Recent behavior of output, unemployment, wages and prices in Colombia: What went wrong?

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Abstract

At the end of the last decade, the real activity in Colombia underwent the sharpest recession of the last fifty years. We postulate a non-triangular structural VAR model (Amisano and Giannini, 1997) to describe the dynamics of output, prices, unemployment and wages during the last two decades. The evidence suggests that, in the long-run, monetary policy has been neutral to both output and unemployment while the main reasons for the increase in the latter have been the lack of credibility of monetary policy, the way in which wages are set and the increase in non-wage labor costs.

Key words: structural VAR, unemployment, monetary policy, wages, expectations.
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1. Introduction

At the end of the 1990s the Colombian economic activity suffered the sharpest recession of the last fifty years to the extent that output decreased about 5% in 1999. In addition, the unemployment rate started to rise consistently reaching 20% in the year 2000. This increase in the unemployment rate was accompanied by a gradual reduction in inflation and an increase in real wages.

In view of these facts, to explain the slowdown of the economic activity several reasons have been put forward. Among the arguments are: i) the tighter monetary policy set by an authority committed with an inflation reduction program; ii) the lack of credibility of monetary policy; iii) the type of expectations formed by agents when setting nominal wages which put wages above their long-run equilibrium level (Arango and Posada, 2001, 2002). Urrutia (2002) provides another explanation of the recession of 1999 which is linked to the deficit of the current account and the sudden stop of capital flows associated to the crises of the international capital markets occurred in 1997. However, we do not emphasize this view in our work. Instead, we highlight the other three explanations since we understand that the most important causes of the recession were internal rather than external.

Accordingly, we postulate a structural \( VAR \) model for a closed economy to describe the dynamics of output, unemployment, wages and prices during the last two decades. Such an approach has been previously used to study some macroeconomic aspects in different economies. For example, Dolado and Jimeno (1997) associate the causes of Spanish unemployment with shocks of different nature which have long lasting effects due to a full hysteresis phenomenon. They find that the “… dismal performance of Spanish unemployment can be explained as the result of a series of adverse shocks, which were difficult to absorb in a context of a rigid system of labor market institutions and disinflationary policies. This finding has relevant policy implications that unless supply side reforms are implemented, deflationary policies will continue to be very costly in unemployment terms” (p. 1285).

Balmaseda et al. (2000) focus on the role played by aggregate demand, productivity, and labor supply shocks in explaining the joint dynamic behavior of real output, real wages and the unemployment rate in the modeling of labor markets in a sample of 16 OECD economies over the period 1950-96. They find that “in most countries the identification scheme based on
unemployment being persistent but stationary yields more reasonable results than those based on full-hysteresis whereby unemployment is considered to be an I(1) variable” (p.22). In addition, the authors find that unemployment fluctuations are dominated by aggregate demand shocks in the short-run and by labor supply and productivity shocks at lower frequencies.

For the case of Colombia, Misas and López (1998, 2001) use a SVAR (Blanchard and Quah, 1989) approach to estimate output and unemployment gaps, whereas Misas and Posada (2000) examine the sources of variation of the unexpected component of output growth. Arango (1998) obtains some evidence on the nature (either nominal or real) of the temporary and permanent components of the Colombian output and prices. Finally, Restrepo (1997) uses a VAR approach to explain the response of some macroeconomic variables (GDP, real exchange rate index, real money balances, money, inflation, and interest rate) to supply, demand, money demand and money supply shocks.

However, our work advances some of them since we present and solve a stylized model for the Colombian economy and introduce, in the empirical model, the long run restrictions provided by the theoretical framework. The evidence provided by this work suggests that, in the long-run, monetary policy has been neutral to both output and unemployment while the main reasons for the increase in the latter have been the lack of credibility of monetary policy, the way in which wages are set (both of which increased the real wage) and the increase in labor costs other than wages, such as those introduced by pension reforms.

The outline of the paper is as follows. Section 2 presents some facts related to the macroeconomic performance of the Colombian economy during the last years. Section 3 introduces the model. Section 4 presents the methodology and the empirical application. Section 5 presents a discussion of our findings and section 6 provides some concluding remarks.

2. The facts

Since 1991 the Colombian central bank has been conducting a program for reducing inflation. Such a program has been characterized by targets that decrease gradually, accompanied, among

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1 See also Algan (2001) and Fabiani et al. (2001).
2 Arango (1998) also presents and solves a small macro model.
3 By using a different approach Cárdenas and Gutiérrez (1998) also underline the increase in nonwage labor costs as one of the determinants of the rise in the unemployment rate. However, other aspects such as the appreciation of the Colombian peso and the increase in the value added tax have also played a role in their view.
other things, by consistent stances of monetary policy\(^4\). On the one hand, the policy was
effective in the sense that, since then, inflation showed a negative-sloped long run component
(see Figure 1).

