This result suggest that this process might be the best tool for exchange rate forecasting in the short and medium term, at least for the time being.

When it comes to exchange rate forecasting in the long-term, a fundamental model might still be the best alternative. A purchasing power parity relationship has been shown to exist for the past 30-year period, even if this is not a parity relationship in the strong sense. If assumed that this relationship is stable, which is not unrealistic, it can indeed be used for forecasts in the long term, let us say for the 10-year horizon.

[42] See, for example, Kim and Mo (1995), MacDonald and Taylor (1984), and Tawadaros (2001).

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