![Figure 1. Annual inflation rate](image)

On the other hand, the program for reducing inflation was somehow intricate in the sense
that no inflation target was reached until 1997 (see Figure 1), a fact that, without any doubt,
derminated the credibility of the monetary authority\(^5\). In 1998 the authorities missed the target
again; in 1999 and 2000 the observed inflation levels were of 9.2\% and 8.8\% while the targets were
15\% and 10\%, respectively. These results might have suggested of a monetary policy stance tighter
than required\(^6\).

To complete the picture, in the late 1990s Colombia experienced a sharp recession as panels
A, B, and C of Figure 2 show. The growth of real output and the employment rate underwent an
abrupt reduction (Panel A) while the unemployment rate of the seven major cities of the
country\(^7\) soared between 1994 and 1999 (Panel B). We use two measures of the unemployment
rate: the first one is defined as “one minus the occupation rate”, which rose from 44\% to about
50\% between 1994 and 2000, whereas the second measure, defined as “one minus the ratio of
the occupation rate to the participation rate”, increased to about 20\% during the same period.

\(^4\) See Urrutia (2002) for an interpretation of the monetary policy during the last decade. See also Hernández and
Tolosa (2001).

\(^5\) In 1997 the inflation target was 18\% while the observed inflation was 17.7\%.

\(^6\) By that time, an intricate international environment for emerging markets together with internal fiscal difficulties and a
current account imbalance were also part of the picture (see Urrutia, 2002). However, our analysis shall not focus
explicitly on these aspects.
We make this distinction since the first measure of unemployment is the one that results from our model and so we use it in the empirical exercise, while the second measure is the one officially published. Finally, Panel C shows the urban employment rate which exhibits a strong reduction during the same period. However, against some beliefs, the period of inflation reduction and increase in the unemployment rate (our second measure) are far from coincident (Figure 3). This is because inflation started to fall in 1991 while unemployment started to rise in 1994.

Figure 2. Evidence of the slump in Colombia in late 1990’s

A. Annual growth rate of output and employment rate

B. Unemployment rates (seven cities)

C. Employment (occupation) rate (seven cities)

Source: DANE-DNP, Banco de la República-SGEE and authors’ calculations

Figure 4 shows the unemployment rate (as measured by “one minus the occupation rate”) and (the log of) the real labor income index, based on labor income data taken from the Encuesta Nacional de Hogares (National Housing Survey) deflated by the CPI, which is the

7 These cities account for about 75% of the total population of the country.
proxy we use for the real wage index. According to the figure, this period was characterized, firstly, by a sharp increase in the unemployment rate: 1994 – 1999, and, secondly, by an inconsistent strong wage growth that started in mid 1992.

The hypothesis we maintain is that the increase in the unemployment rate was a reaction to a real wage growth out of the equilibrium path (Arango and Posada, 2002) that converted labor in a very costly factor8. The behavior of the real wage might be the result, on the one hand, of an unexpected reduction in the inflation rate, perhaps due to the combination of a backward-formed expectations phenomenon and low credibility of the monetary policy. On the other hand, the behavior of the real wage might be the result of a minimum wage policy that sometimes leads other nominal wage settings in the country, whose level is established on political rather than factual (economic) grounds.

Just before the unemployment rate started to rise in 1994, the Law 100 of 1993, a new labor market and social security legislation was enacted. Under this new scheme, some of the labor costs related to health and pension plans were augmented (see Arango and Posada, 2001). Figure 5 shows the behavior of the labor costs, other than real wages, and the unemployment rate. Accordingly, the above hypothesis -related to the expectations and effects of the minimum wage policy- could be amended to consider the impact of the increase in labor costs introduced

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8 Iregui and Otero (2002), through nonlinear techniques, make the point that wages above their long-run equilibrium level do increase unemployment, but wages below this level do not reduce it. This result supports the view that factors that increase unemployment are not the same as those reducing it.
by such legislation. As a result, labor became rather costly at the beginning of the second half of the nineties.

**Figure 4. Unemployment rate and real wage index**

Source: DANE-DNP, authors' calculations

**Figure 5. Unemployment rate and labor costs other than wages**


### 3. A stylized model

To rationalize the above facts we use a model consisting of a set of structural equations (all variables are in logs):

\[
y_t^d = m_t - p_t
\]

(1)

\[
y_t^s = \gamma (p_t - E_{t-1} p_t) + \theta_t
\]

(2)

\[
n_t^d = -\alpha (w_t - p_t) + \beta y_t - \varphi c_t
\]

(3)

\[
n_t^s = \delta (w_t - p_t) + \tau_t
\]

(4)
where \( y_t^d \) stands for aggregate demand in period \( t \), \( y_t^s \) for aggregate supply, \( p_t \) for the price level, \( \theta_t \) for the technology process, \( n_t^d \) for labor demand, \( n_t^s \) for labor supply, \( w_t \) for nominal wage, \( c_t \) for the labor costs different from the wage, and \( \tau_t \) for a labor-supply shift factor (Balmaseda et al, 2000). \( E_t \) is the expectations operator.

Equation (1) suggests that aggregate demand responds to real balances as in the quantity theory setting. Equation (2) assumes that aggregate supply is moved by two factors: technology and surprises in the price level or, in other words, deviations of observed prices from expected prices (Sargent and Wallace, 1975). Equation (3) sets that labor demand depends on real wages, economic activity and labor costs different from real wages. Variable \( c_t \) reflects costs such as social security contributions (health and pension plans) and other payroll taxes such as the contributions to the Instituto Colombiano de Bienestar Familiar (ICBF), the Servicio Nacional de Aprendizaje (SENA) and the Cajas de Compensación Familiar\(^9\) (see Figure 5).

In addition, the model contains the following two definitions:

\[
\begin{align*}
    u_t &= n_t^s - n_t^d \\
    w_t &= E_{t-1} p_t - \rho \tau_t + \lambda \theta_t
\end{align*}
\]  

where \( u_t \) is the unemployment rate.

Finally, it is assumed that \( m_t \), \( \theta_t \), \( c_t \) and \( \tau_t \) behave as random walks\(^{10}\):

\[
\begin{align*}
    m_t &= m_{t-1} + \epsilon_t^m \\
    \theta_t &= \theta_{t-1} + \epsilon_t^\theta \\
    \tau_t &= \tau_{t-1} + \epsilon_t^\tau \\
    c_t &= c_{t-1} + \epsilon_t^c
\end{align*}
\]  

The solution of the model is given by:

\(^9\) The ICBF attends issues related to the welfare of family, women, childhood and third age, The SENA is an official agency committed to training of labor force; the Cajas receive contributions for leisure, training and health of the labor force and families of the workers.

\(^{10}\) We also included a drift in equation (8). However, the long run restrictions of the system did not change.
\[ \Delta y_i = \frac{\gamma}{1 + \gamma} (\epsilon_i^m - \epsilon_{i-1}^m) + \frac{1}{1 + \gamma} \epsilon_i^\theta + \frac{\gamma}{1 + \gamma} \epsilon_{i-1}^\theta \] (11)

\[ \Delta u_i = -A (\epsilon_i^m - \epsilon_{i-1}^m) + A (\epsilon_i^\theta - \epsilon_{i-1}^\theta) + (\alpha \lambda + \delta \lambda - \beta) \epsilon_i^\theta 
+ (1 - \delta \rho - \alpha \rho) \epsilon_i^\tau + \phi \epsilon_i^c \] (12)

\[ \Delta w_i = \epsilon_{i-1}^m + \lambda \epsilon_i^\theta - \epsilon_{i-1}^\theta - \rho \epsilon_i^c \] (13)

\[ \Delta p_i = \frac{1}{1 + \gamma} \epsilon_i^m + \frac{\gamma}{1 + \gamma} \epsilon_{i-1}^m - \frac{1}{1 + \gamma} \epsilon_i^\theta - \frac{\gamma}{1 + \gamma} \epsilon_{i-1}^\theta \] (14)

where \( A = [\alpha + \beta \gamma + \delta / 1 + \gamma] \), and \([1/(1 + \gamma)][\epsilon_i^m - \epsilon_i^\theta]\) corresponds to the contemporary inflationary surprise.

Accordingly, in this economy only technology shocks have permanent effects on \( y \) \(^{11}\). Technology, labor supply and ex-wage labor costs shocks have permanent effects on \( u \). Technology, nominal and labor supply shocks have permanent effects on \( w \) and both technology and nominal shocks have permanent effects on \( p \). By looking at the restrictions that emerge from the model, it is obvious that no triangular matrix is useful for identifying the shocks. Next we explain the methodology.

4. Empirical approach

We use a Blanchard-Quah (1989) style non-triangular decomposition to identify a SVAR model of output \( (y) \), unemployment rate \( (u) \), nominal wages \( (w) \) and prices \( (p) \) for the period 1984:1 – 2000:4. Each variable was entered into the model in log levels, with the exception of the unemployment rate which was entered as a fraction.

The statistical tests indicate that these series are integrated of order one each\(^{12}\) and that no cointegrating relationship arises among them. Taking into account these stochastic properties

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\(^{11}\) The long run restriction of the nominal shock stems from the fact that the polynomial of lags that relates this shock to the first difference of \( y \) is zero when it is evaluated with the lag operator equal to one. This type of restriction is used by Blanchard and Quah (1989) among others.

\(^{12}\) The persistence of unemployment rate is related to the so-called hysteresis phenomenon (Blanchard and Summers, 1986). However, the persistence of unemployment rate in Colombia seems to arise because of the behaviour during the second half of 1990’s (Arango and Posada, 2001).
we estimate a VAR model of order two for the first difference of the selected series. The selected reduced-form VAR model can be expressed as:

\[ A(L)\Delta X_t = e_t, \quad t = 1,2,\ldots,T \]  

(15)

where \( X'_t = (y_t, u_t, w_t, p_t) \), \( A(L) = I - A_1 L - \ldots - A_p L^p \), with \( L \) as the lag operator, \( p=2 \) and \( \{e_t\} \) a Gaussian white noise process with covariance matrix \( \Sigma \). The model (15) can also be noted in terms of structural shocks as:

\[ B(L)\Delta X_t = \epsilon_t \]  

(16)

where \( B(L) = B_0 - B_1 L - \ldots - B_p L^p \) and \( \{\epsilon_t\} \) is a Gaussian white noise process with covariance matrix \( I \). In our case the structural shocks correspond to \( \epsilon_t' = (\epsilon_t^\theta, \epsilon_t^m, \epsilon_t^\tau, \epsilon_t^c) \). By using the Wold theorem, expression (15) can be written in terms of reduced form shocks:

\[ \Delta X_t = C(L)e_t \]  

(17)

or in terms of structural shocks as:

\[ \Delta X_t = \Phi(L)\epsilon_t \]  

(18)

where, \( C(L) = I + C_1 L + C_2 L^2 + \ldots \) and \( \Phi(L) = \phi_0 + \phi_1 L + \phi_2 L^2 + \ldots \)

Based on expressions (17) and (18) it can be shown that structural and reduced form shocks are related to each other as follows:

\[ e_t = \phi_0 \epsilon_t \]  

(19)

Since the series included in the model are I(1) processes, the long-run restrictions, in terms of the structural shocks, can be formulated using the elements of \( \Phi(1) \):

\[ \Phi(1) = \begin{bmatrix}
\sum_{i=0}^{\infty} \phi_{1,11} & \sum_{i=0}^{\infty} \phi_{1,12} & \sum_{i=0}^{\infty} \phi_{1,13} & \sum_{i=0}^{\infty} \phi_{1,14} \\
\sum_{i=0}^{\infty} \phi_{i,21} & \sum_{i=0}^{\infty} \phi_{i,22} & \sum_{i=0}^{\infty} \phi_{i,23} & \sum_{i=0}^{\infty} \phi_{i,24} \\
\sum_{i=0}^{\infty} \phi_{i,31} & \sum_{i=0}^{\infty} \phi_{i,32} & \sum_{i=0}^{\infty} \phi_{i,33} & \sum_{i=0}^{\infty} \phi_{i,34} \\
\sum_{i=0}^{\infty} \phi_{i,41} & \sum_{i=0}^{\infty} \phi_{i,42} & \sum_{i=0}^{\infty} \phi_{i,43} & \sum_{i=0}^{\infty} \phi_{i,44}
\end{bmatrix} = \begin{bmatrix}
limit_{k\to\infty} \frac{\partial y_t}{\partial e_{t-k}^\theta} & \lim_{k\to\infty} \frac{\partial y_t}{\partial e_{t-k}^m} & \lim_{k\to\infty} \frac{\partial y_t}{\partial e_{t-k}^\tau} & \lim_{k\to\infty} \frac{\partial y_t}{\partial e_{t-k}^c} \\
\lim_{k\to\infty} \frac{\partial u_t}{\partial e_{t-k}^\theta} & \lim_{k\to\infty} \frac{\partial u_t}{\partial e_{t-k}^m} & \lim_{k\to\infty} \frac{\partial u_t}{\partial e_{t-k}^\tau} & \lim_{k\to\infty} \frac{\partial u_t}{\partial e_{t-k}^c} \\
\lim_{k\to\infty} \frac{\partial w_t}{\partial e_{t-k}^\theta} & \lim_{k\to\infty} \frac{\partial w_t}{\partial e_{t-k}^m} & \lim_{k\to\infty} \frac{\partial w_t}{\partial e_{t-k}^\tau} & \lim_{k\to\infty} \frac{\partial w_t}{\partial e_{t-k}^c} \\
\lim_{k\to\infty} \frac{\partial p_t}{\partial e_{t-k}^\theta} & \lim_{k\to\infty} \frac{\partial p_t}{\partial e_{t-k}^m} & \lim_{k\to\infty} \frac{\partial p_t}{\partial e_{t-k}^\tau} & \lim_{k\to\infty} \frac{\partial p_t}{\partial e_{t-k}^c}
\end{bmatrix} \]

13 The number of lags was chosen to be the minimum for which we obtain Gaussian white noise residuals. The diagnostic statistics for the residuals of the model are presented in Appendix 1.
Accordingly, with the economic constraints implied by the model of the previous section, we have the following restrictions:

\[
\Phi(l) = \begin{bmatrix}
\Phi_{11}(l) & 0 & 0 & 0 \\
\Phi_{21}(l) & 0 & \Phi_{23}(l) & \Phi_{24}(l) \\
\Phi_{31}(l) & \Phi_{32}(l) & \Phi_{33}(l) & 0 \\
\Phi_{41}(l) & \Phi_{42}(l) & 0 & 0 \\
\end{bmatrix}
\]  

(20)

where the first row shows the long run response of \( y \) to the shocks \( \varepsilon^\theta, \varepsilon^m, \varepsilon^\tau \) and \( \varepsilon^c \) respectively; the second, third and fourth row shows the response of \( u, w, \) and \( p \), respectively, to the same shocks in the same order.

It is convenient to re-express these restrictions in the following form:

\[
R \text{vec}(\phi_0) = d
\]

(21)

where \( \text{vec} \) is an operator that stacks the columns of a matrix into a single column vector, \( R \) is a full-rank matrix of dimension \( n \times k^2 \), \( d \) is a vector \( n \times 1 \), \( k \) the number of variables in the model (four in this case) and \( n \) is the number of restrictions\(^{14}\).

Given that the number of distinct reduced form parameters in equation (15), \( pk^2 + k(k+1)/2 \), is less than the number of the structural form parameters in (16), \( (p+1)k^2 \), the usual order conditions state that at least \( k(k-1)/2 \) restrictions are necessary in order to achieve identification of the structural form\(^{15}\).

As it is customary, for identification the order conditions are necessary but not sufficient. Hence the constraints must also satisfy the rank conditions to be able to generate an identified or over-identified model. By assuming invertibility of the \( \phi_0 \) matrix, the true vector \( \text{vec}(\phi_0^*) \) is locally identified if and only if the system \( R(I \otimes \phi_0)D_n x = [0] \), with the matrix \( R(I \otimes \phi_0)D_n \) evaluated at \( \phi_0^* \), has only \( x = [0] \) as admissible solution\(^{16}\).

Under over-identification of the model it is possible to construct a test based on the likelihood ratio principle (\( LR \)) to check the validity of the restrictions:

\[
LR = 2(L(\hat{\Sigma}) - L(\Sigma))
\]

(22)

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\(^{14}\) For the model at hand, this representation, including the process of identification, is presented in detail in Appendix 2.

\(^{15}\) The classical Blanchard-Quah approach uses restrictions that imply an upper triangular form for \( \Phi(l) \) which gives six restrictions [from \( k(k-1)/2 \)] since \( k=4 \). Instead, for our model we have seven restrictions.

\(^{16}\)
where $L$ is the logarithm of the likelihood function and $\Sigma$ is an estimator of $\Sigma$ for the restricted model. Under the null hypothesis (i.e., the validity of the restrictions being imposed), the test is asymptotically distributed as $\chi^2$ with degrees of freedom equal to the number of constraints minus $k(k-1)/2$.

As shown in Appendix 2, our system, including the constraints presented in (21), is over-identified. Then, it is possible to compute the test described in (22), which gives $LR = 1.36$ with a $p$-value of 0.243, suggesting that we cannot reject the validity of the constraints being imposed at the usual significance levels. If the model happens to be identified or over-identified the estimation stage is possible.

This type of SVAR model is called C-model by Amisano and Giannini (1997) who provide the following two-step estimation technique. First, the reduced-form VAR is estimated by OLS; second, the coefficients in $\phi_0$ are determined by imposing the long-run restrictions, estimating the remaining free elements by maximization of the following log-likelihood function:

$$L(\phi_0) = a - \frac{T}{2} \log|\phi_0|^2 - \frac{T}{2} \text{tr}\left(\phi_0^{-1}\phi_0 \Sigma\right)$$

where $a$ is a constant, $T$ is the sample size and $\Sigma$ is a consistent estimator of $\Sigma$. Then, in order to achieve local identification this maximization is subject to the restrictions summarized in (21).

5. Results

We use quarterly data on the real output ($y$), the unemployment rate ($u$), nominal wages ($w$) and prices ($P$) for the period 1984:1 – 2000:4. Real output corresponds to Gross Domestic Product in 1994 prices; the unemployment rate was calculated as “one minus the occupation rate”; nominal wages were computed as labor income (taken from the National Housing Survey); finally, prices correspond to the CPI. The SVAR model has the restrictions set in expression (20) above.

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16 Matrix $D_n$ is specified in Appendix 2.
Figure 6 shows the response functions of $y$, $u$, $w$ and $p$ (in levels) after receiving a shock of one standard error each in $(\varepsilon^\theta, \varepsilon^m, \varepsilon^\tau$ or $\varepsilon^c)^{17}$. As expected, we observe that neither a nominal shock, nor a labor supply shock nor a non-wage labor cost shock had a long-run effect on output. However, a productivity shock increased output both in the short and long-run. In the short run nominal shocks had a positive effect on output but it vanished in about three quarters. Regarding unemployment, a productivity shock reduced it both in the long-run and short-run. These responses seem counterintuitive at first since one may expect that, other things being equal, the higher the productivity, the higher the wages and lower the employment level, from the point of view of the firms. However, what such responses are showing is that $\beta > (\alpha \lambda + \delta \lambda)$ in the third element of the right-hand side of equation (12). Recall that coefficient $\beta$ is the loading factor of economic activity in equation (3) of demand for labor while $\alpha$ and $\delta$ are the coefficients that relate real wage with labor demand and supply both weighted by $\lambda$, the coefficient that links productivity to nominal wage. Hence, for having the increase in unemployment that Colombia had we needed either a raise in the real wage or a bad performance of the economic activity$^{18}$ or both. On the other hand, nominal shocks reduced unemployment in the short-run but did not have any effect in the long-run. Unemployment increased in the short run when facing labor supply shocks. When the shock to unemployment came from the non-wage labor costs the result was a permanent increase in the former$^{19}$.

As to nominal wages, given the significance of the response, productivity shocks did not produce any effect neither in the short-run nor in the long run. A nominal shock increases nominal wages in both the short-run and long run. A shock increasing the labor supply should have reduced real wages; however, we obtained the opposite response in the short as well as in the long-run. Regarding prices, a productivity shock did not affect prices neither in the short nor in the long-run while a nominal shock increased prices in both terms. In summary, according to the impulse response functions the model performs well since only one of them, the response of nominal wages to a labor supply shock, goes against the intuition.

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$^{17}$ The log-likelihood function is maximized using a numerical iterative procedure; for this purpose we use the computation program MALCOM which uses the score algorithm. The confidence intervals of the impulse response functions were coded by the authors.

$^{18}$ This response of economic activity may be driven by productivity shocks which were the only ones with permanent effects on output.
From this exercise it seems that, according to the magnitude of the response of nominal wages and prices to nominal shocks, there is an increase of the real wage caused by a shock of this source. Thus the next exercise we undertake is to modify the model to include the real wage instead of the nominal one. In this case, after simple algebra manipulation, equation (13) is replaced by:

$$\Delta(w_t - p_t) = \Delta w^r_t = -\frac{1}{1 + \gamma} (\epsilon^m_{t-1} - \epsilon^m_{t+1}) + \frac{1}{1 + \gamma} (\epsilon^\theta_{t-1} - \epsilon^\theta_{t+1})$$

$$+ \lambda \epsilon^\theta_t - \rho \epsilon^r_t$$

(13’)

where $w^r_t = w_t - p_t$ is the real wage. From equation (13’) we can see that no long run response should be expected in the real wage caused by nominal shocks. With this modification, the restrictions to the system are now given by:

$$\Phi(1) = \begin{bmatrix} \Phi_{11}(1) & 0 & 0 & 0 \\ \Phi_{21}(1) & 0 & \Phi_{23}(1) & \Phi_{24}(1) \\ \Phi_{31}(1) & 0 & \Phi_{33}(1) & 0 \\ \Phi_{41}(1) & \Phi_{42}(1) & 0 & 0 \end{bmatrix}$$

(20’)

where, as before, the rows show the response in the long run of $y$, $u$, $w^r$ and $p$ to the shocks $\epsilon^\theta$, $\epsilon^m$, $\epsilon^r$ and $\epsilon^c$ respectively.

The system corresponding to this version of the model is also over-identified. In this case, the test described in (21), which gives $LR = 3.46$ with a $p$-value of 0.178, indicates that we cannot reject the validity of the constraints at usual significance levels. The conclusion of this exercise is that responses of the variables remain almost the same as in our benchmark case (see Figure 7). However, notice that the response of unemployment to a labor supply shock is now significant in both long run and short run. Observe also the short run responses of output, unemployment and real wages to nominal shocks: the first increases while the second as well as the real wage reduce.

We retain this version of model (with real instead of nominal wages) to do two additional empirical tests. The exercises are the following. Firstly, we run the SVAR model without imposing any long run restriction in the response of the real wage to a nominal shock.

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19 This evidence should draw some attention from the policy makers that try to solve the pension problem by raising the contributions that firms make to the system. The effect is a permanent increase in the unemployment rate.
Secondly, after reestablishing the version of the model with the real wage, we run the SVAR without imposing long run restrictions of nominal shocks on the unemployment rate.

The evidence of these exercises suggests that nominal shocks, of the nature though for this empirical model, does not have any effect on the real wage in the long run but a short term response is captured by the model (see Figure 8).

Interestingly, for the second exercise, a nominal shock has the effect of reducing the unemployment rate in the short run. In this version of the model output also reacts in the short run while the real wage moves downward. Notice, however, that these effects last only about one year then the responses are not significant (see Figure 9). These reactions of the variables are fully consistent with backwards-formed expectations by the Colombian agents as it is sometimes argued.

Now, let us think about an economy where a negative nominal shift announced every year and made effective by the monetary authority but not believed by agents. The result is a negative response of output and an increase in unemployment and the real wage in the short run. No long run effect on any of these variables is possible according to our results. However, this lack of credibility of monetary policy made feasible this joint behavior of the variables (and agents) in the last years of the 1990s. Thus, to get a better performance of the economy, the announcements of the monetary authority should be credible. If policy announcements had been believed and taken into account when setting wages, unemployment had only increased because of the impact of labor supply and non-wage labor costs.

As it might be obvious, these two last exercises would not be necessary since the long run restrictions have not been rejected according to the statistics. However, some analysts of the Colombian economy insist on a more active monetary policy to take the economy out of the recession. Accordingly, these results suggest that the economy would have only a short run reaction in output, unemployment and real wages. The cost of this behavior is to have a higher price level both in short and long run (Figure 9). Of course if we assume that agents form expectations rationally this policy could be undertaken only once, but if agents form expectations backwards there is room for “managing the demand” over a few periods at a cost of higher inflation.
6. Final remarks

In this work we present a small model for the Colombian economy with the aim of understanding some of the possible causes of the deepest recession the country underwent during the last fifty years. The model consists of structural equations for the product and labor markets and some other definitions and assumptions. By using a non-triangular SVAR empirical approach and quarterly data form 1984 up to 2000 we obtain sensible impulse response functions for output, unemployment, and prices but less satisfactory for both nominal and real wages. The result of the tests suggest that the long run restrictions, implied by the model, on the response of the variables to productivity, nominal, labor supply, and non-wage labor costs shocks can be imposed.

The evidence suggests that among the causes of the recession, the lack of credibility of monetary policy seem to have a privileged place. Other shocks such as the labor supply and non-wages labor costs also explain the increase in the unemployment rate. Most tellingly, the exercise also shows the long run neutrality of nominal shocks to both output and unemployment. However, these variables react in the short run to shocks of the same source.

Now, it is time for the authorities to analyze the way in which agents form their expectations to exploit differences between expected and observed inflation or not given the reaction in prices produced by these types of policies.

References


Appendix 1: Diagnostics tests for the residuals of the reduced-form VAR

For system with nominal wages

<table>
<thead>
<tr>
<th>Test</th>
<th>Value</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jarque-Bera</td>
<td>8.64</td>
<td>0.37</td>
</tr>
<tr>
<td>Adjusted-Q statistic (^1/)</td>
<td>248.78</td>
<td>0.03</td>
</tr>
<tr>
<td>Serial correlation LM test (^1/)</td>
<td>26.45</td>
<td>0.05</td>
</tr>
</tbody>
</table>

\(^1/\) Evaluated up to 15 lags.
Note: these statistics correspond to the system graphed in Figure 6.

For system with real wages

<table>
<thead>
<tr>
<th>Test</th>
<th>Value</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jarque-Bera</td>
<td>10.72</td>
<td>0.22</td>
</tr>
<tr>
<td>Adjusted-Q statistic (^1/)</td>
<td>229.40</td>
<td>0.15</td>
</tr>
<tr>
<td>Serial correlation LM test (^1/)</td>
<td>16.60</td>
<td>0.41</td>
</tr>
</tbody>
</table>

\(^1/\) Evaluated up to 15 lags.
Note: these statistics correspond to the system graphed in Figure 7.

Appendix 2: Identification of a structural VAR with long-run constrains

Amisano and Giannini (1997) show that the identification of the model, including the restrictions in (21), can be analyzed in terms of the following systems of equations:

\[
N_\phi y = [0] \tag{A.1}
\]

\[
R(I \otimes \phi_0) y = [0] \tag{A.2}
\]

The system (A.1) has \(k^2\) equations in \(k^2\) unknowns and the system (A.2) has \(n\) equations in \(k^2\) unknowns. The two systems are connected because they share the same \(k^2\) unknowns.

Amisano and Giannini (1997) suggest to insert the general solution of (A1) in (A2). The vector representing the general solution of system (A1) can be written as:

\[
y = \bar{D}_k x \tag{A.3}
\]
where \( x \) is a \( k(k-1)/2 \) vector of free elements, \( \bar{D}_k \) is a \( k^2 \times k(k-1)/2 \) full column rank matrix which columns are associated with the eigenvectors corresponding to the zero eigenvalues of the \( N_k \) matrix with \( N_k = \frac{1}{2} \left( I_{k^2} + \Theta_{kk} \right) \) and \( \Theta_{kk} \) is the commutation matrix\(^{20} \) of dimensions \( k^2 \times k^2 \).

Inserting (A.3) in the system (A2) we obtain:

\[
R(I \otimes \phi_0)\bar{D}_k x = [0] 
\] (A.4)

Therefore, the system is identified if and only if (A.4) with the matrix \( R(I \otimes \phi_0)\bar{D}_k \) evaluated at \( \phi_0^* \) has only admissible solution \( x = [0] \). In order to evaluate the identification of our model, we need to define the matrices \( R \) and \( \bar{D}_k \).

**Computation of the \( R \) matrix**

The \( R \) matrix is associated to the restrictions that are imposed to the reduced-form shocks. For the model with real wage, we have the following long-run constraints:

\[
\Phi(1) = \begin{bmatrix}
\Phi_{11}(1) & 0 & 0 & 0 \\
\Phi_{21}(1) & 0 & \Phi_{23}(1) & \Phi_{24}(1) \\
\Phi_{31}(1) & 0 & \Phi_{33}(1) & 0 \\
\Phi_{41}(1) & \Phi_{42}(1) & 0 & 0
\end{bmatrix} 
\] (A.5)

From equations (17), (18) and (19) it is straightforward to obtain the relationship:

\[
\Phi(1) = C(1) \phi_0 
\] (A.6)

Then, using (A.6) the restrictions implied in (A5) can be noted as:

\[
R \text{vec}(\phi_0) = d 
\] (A.7)

Specifically (A.6) implies:

Then, the long-run constrains of the previous expression imply the following equations:

\[^{20}\text{Let } A \text{ be a } p \times q \text{ matrix. The vectors } \text{vec}(A) \text{ and } \text{vec}(A') \text{ clearly contain the same } pq \text{ components, but in different order. Hence, there exists a unique } pq \times pq \text{ permutation matrix which transforms } \text{vec}(A) \text{ into } \text{vec}(A'). \text{ This matrix is called the commutation matrix and is denoted } \Theta. \text{ Thus: } \Theta_{pq} \text{vec}(A) = \text{vec}(A'). \]
These eight equations can be expressed in the notation of (A.7) with $d$ as a vector of zeros and the following $R$ matrix:

$$R = \begin{bmatrix}
C_{11}(I) & C_{12}(I) & C_{13}(I) & C_{14}(I) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & C_{12}(I) & C_{13}(I) & C_{14}(I) & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

The components of the $R$ matrix can be easily estimated since they come from the reduced-form VAR.\footnote{The fact that the $R$ matrix involves non-constant terms but the reduced form cumulated impulse response functions coefficients, $C(I)$, which are not known and must be estimated causes some complications in the SVAR analysis. However, Amisano and Giannini (1997) show that under this situation we still can have consistent estimators for the optimization process.}

**Computation of the matrix $\bar{D}_k$**

As stated above, the columns of $\bar{D}_k$ are associated with the eigenvectors that correspond to zero eigenvalues of the $N_k$ matrix with $N_k = \frac{1}{2} (I_k + \Theta_{kk})$ and $\Theta_{kk}$ as the commutation matrix. For our case $k=4$, hence the commutation matrix $\Theta_{44}$ is:

$$\Theta_{44} = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}$$
And the $\bar{D}_4$ matrix is the following:

\[
\bar{D}_4 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

With the previous definitions of $R$ and $\bar{D}_4$ and using some algebra it is easy to prove that for any vector $x$, of dimension $6\times1$, the system $R(I \otimes \phi^\prime_\phi)\bar{D}_k x = [0]$ has only the admissible solution $x = [0]$. Then, given the result of the order condition, number of restrictions greater than $k(k-1)/2$, our system (with real wage) is over-identified.
Figure 6. Impulse response functions with nominal wages in the system

<table>
<thead>
<tr>
<th>Response of output</th>
<th>To a productivity shock</th>
<th>To a nominal shock</th>
<th>To a labor supply shock</th>
<th>To a labor cost shock</th>
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<tr>
<td>Response of unemployment</td>
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<tr>
<td>Response of nominal wage</td>
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<tr>
<td>Response of prices</td>
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</table>
Figure 7. Impulse response functions with real wages in the system

<table>
<thead>
<tr>
<th>Response</th>
<th>To a productivity shock</th>
<th>To a nominal shock</th>
<th>To a labor supply shock</th>
<th>To a labor cost shock</th>
</tr>
</thead>
<tbody>
<tr>
<td>of output</td>
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<td>of unemployment</td>
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<td>of real wage</td>
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<tr>
<td>of prices</td>
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</table>
Figure 8. Impulse response functions with real wages in the system but without long run restrictions on real wages to a nominal shock.

To a productivity shock  
To a nominal shock  
To a labor supply shock  
To a labor cost shock

Response of output

Response of unemployment

Response of real wage

Response of prices
Figure 9. Impulse response functions with real wages in the system but without long run restrictions on unemployment to a nominal shock.

<table>
<thead>
<tr>
<th>Response of output</th>
<th>To a productivity shock</th>
<th>To a nominal shock</th>
<th>To a labor supply shock</th>
<th>To a labor cost shock</th>
</tr>
</thead>
<tbody>
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</table>

| Response of unemployment |                         |                    |                        |                       |
|--------------------------|-------------------------|--------------------|------------------------|                       |

| Response of real wage    |                         |                    |                        |                       |
|--------------------------|-------------------------|--------------------|------------------------|                       |

| Response of prices       |                         |                    |                        |                       |
|--------------------------|-------------------------|--------------------|------------------------|                       |